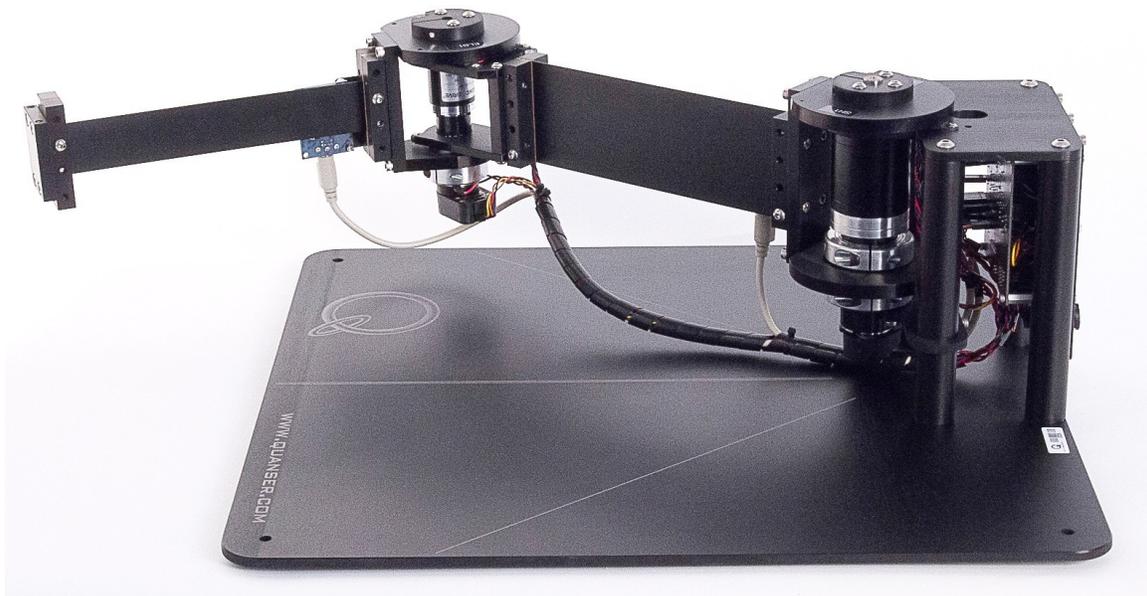


*Specialty Plant: 2DSFL Robot*

# **2-DOF Serial Flexible Link Robot**

*(With Two Strain Measurements)*



## **Reference Manual**

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## 1. Running Experiments

### 1.1. About QUARC

QUARC is Quanser's, state-of-the-art rapid prototyping and production system for real-time control. QUARC integrates seamlessly with Simulink to allow Simulink models to be run in real-time on a variety of targets, such as Windows, and QNX. For details on installing QUARC, please see [2]. For information about using the QUARC software please see [3]. Use QUARC with the supplied Simulink models to interface to the 2 DOF Flexible Link system's hardware and run controllers.

### 1.2. Supplied Sample Controllers

The Simulink models supplied with the 2 DOF Serial Flexible Link system are summarized in Table 1. Make sure you **run setup\_2dsfl.m prior to running any of these files.**

<i>File Name</i>	<i>Description</i>
s_2dsfl_pos_cntrl.mdl	Simulates the closed-loop 2-DOF Serial Flexible Link system using the state-space models of stage 1 and 2.
q_2dsfl_open_loop.mdl	Implements a state-feedback control on the actual 2-DOF Serial Flexible Link system using QUARC.
q_2dsfl_pos_cntrl.mdl	Applied a step input to the 2DSFL system and reads the corresponding joint and link angles of stage 1 and 2. Can be used for model validation.

Table 1: Sample controllers supplied with the 2DSFL.

### 1.3. Example: Vibration Control

This Section outlines the design of a control system to reduce the vibration of the Two-Degree-Of-Freedom Serial Flexible Link robot. A decoupled approach is used, which is to say that link coupling of the serial mechanism is neglected. Both drives are commanded independently of each other, each using a separate state-feedback control loop.

### 1.3.1. Operating Procedure



Note: Before running the following example, ensure that the system is cabled and configured as detailed in Section Error: Reference source not found.1. Also ensure that all encoders and strain gauge sensors of the 2DSFL system are working properly before proceeding. The controller tuning described hereafter assumes that no load is attached to the 2DSFL end-effector.

1. Open the Simulink diagram called `q_2dsfl_pos_cntrl.mdl`.
2. Align the two flexible links in their central position.
3. Run `setup_2dsfl.m` script to set the required parameters for proper operation of the Simulink model and the hardware itself.
4. In the Simulink diagram, go to QUARC | Build to build the controller.
5. Make sure both Manual Switch blocks are set to the upward **full-state feedback** control positions.
6. Go to QUARC | Start to start the QUARC controller.
7. Each of the two flexible links (i.e., stage 1 and stage 2) should now be tracking two angular position trajectories in the form of square waves. Typical system responses should look similar to the ones represented in the Scopes shown in Figures 1 and 2, where the square waves frequency is 0.1 Hz. In order to command each link tip of the device to a desired position, each of the two actuators (harmonic drives) has its own position control loop. Each uses a state-feedback control scheme tuned with the Linear-Quadratic Regulator (LQR) algorithm. The vibration of both links should be significantly minimized by the two state-feedback controllers.
8. Figures 1 and 2 depict the responses of the 2DSFL system from the same run. For example Figure 1 corresponds to the QUARC Scope located at `QUARC_2DSFL_robot/2-DOF SFL Robot: Actual Plant/Stage 1/Scopes/theta11 (deg)`, which plots the reference/desired (yellow trace) and actual position (purple trace) responses of the 2DSFL  $\theta_{11}$  angular output in degrees. It can be seen in Figures 1 and 2 that both square wave trajectories are out of phase. This is done so that link coupling can be best observed. It is uncompensated for, as the implemented controller design assumed a decoupled system.



Figure 1 Drive #1 Load Shaft Angular Response.



Figure 2 Drive #2 Load Shaft Angular Response.

- The *full-state feedback* response of stage 1 and 2 are depicted in Figure 3 and Figure 4. In each figure, the top plot shows the reference and measured drive angle, the middle plot is the measured flexible link angle, and the bottom plot is the motor current.

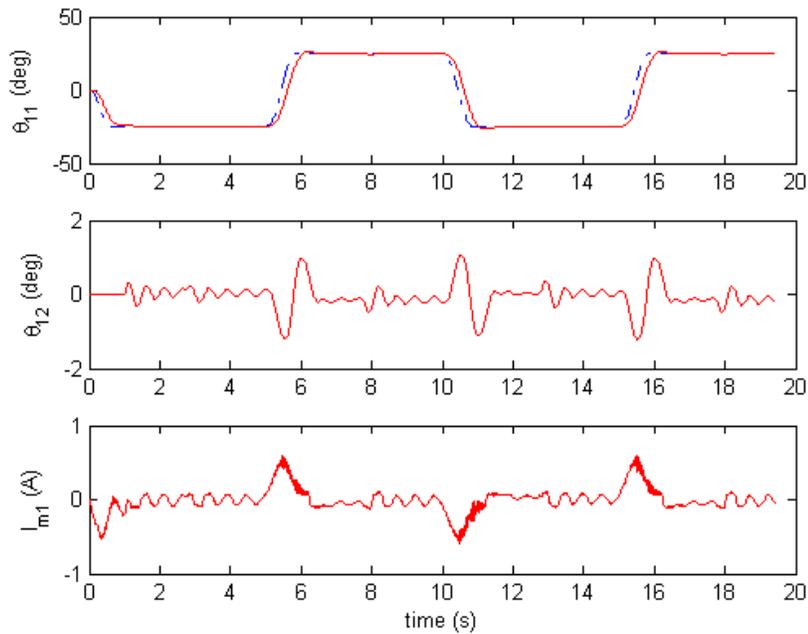


Figure 3: Full-state feedback response - stage 1.

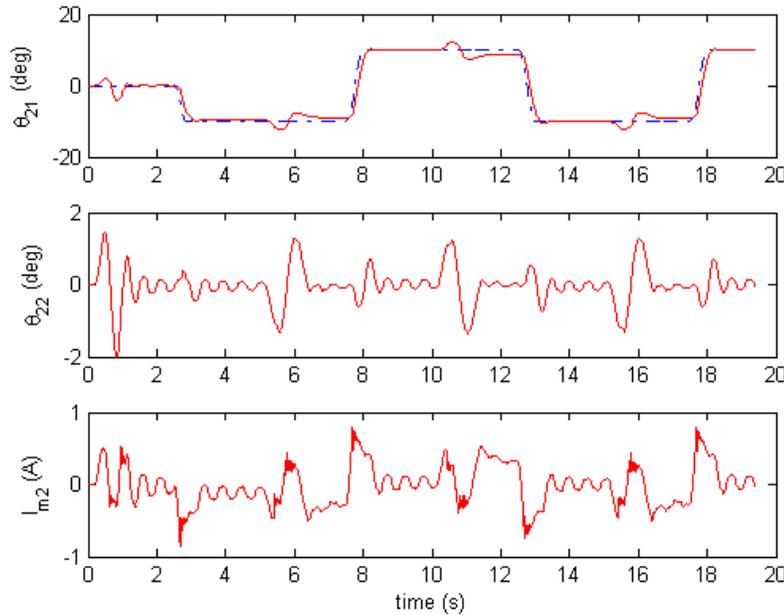


Figure 4: Full-state feedback response - stage 2.

10. Moreover, the square wave setpoint generation for each drive is done using the Quanser *Continuous Sigmoid* block (which is located in the QUARC Targets | Sources | *Sigmoids* sub-toolbox) in order to limit the setpoint maximum velocity and maximum acceleration. This is done so that the physical limitations of the system are respected. It results that both command currents never go into saturation. Also the deflection of each flexible link tip is limited to a given maximum.
11. There are two control modes: *full-state feedback* and *partial-state feedback*. Full-state feedback uses all the states of the system to command the motors to different reference positions while minimizing the deflection of the flexible links. See the sample full-state response given in Figure 3 and Figure 4. In partial-state feedback, only the motor angles are fed back to the controller (i.e. the link angles measured by the strain gage are ignored). **To run in partial-state feedback mode, set the stage 1 and stage 2 Manual Switch blocks in `q_2dsfl_pos_cntrl.mdl` to the downward position.** The sample partial-state feedback response shown in Figure 5 and Figure 6. Notice how both the stage 1 and 2 flexible links (i.e. middle plots) tend to oscillate more in this mode.

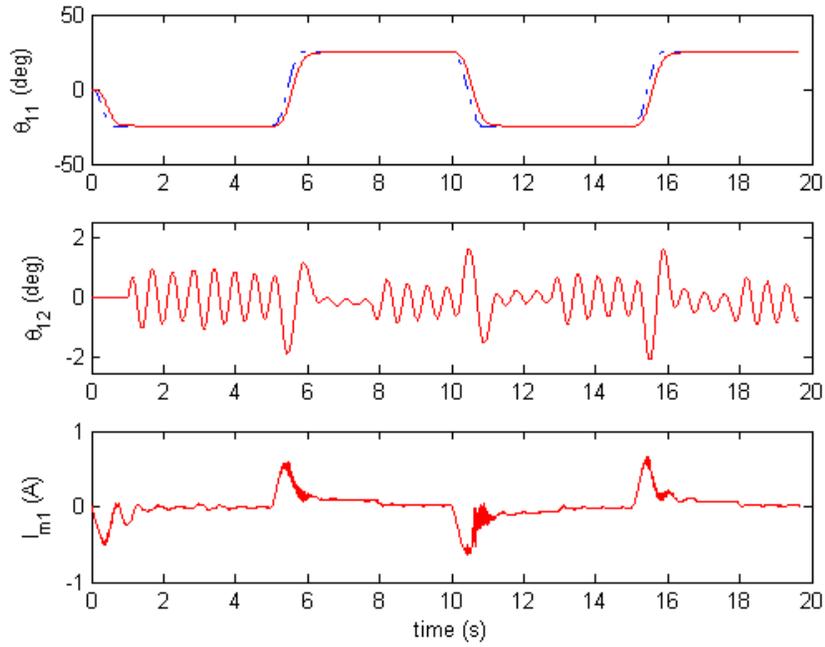


Figure 5: Partial-state feedback response - stage 1.

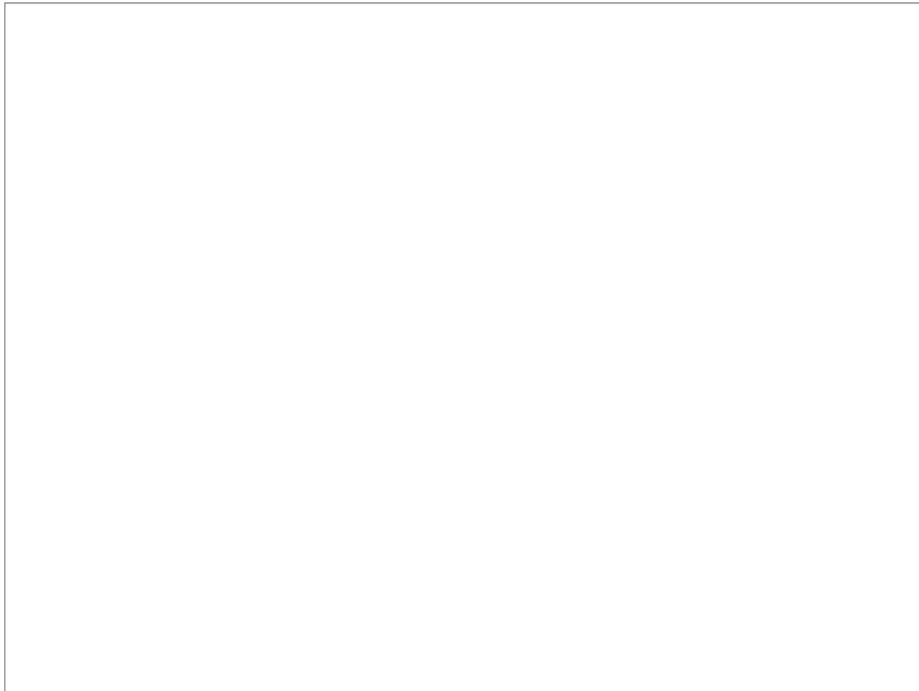


Figure 6: Partial-state feedback response - stage 2.

12. The two state-feedback controller gain vectors and the model parameters are initialized in the MATLAB workspace by running the file `setup_2dsfl.m` in the MATLAB prompt. In order to control each flexible link tip to a desired position, the LQR algorithm is used. You can modify both `q_2dsfl_pos_cntrl.mdl` and/or `setup_2dsfl.m` files and re-generate the corresponding real-time code using QUARC. To compile the real-time code corresponding to the controller diagram, use the `QUARC | Build` option from the Simulink menu bar. After successful compilation and download to the QUARC target, you can click the `start real-time code` button to run in real-time the actual system. You can also open a few QUARC Scopes to visualize on-the-fly your system response signals.
13. The Simulink-implemented controller model comes with two position watchdogs. They would stop the controller if the first flexible link tip deflection exceeds  $\pm 10^\circ$  (as set by default by the `DTH1_MAX` variable in the setup script) or if the second flexible link tip goes beyond  $\pm 10^\circ$  (as set by default by the `DTH2_MAX` variable in the setup script).
14. Running the `setup_2dsf.m` script can also, if the corresponding flag variables (e.g., `SYS_ANALYSIS_1`, `SYS_ANALYSIS_2`) are enabled, simulate and analyse the Two-Degree-Of-Freedom Serial Flexible Link system responses, using the MATLAB Control System Toolbox.

### 1.3.2. From Link Base Strain To End-Effector Deflection And Rotational Stiffness

Each flexible link base strain,  $E_b$  (in meters per meters), is measured at the clamped end of the link and can be expressed by the following equation:

$$E_b = \frac{6FL_b}{M_eXT^2}$$

where  $M_e$  is the modulus of elasticity (a.k.a. Young's modulus) for steel (in Pascal),  $X$  the flexible link width (in meters),  $T$  the flexible link thickness (in meters),  $L_b$  the distance from the load (i.e. link tip) to the strain gauge sensor on the clamped end of the link (in meters), and  $F$  the load force applied at the tip of the link (in Newtons).

For a given beam geometry and material, the deflection of the tip of the beam from its base strain gauge (at its clamped end) is a function of the load applied  $F$  and the position along the length of the beam  $L_b$ . The deflection of the link tip,  $Y$  (in meters), can be expressed as follows:

$$Y = \frac{4 F L_b^3}{M_e X T^3}$$

The link deflection at the tip,  $Y$ , can also be expressed as a function of the link base strain (i.e., the strain at the link base gauge position),  $E_b$ , as follows:

$$Y = \frac{2 E_b L_b^2}{3 T}$$

It results that the equivalent linear stiffness of the flexible link tip  $K_l$  (in Newtons per meters) can be expressed by:

$$K_l = \left( \frac{F}{Y} = \frac{1}{4} \frac{M_e X T^3}{L_b^3} \right)$$

The resulting equivalent rotational stiffness of the flexible link tip  $K_r$  (in Newtons meters per radians) can be expressed by:

$$K_r = \left( K_l L_b^2 = \frac{1}{4} \frac{M_e X T^3}{L_b} \right)$$

### 1.3.3. LQR Controller Design

A schematic of the Two-Degree-Of-Freedom Serial Flexible Link (2DSFL) system is represented in Figure 7. It depicts the two flexible joints connected in series and each actuated by its own drive system.

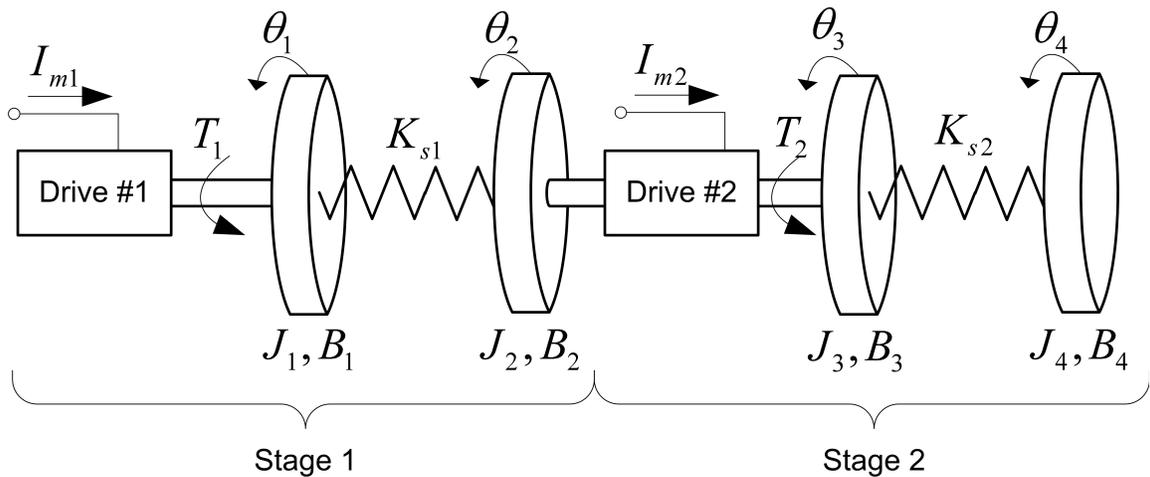


Figure 7 Schematic of the Two-Degree-Of-Freedom Serial Flexible Link (2DSFL) System where  $K_{s1}$  and  $K_{s2}$  are the first and second flexible link torsional stiffness constants,  $I_{m1}$  and  $I_{m2}$  the drive currents,  $J_i$  (for  $i = 1, 2, 3, 4$ ) the intermediary load moments of inertia, and  $B_i$  (for  $i = 1, 2, 3, 4$ ) the intermediary load viscous damping coefficients.

**Sign Convention:**

The positive direction of rotation, as illustrated in Figure 7 for all four load angles  $\theta_i$  (for  $i = 1, 2, 3, 4$ ) is chosen to be CounterClockWise (CCW) when looking at the robot from top.

In the controller design procedure described hereafter, the 2DSFL system is considered decoupled and split into two separate and independent stages: Stage 1 and Stage 2, as depicted in Figure 7. Each stage has its own LQR state-feedback control loop.

Let us first consider the stage 1 system of the 2DSFL plant. Its schematic is represented in Figure 8.

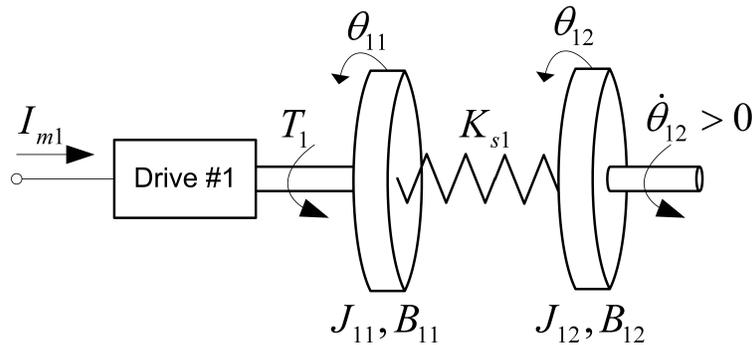


Figure 8 Schematic of the 2DSFL Robot Stage 1 System

Table 2 provides a nomenclature of the symbols used in the 2DSFL Stage 1 system mathematical modeling, as presented in this manual.

<i>Symbol</i>	<i>Description</i>	<i>Units</i>
$I_{m1}$	First (Shoulder) Motor Armature Current	A
$K_{t1}$	First (Shoulder) Drive Torque Constant	N.m/A
$T_1$	Torque Produced by Drive #1, at the Load Shaft	N.m
$\theta_{11}$	First (Shoulder) Driving Shaft Absolute Angular Position	rad
$\frac{d}{dt}\theta_{11}(t)$	First (Shoulder) Driving Shaft Absolute Angular Velocity	rad/s
$\theta_{12}$	First Flexible Link Relative End-Effector Angular Position	rad
$\frac{d}{dt}\theta_{12}(t)$	First Flexible Link Relative End-Effector Angular Velocity	rad/s
$J_{11}$	First Joint Equivalent Moment Of Inertia	kg.m <sup>2</sup>
$B_{11}$	First Joint Equivalent Viscous Damping Coefficient	N.m.s/rad
$J_{12}$	First Flexible Link End-Effector Equivalent Moment Of Inertia (Compounded With The Stage 2 System)	kg.m <sup>2</sup>
$B_{12}$	First Flexible Link End-Effector Equivalent Viscous Damping Coefficient (Compounded With The Stage 2 System)	N.m.s/rad
$K_{s1}$	First Flexible Link Torsional Stiffness Constant	N.m/rad

Table 2 First Stage Of The 2-DOF SFJ Robot Model Nomenclature

Reference [4] details and derives the general dynamic equations of the 2DSFL Stage 1 system. The Lagrange's method is used to obtain the dynamic model of the system. The described modeling assumes that both actuator (drive #1 shaft) and link deflection sensor (base strain gauge sensor) are collocated. The system's state vector,  $X_1$ , is chosen to include the generalized coordinates as well as their first-order time derivatives. It is defined by its transpose, as shown below:

$$X_1^T = \left[ \theta_{11}(t), \theta_{12}(t), \frac{d}{dt} \theta_{11}(t), \frac{d}{dt} \theta_{12}(t) \right]$$

The system input,  $U_1$ , is the current to the first motor, that is to say:

$$U_1 = I_{m1}$$

The state-space matrices  $A_1$  and  $B_1$  are defined to give a dynamic representation of the 2DSFL Stage 1 system, such that:

$$\frac{\partial}{\partial t} X_1 = A_1 X_1 + B_1 U_1$$

From the system's two equations of motion, the  $A_1$  matrix can be determined as follows:

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{K_{sl}}{J_{11}} & -\frac{B_{11}}{J_{11}} & \frac{B_{12}}{J_{11}} \\ 0 & -\frac{(J_{11} + J_{12}) K_{sl}}{J_{11} J_{12}} & \frac{B_{11}}{J_{11}} & -\frac{B_{12} (J_{11} + J_{12})}{J_{11} J_{12}} \end{bmatrix}$$

Likewise, the transpose of the  $B_1$  matrix characterizing the system can be seen below:

$$B_1^T = \begin{bmatrix} 0 & 0 & \frac{K_{tl}}{J_{11}} & -\frac{K_{tl}}{J_{11}} \end{bmatrix}$$

To control the stage 1 system position, a state-feedback controller is implemented according to the following feedback control law:

$$I_{m1} = -K_1 X_1$$

where  $K_l$  is the gain vector for the stage 1 system.

The design file *setup\_2dsfl.m* calculates the state-feedback gain  $K_l$  using the LQR tuning algorithm. You may edit the file to change the system closed-loop behaviour. Moreover, some additional hand-tuning of each of the state-feedback gain vector elements can also be carried out to further eliminate the effects of dynamic link coupling as well as parameter estimation errors.

Likewise, let us now consider the stage 2 system of the 2DSFL plant. Its schematic is represented in Figure 9.

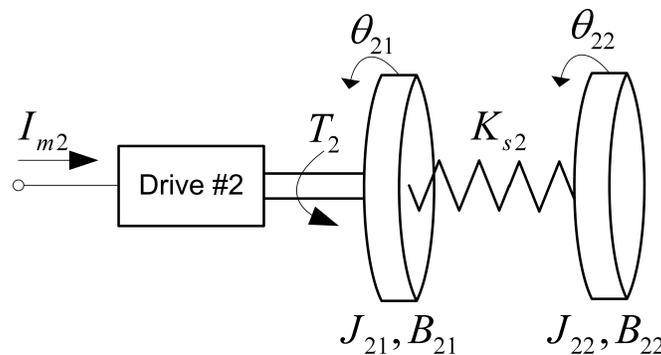


Figure 9 Schematic of the 2DSFL Robot Stage 2 System

Table 3 provides a nomenclature of the symbols used in the 2DSFL Stage 2 system mathematical modeling, as presented in this manual.

<i>Symbol</i>	<i>Description</i>	<i>Units</i>
$I_{m2}$	Second (Elbow) Motor Armature Current	A
$K_{t2}$	Second (Elbow) Drive Torque Constant	N.m/A
$T_2$	Torque Produced by Drive #2, at the Load Shaft	N.m
$\theta_{21}$	Second (Elbow) Driving Shaft Angular Position Relative To Link #1	rad
$\frac{d}{dt} \theta_{21}(t)$	Second (Elbow) Driving Shaft Angular Velocity Relative To Link #1	rad/s
$\theta_{22}$	Second Flexible Link End-Effector Angular Position Relative To Link #1	rad
$\frac{d}{dt} \theta_{22}(t)$	Second Flexible Link End-Effector Angular Velocity Relative To Link #1	rad/s
$J_{21}$	Second Joint Equivalent Moment Of Inertia	kg.m <sup>2</sup>
$B_{21}$	Second Joint Equivalent Viscous Damping Coefficient	N.m.s/rad
$J_{22}$	Second Flexible Link End-Effector Equivalent Moment Of Inertia	kg.m <sup>2</sup>
$B_{22}$	Second Flexible Link End-Effector Equivalent Viscous Damping Coefficient	N.m.s/rad
$K_{s2}$	Second Flexible Link Torsional Stiffness Constant	N.m/rad

Table 3 Second Stage Of The 2-DOF SFJ Robot Model Nomenclature

Reference [5] details and derives the general dynamic equations of the 2DSFL Stage 2 system. The Lagrange's method is used to obtain the dynamic model of the system. The described modeling assumes that both actuator (drive #2 shaft) and link deflection sensor (base strain gauge sensor) are collocated. The system's state vector,  $X_2$ , is chosen to include the generalized coordinates as well as their first-order time derivatives. It is defined by its transpose, as shown below:

$$X_2^T = \left[ \theta_{21}(t), \theta_{22}(t), \frac{d}{dt} \theta_{21}(t), \frac{d}{dt} \theta_{22}(t) \right]$$

The system input,  $U_2$ , is the current to the second motor, that is to say:

$$U_2 = I_{m2}$$

The state-space matrices  $A_2$  and  $B_2$  are defined to give a dynamic representation of the 2DSFL Stage 2 system, such that:

$$\frac{\partial}{\partial t} X_2 = A_2 X_2 + B_2 U_2$$

From the system's two equations of motion, the  $A_2$  matrix can be determined as follows:

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{K_{s2}}{J_{21}} & -\frac{B_{21}}{J_{21}} & \frac{B_{22}}{J_{21}} \\ 0 & -\frac{(J_{21} + J_{22}) K_{s2}}{J_{21} J_{22}} & \frac{B_{21}}{J_{21}} & -\frac{B_{22} (J_{21} + J_{22})}{J_{21} J_{22}} \end{bmatrix}$$

Likewise, the transpose of the  $B_2$  matrix characterizing the system can be seen below:

$$B_2^T = \begin{bmatrix} 0 & 0 & \frac{K_{t2}}{J_{21}} & -\frac{K_{t2}}{J_{21}} \end{bmatrix}$$

To control the stage 2 system position, a state-feedback controller is implemented according to the following feedback control law:

$$I_{m2} = -K_2 X_2$$

where  $K_2$  is the gain vector for the stage 2 system.

The design file *setup\_2dsfl.m* calculates the state-feedback gain  $K_2$  using the LQR tuning algorithm. You may edit the file to change the system closed-loop behaviour. Moreover, some additional hand-tuning of each of the state-feedback gain vector elements can also be carried out to further eliminate the effects of dynamic link coupling as well as parameter estimation errors.

## 2. References

- [1] Q8 Data Acquisition System – QUARC Support And Installation Guide.
- [2] QUARC Installation Guide.
- [3] QUARC help pages (access by typing `doc quarc` in Matlab prompt)
- [4] Dynamic Equations For The First Stage Of The Serial Flexible Link (2DSFL) Robot – Maple Worksheet or HTML File.
- [5] Dynamic Equations For The Second Stage Of The Serial Flexible Link (2DSFL) Robot – Maple Worksheet or HTML File.

## 3. Obtaining Support

To obtain support from Quanser, go to <http://www.quanser.com> and click on the Tech Support link. Fill in the form with all the requested software and hardware information as well as a description of the problem encountered. Also, make sure your e-mail address and telephone number are included. Submit the form and a technical support person will contact you.