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LABORATORY GUIDE

3 DOF Gyroscope Experiment for LabVIEW™ Users

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1 INTRODUCTION

Gyroscopes have become of great practical interest as they are used in control and guidance systems for air, sea, and space vehicles. The Quanser 3 DOF Gyroscope system can be actuated about all of its frames using the mounted motors while encoders measure the angle about each axis. In addition, the rotor itself is actuated and measured in the same manner.

The outer rectangular frame, outer red gimbal and the inner blue gimbal are designed such that they can be individually fixed in place upon desire. This allows the users to perform a variety of different experiments using the device. In this laboratory, the gyroscopic effect will be employed to control the angle of the red gimbal by applying the control command about the blue gimbal. The gray outer rectangular frame will be fixed (see the 3 DOF Gyroscope User Manual [2] for instructions on how to fix each frame). In order to do this, the rotor has to have acquired enough angular momentum (RPM) for the gyroscopic effect to take place. Therefore a controller is required to control the angular speed of the disk while another is required to control the red gimbal angle.

Topics Covered

- Obtain a state-space representation of the open-loop system.
- Design a state-feedback gain for the closed-loop system using the Linear-Quadratic Regulator (LQR) optimization.
- Simulate the system and ensure it is stabilized using the designed state-feedback control.
- Implement your state-feedback controller on the 3D GYRO system and evaluate its actual performance.

Prerequisites
In order to successfully carry out this laboratory, the user should be familiar with the following:

1. Hardware and software requirements given in Section 4.
2. Modeling and state-space representation.
3. State-feedback design using Linear-Quadratic Regulator (LQR) optimization.
4. Basics of LabVIEW™ .

2 BACKGROUND

2.1 Modeling

2.1.1 Model Convention

The reference coordinate frame for the 3 DOF Gyroscope is shown in Figure 2.1.

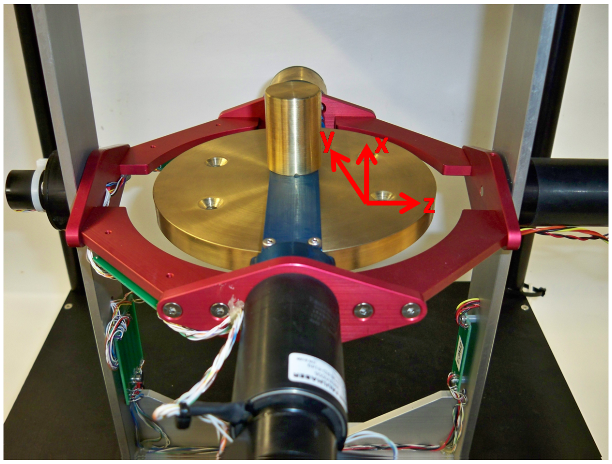

Figure 2.1: 3 DOF Gyroscope coordinate frame

2.1.2 Equations of Motion

The equations of motion representing the angular rate of the red gimbal, ψ, and the outer blue gimbal, θ, are ([1]):

$$\begin{aligned} J_y\ddot{\theta} + h\dot{\psi} &= \tau_y \\ J_z\ddot{\psi} - h\dot{\theta} &= 0 \end{aligned} \tag{2.1}$$

where

$$\begin{aligned} J_y &= 0.0039 \text{ kg-m}^2 \\ J_z &= 0.0223 \text{ kg-m}^2 \\ h &= 0.44 \text{ kg-m}^2\text{/s.} \end{aligned}$$

The moment of inertia about the y-axis angle, θ, is J_y and the moment of inertia about the z-axis angle (red gimbal), ψ, is denoted as J_z. The constant h is calculated based on the moment of inertia of the gyroscope rotor about its own axis and its velocity. Because the outer gray rectangular frame is fixed, the only actuated axis is the y-axis. The control input in the single-input single-output (SISO) system is the torque applied in the y-axis, τ_y.

2.1.3 Linear State-Space Model

The linear state-space equations are

$$\dot{x} = Ax + Bu \tag{2.2}$$

and

$$y = Cx + Du \tag{2.3}$$

where x is the state, u is the control input, A, B, C, and D are state-space matrices.

For this system, the state and output are defined

$$x^{\top} = \begin{bmatrix}\dot{\theta} & \psi & \dot{\psi} & \int \psi\end{bmatrix} \quad (2.4)$$

and

$$y = \psi$$

Solving for the acceleration terms in the linear equations of motion given in Equation 2.1 we get

$$\begin{aligned}\ddot{\theta} &= \frac{-h}{J_y}\dot{\psi} + \frac{1}{J_y}\tau_y \\ \ddot{\psi} &= \frac{h}{J_z}\dot{\theta}\end{aligned}$$

Substituting the state in , we obtain the following state-space matrices:

$$A = \begin{bmatrix} 0 & 0 & -\frac{h}{J_y} & 0 \\ 0 & 0 & 1 & 0 \\ \frac{h}{J_z} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

and

$$B = \begin{bmatrix} \frac{1}{J_y} \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The system output (control variable) is the red gimbal angle which is the second entry in the system state vector i.e., ψ. Based on this, the C and D matrices in the output equation are

$$C = \begin{bmatrix}0 & 1 & 0 & 0\end{bmatrix}$$

and

$$D = [0]$$

The velocities of the red and blue gimbal angles can be computed in the digital controller, e.g., by taking the derivative and filtering the result though a high-pass filter. The integral state can be computed by integrating the red gimbal angle measurement in the digital controller.

2.2 Control

In Section 2.1, we found a linear state-state space model that represents the 3 DOF Gyroscope system. This model is used to investigate the stability properties of the system in Section 2.2.1. In Section 2.2.2, the notion of controllability is introduced. The Linear Quadratic Regulator (LQR) algorithm is a common way to find the control gain and is discussed in Section 2.2.3. Lastly, Section 2.2.4 describes the state-feedback control used to control the red gimbal position.

2.2.1 Stability

The stability of a system can be determined from its poles ([3]):

- Stable systems have poles only in the left-hand plane.
- Unstable systems have at least one pole in the right-hand plane and/or poles of multiplicity greater than 1 on the imaginary axis.

- Marginally stable systems have one pole on the imaginary axis and the other poles in the left-hand plane.

The poles are the roots of the system's characteristic equation. From the state-space, the characteristic equation of the system can be found using

$$\det(sI - A) = 0 \tag{2.5}$$

where *det()* is the determinant function, s is the Laplace operator, and I the identity matrix. These are the *eigenvalues* of the state-space matrix A.

2.2.2 Controllability

If the control input, u, of a system can take each state variable, x_i where $i = 1 \ldots n$, from an initial state to a final state then the system is controllable, otherwise it is uncontrollable ([3]).

Rank Test The system is controllable if the rank of its controllability matrix

$$T = \left[B \; AB \; A^2B \ldots A^nB\right] \tag{2.6}$$

equals the number of states in the system,

$$\text{rank}(T) = n. \tag{2.7}$$

2.2.3 Linear Quadratic Regular (LQR)

If (A,B) are controllable, then the Linear Quadratic Regulator optimization method can be used to find a feedback control gain. Given the plant model in Equation 2.2, find a control input u that minimizes the cost function

$$J = \int_0^\infty x(t)'Qx(t) + u(t)'Ru(t)\, dt, \tag{2.8}$$

where Q and R are the weighting matrices. The weighting matrices affect how LQR minimizes the function and are, essentially, tuning variables.

Given the control law $u = -Kx$, the state-space in Equation 2.2 becomes

$$\begin{aligned}\dot{x} &= Ax + B(-Kx) \\ &= (A - BK)x\end{aligned}$$

2.2.4 Feedback Control

The feedback control loop in Figure 2.2 is designed to stabilize the red gimbal to a desired position, ψ_d.

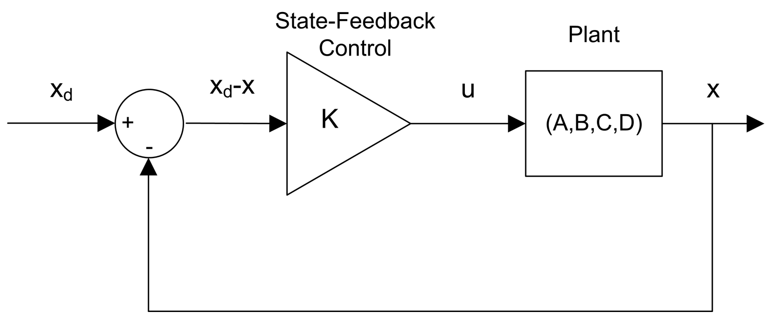

Figure 2.2: State-feedback control loop

The reference state is defined

$$x_d = \begin{bmatrix}0 & \psi_d & 0 & 0\end{bmatrix}$$

and the controller is

$$u = K(x_d - x). \tag{2.9}$$

Note that if $x_d = 0$ then $u = -Kx$, which is the control used in the LQR algorithm.

3 LAB EXPERIMENTS

3.1 Simulation

In this section we will use the LabVIEW VI shown in Figure 3.1 to simulate the closed-loop control of the *3 DOF Gyroscope* system. The VI uses the state-feedback control described in Section 2.2.4. The feedback gain K is found using the LQR VI from the LabVIEW *Simulation and Control Design Module* (LQR is described briefly in Section 2.2.3).

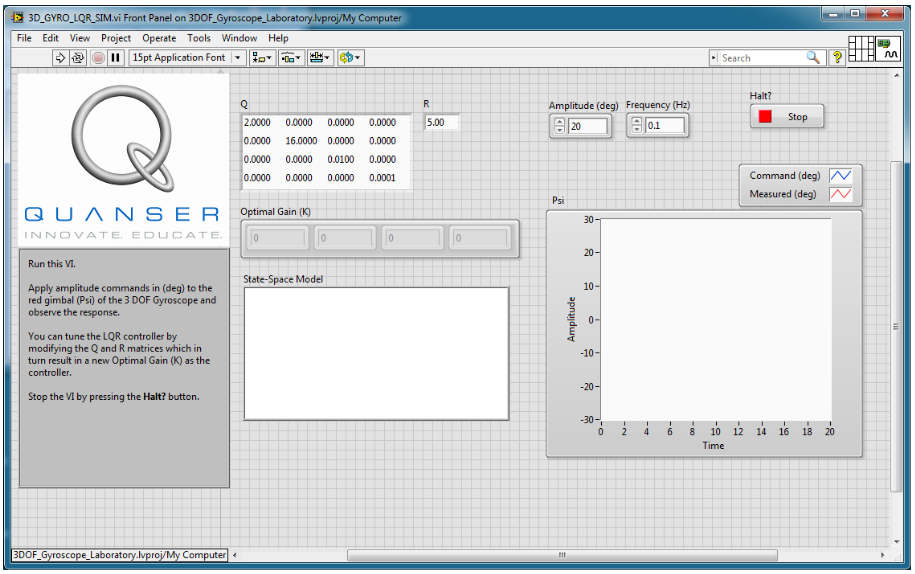

Figure 3.1: VI used to simulate the 3 DOF Gyroscope.

IMPORTANT: Before you can conduct these experiments, you need to make sure that the lab files are configured according to your setup. If they have not been configured already, then you need to go to Section 4 to configure the lab files first.

3.1.1 Procedure

Follow these steps to simulate the 3 DOF Gyroscope:

1. Open and run the *3D_GYRO_LQR_SIM.vi* as described in Section 4.
2. By default, the Q matrix is sent to identity matrix. Set the LQR weighting matrices in to

$$Q = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 16 & 0 & 0 \\ 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0.0001 \end{bmatrix}$$

   and $R = 5$.
3. This automatically generates the gain

$$K = \begin{bmatrix} 0.65 & 1.79 & 0.12 & 0.004 \end{bmatrix}.$$

Remark: When tuning the LQR, $Q(2,2)$ effects the red gimbal proportional gain while $Q(1,1)$ effects the red gimbal derivative gain (which reduces the overshoot). $Q(4,4)$ affects the red gimbal integral gain which is used to minimize the steady state error.

4. The *Frequency* control in the VI is already setup to generate a 0.1 Hz square wave reference.
5. Start the simulation by clicking on the white arrow button found on the top right hand corner of the VI (CTRL + R).
6. Set the *Amplitude* control to 20 degrees to generate a step with an amplitude of 20 degrees (i.e., square wave goes between ± 20 which results in a step amplitude of 20).
7. Go to the *Psi Scope* to view the red gimbal position command and actual values.
8. The scope should be displaying a response similar to Figure 3.2. Note that in the *Psi* scope, the blue trace is the setpoint position and the red trace is the simulated position.

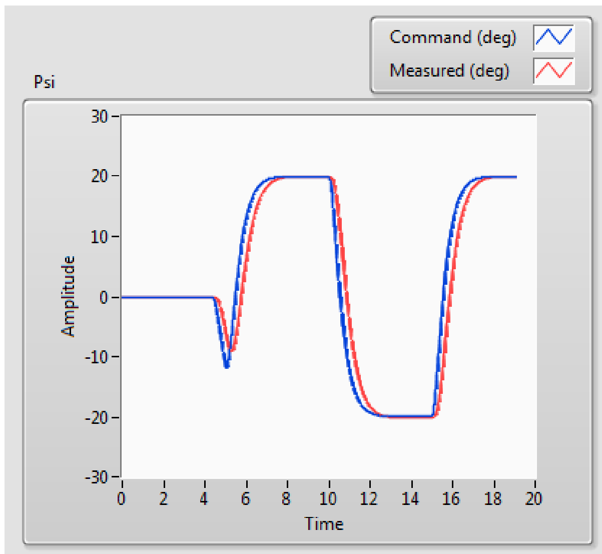

Figure 3.2: Simulated closed-loop response.

9. This is an iterative design process. You can go back and change your Q and R matrices, acquire a new control gain K, on the fly while the simulation is running and see the resulting performance.
10. Click on the *STOP* button to stop running the VI.

3.2 Implementation

The *3D_GYRO_LQR* VI shown in Figure 3.3 is used to perform the red gimbal position control on the 3D GYRO. The VI contains drivers that interface with the DC motor and sensors of the 3D GYRO system.

IMPORTANT: Before you can conduct these experiments, you need to make sure that the lab files are configured according to your setup. If they have not been configured already, then you need to go to Section 4 to configure the lab files first.

3.2.1 Procedure

Follow this procedure:

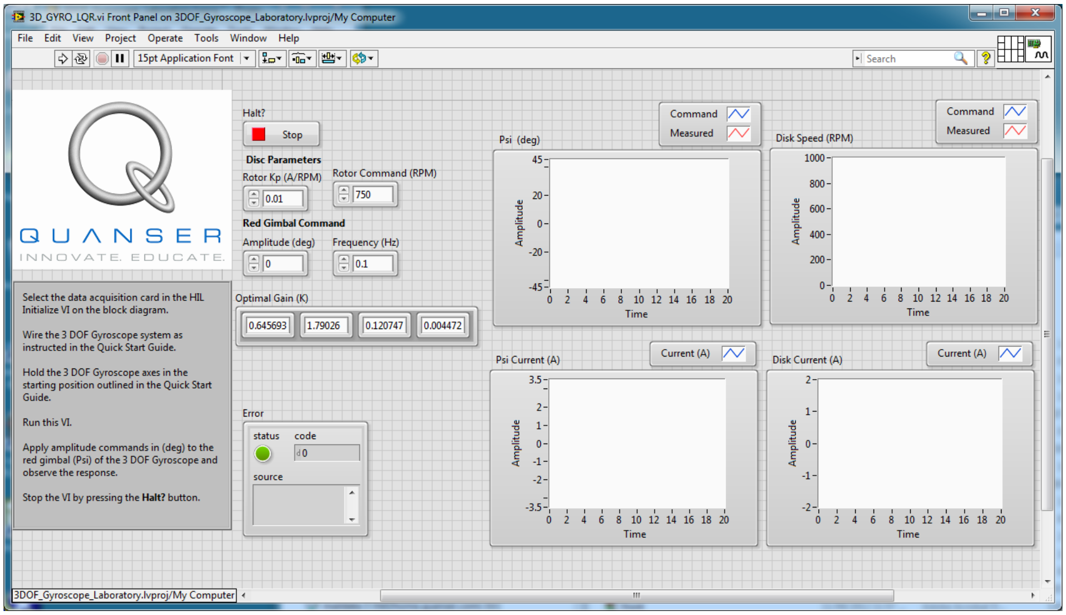

Figure 3.3: VI used to run controller on the 3D GYRO.

1. Place the 3 DOF Gyroscope flywheel, blue and red gimbals in their home starting position. Ensure that the flywheel's motor and the mass counter balances on the red gimbal (identified with red squares in Figure 3.4) are pointing toward the same direction (e.g., all facing up or all facing down).

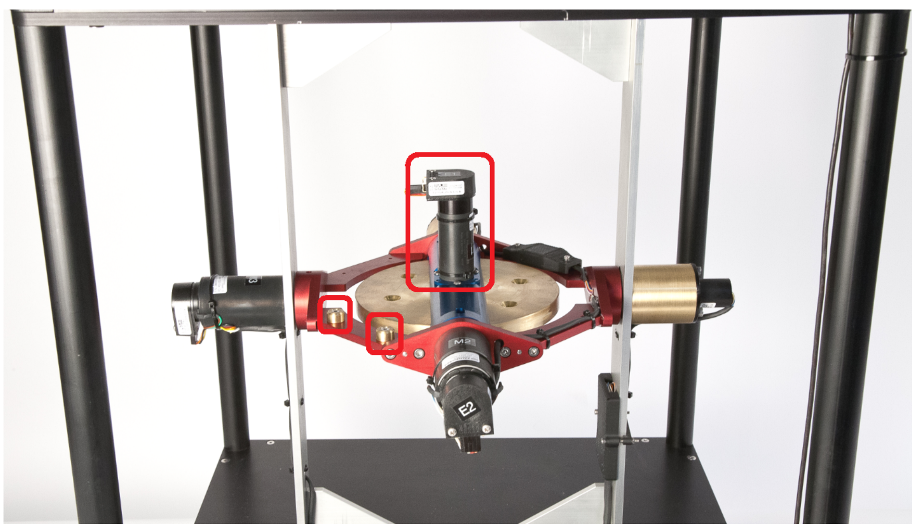

Figure 3.4: Starting position for the 3D GYRO.

2. You will need to gently hold on to the blue and red gimbals and keep them in place for the first few seconds after the VI has started running while the flywheel speeds up tp the required RPM as show in Figure 3.5
3. Make sure the VI is set to use the same gain K found in the simulation in Section 3.1. See Section 4 for more information.
4. Set the *Frequency* control in the VI front panel to 0.1 Hz.
5. Run the VI by clicking on the white arrow found in the top left hand corner of the VI (CTRL+R) while holding the blue and red gimbals in their home position as instructed above.
6. Monitor the flywheel speed in the *Disc Speed* scope. Once it reaches the commanded speed (e.g., 750 RPM), set the *Amplitude* control to 20 degrees. This will generate a ± 20 degree 0.1 Hz square wave reference.

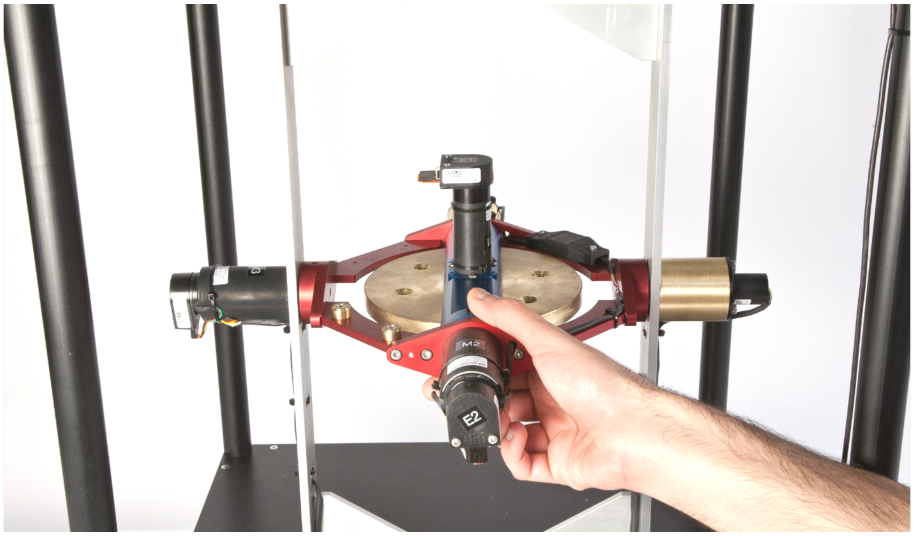

Figure 3.5: Holding the blue and red gimbals

7. The red gimbal should now be going back and forth between the commanded positions at the frequency specified. Examine the position of the red gimbal in the *Psi* scope. You can also view the commanded motor currents in the *Disk Current* and *Psi Current* scopes. The scopes should be displaying responses similar to Figure 3.6.

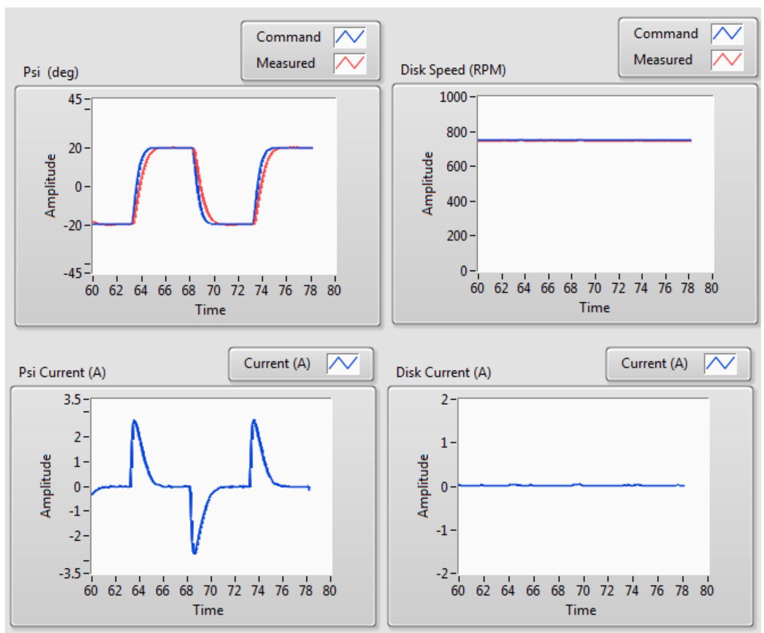

Figure 3.6: Typical response when running control on 3D GYRO system

4 SYSTEM REQUIREMENTS

Required Software

Make sure LabVIEW™ is installed with the following required add-ons:

1. LabVIEW™
2. NI-DAQmx
3. NI LabVIEW™ Control Design and Simulation Module
4. NI LabVIEW™ MathScript RT Module
5. Quanser Rapid Control Prototyping Toolkit®

Note: Make sure the Quanser Rapid Control Prototyping (RCP) Toolkit is installed after LabVIEW. See the RCP Toolkit Quick Start Guide for more information.

Required Hardware

- Data acquisition (DAQ) device **with four encoder inputs** and that is compatible with Quanser Rapid Control Prototyping Toolkit®. This includes Quanser DAQ boards such as Q8-USB, QPID, and QPIDe and some National Instruments DAQ devices.
- Quanser 3 DOF Gyroscope
- Quanser AMPAQ-L4 power amplifier, or equivalent.

Before Starting Lab

Before you begin this laboratory make sure:

- LabVIEW™ is installed on your PC.
- DAQ device has been successfully tested (e.g., using the test software in the Quick Start Guide or the *Analog Loopback Demo*).
- 3 DOF Gyroscope and amplifier are connected to your DAQ board as described its User Manual [2].

4.1 Overview of Files

File Name	Description
3D GYRO User Manual.pdf	This manual describes the hardware of the 3D GYRO system and explains how to setup and wire the system for the experiments.
3D GYRO Laboratory Guide.pdf	This document demonstrates how to obtain the linear state-space model of the system, simulate the closed-loop system, and implement controllers on the 3D GYRO plant using LabVIEW™ .
3DOF_Gyroscope_Laboratory.lvproj	3 DOF Gyroscope LabVIEW project that contains all the VIs required for the lab.
3D_GYRO_LQR_SIM.vi	VI used to design the LQR state-feedback gain and simulate the 3 DOF Gyroscope system.
3D_GYRO_LQR.vi	VI that implements the state-feedback control on the 3D GYRO system.
3D_GYRO_LQR (cRIO).vi	VI that implements the state-feedback control on the 3D GYRO system using the NI CompactRIO and Quanser Q1-cRIO configuration.

Table 4.1: Files supplied with the 3D GYRO

4.2 Setup for Simulation

Before beginning the in-lab procedure outlined in Section 3.1, the *3D_GYRO_LQR_SIM* VI must be configured.

1. Load LabVIEW™ .
2. Open the *3DOF_Gyroscope.lvproj* LabVIEW project, shown in Figure 4.1.

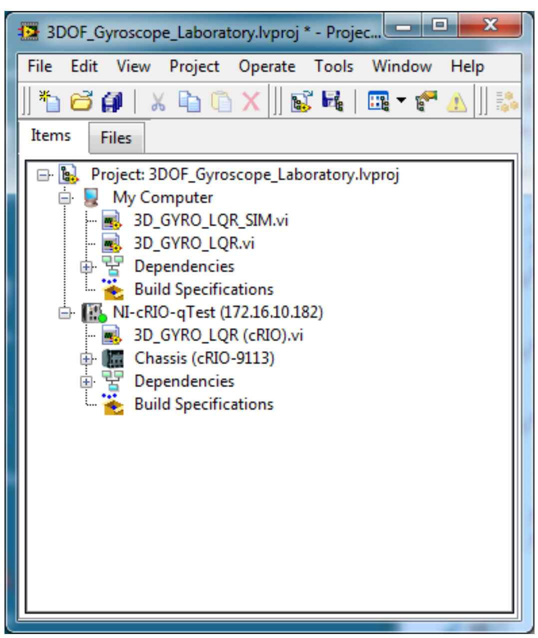

Figure 4.1: 3DOF Gyroscope LabVIEW project

3. Open the *3D_GYRO_LQR_SIM.vi* shown in Figure 4.2.

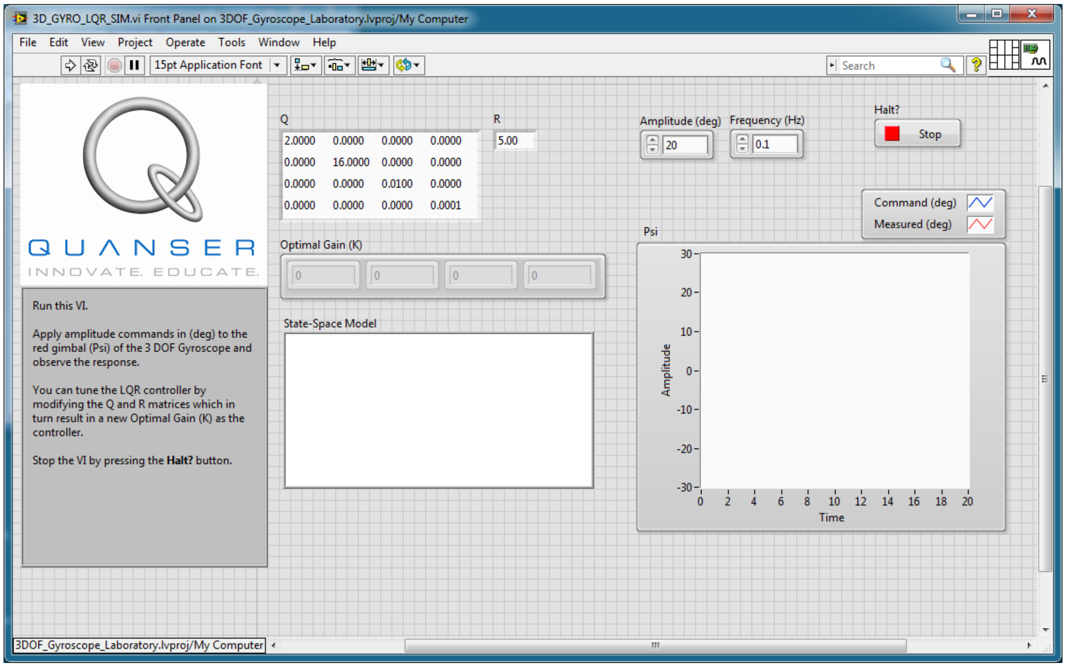

Figure 4.2: 3DOF Gyroscope Simulation VI

4. The state space matrices are already loaded. The Q and R matrices are also set to the original values mentioned in Section 3.1.1.
5. Run the VI. You are now ready to design your LQR control and simulate the closed-loop response.

4.3 Setup for Running on 3D GYRO

Before performing the in-lab exercises in Section 3.2, the 3D GYRO system and the *3D_GYRO_LQR.vi* must be configured properly.

Follow these steps to get the system ready for this lab:

1. Lock the silver rectangular frame of the 3D GYRO in place as detailed in the 3D GYRO User Manual [2].
2. Open the *3D_GYRO_LQR.vi*, shown in Figure 3.3.
   NI CompactRIO Users: Open the *3D_GYRO_LQR (cRIO).vi* under the NI-CompactRIO device in the LabVIEW Project. Make sure the NI CompactRIO in the project is configured for the cRIO you will be using (e.g., correct IP address). See the example in the *RCP Installation Guide for NI CompactRIO* for more information.
3. **Configure DAQ:** Ensure the HIL Initialize block is configured for the DAQ device that is installed in your system. To do this, go to the block diagram (CTRL-E) and double click on the **HIL Initialize** Express VI shown in Figure 4.3.

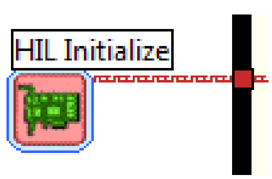

Figure 4.3: HIL Initialize Express VI

4. Under the Main tab, select the data acquisition device that is installed on your system in the *Board type* section. For example, in Figure 4.4 the Q8-USB is chosen.

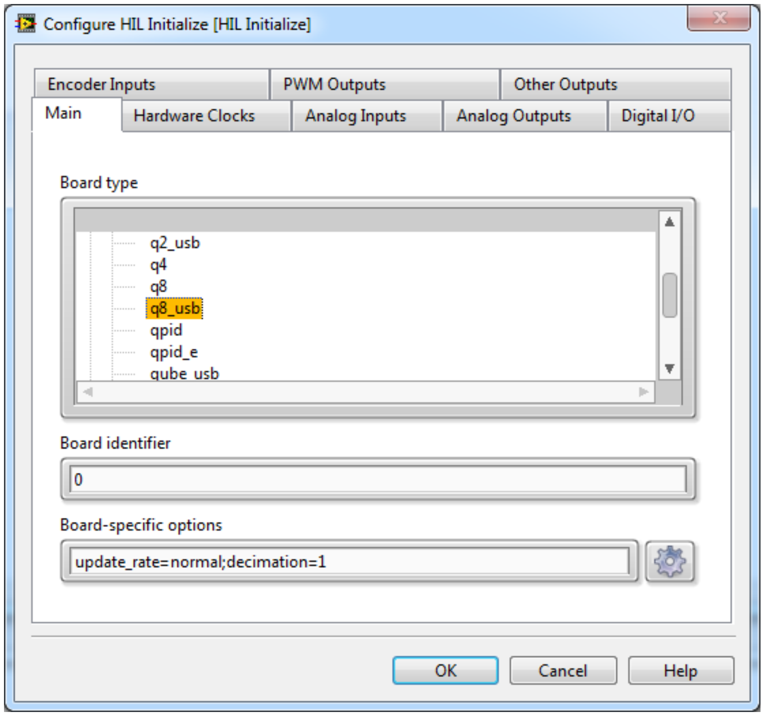

Figure 4.4: Selecting the q8_usb board

5. You are now ready to run and tune the LQR controller as outlined in Section 3.2.1.

REFERENCES

[1] Robert H. Canon. *Dynamics of Physical Systems*. McGraw Hill Book Company, New York.

[2] Quanser Inc. *3 DOF Gyroscope User Manual*, 2012.

[3] Norman S. Nise. *Control Systems Engineering*. John Wiley & Sons, Inc., 2008.

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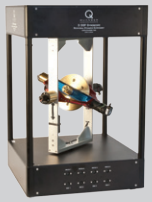
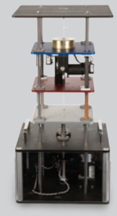
3 DOF Gyroscope



Omni Bundle

2 DOF Planar Robot

Active Suspension



Hexapod*

IMDU Base Unit

IMDU-Web Winding

IMDU-Multi DOF Torsion

* **Please note:** The Hexapod is not available for purchase in North America, Japan and Taiwan.

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