LABORATORY GUIDE

3 DOF Helicopter Experiment for LabVIEW™ Users

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CONTENTS

1 PRESENTATION

1.1 Description

The 3 DOF Helicopter plant is depicted in Figure 1.1. Two DC motors are mounted at each end of a rectangular frame and drive two propellers. The motors' axes are parallel and the thrust vector is normal to the frame. The helicopter frame is suspended from an instrumented joint mounted at the end of a long arm and is free to pitch about its centre. The arm is installed on an additional 2-DOF instrumented joint which allows the helicopter body to move in the elevation and yaw directions. The other end of the arm carries a counterweight such that the effective mass of the helicopter is light enough for it to be lifted using the thrust from the motors. The system is analogous to a tandem rotor helicopter oriented perpendicular to the support arm, as shown in Figure 3.2.

A positive voltage applied to the front motor causes a positive pitch while a positive voltage applied to the back motor causes a negative pitch. A positive voltage to either motor also causes an elevation of the body. If the body pitches, the thrust vectors result in a travel of the body (i.e., yaw of the arm) as well. The vertical base is equipped with an eight-contact slip ring. Electrical signals to and from the arm and helicopter are channelled through the slip ring to eliminate tangled wires, reduce friction, and allow for unlimited and unhindered travel.

Figure 1.1: 3 DOF Helicopter when running.

The objective of this experiment is to design a control system to track and regulate the elevation and travel angles of the 3 DOF Helicopter. The system is supplied with a complete mathematical model, the system parameters, and a sample state-feedback controller.

As shown in Figure 1.2, the 3 DOF Helicopter can also be fitted with an Active Mass Disturbance System (ADS). The ADS is comprised of a lead-screw, a DC motor, an encoder, and a moving mass. The lead-screw is wound through the mass such that when lead is rotated the mass moves along the helicopter arm linearly. One end of the lead-screw is connected to a DC motor and the other end has an encoder. As the motor is driven, the lead-screw rotates and causes the mass to move. Using the encoder measurement and a position controller, the user can move the mass to a desired position and actively disturb the helicopter.
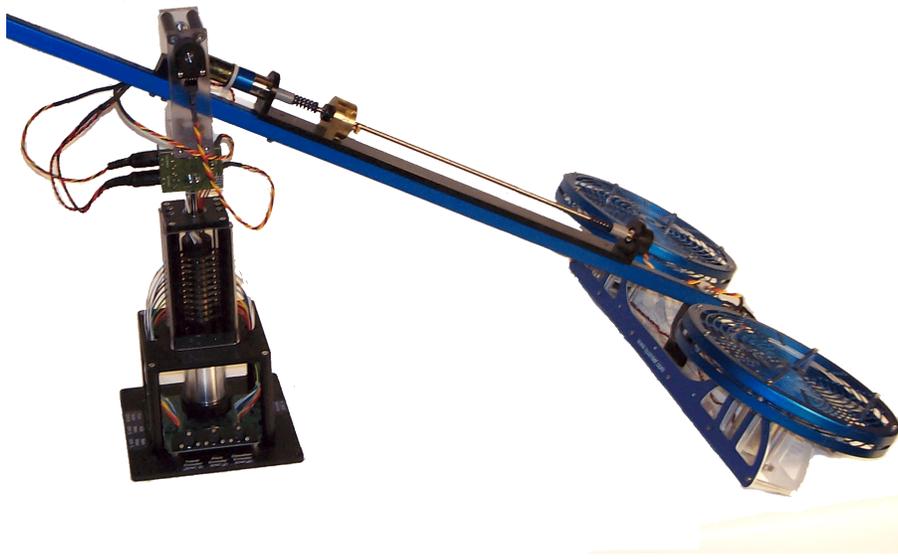
Figure 1.2: Active Disturbance System on the 3 DOF Helicopter .

1.2 Prerequisites

In order to successfully carry out this laboratory, the user should be familiar with the following:

- 3 DOF Helicopter main components (e.g. actuator, sensors), the data acquisition card (e.g. Q8-USB), and the power amplifier (e.g. VoltPAQ), as described in [1].

- Wiring the 3 DOF Helicopter plant with the amplifier and DAQ device, as discussed in [1].

- Designing a state-feedback control using Linear-Quadratic Regulator (LQR).

- Using LabVIEW™ to design, control, and monitor a plant in real-time.

2 EXPERIMENT FILES OVERVIEW

Table Table 2.1 below lists and describes the various files supplied with the 3 DOF Helicopter experiment.

File Name #	Description
3 DOF Helicopter - User Manual.pdf	This manual is the user guide for the Quanser 3 DOF Helicopter specialty aerospace plant. It contains information about the hardware components, specifications, and the information to setup and configure the hardware.
3 DOF Helicopter - Laboratory Manual.pdf	This manual is the laboratory guide for the Quanser 3 DOF Helicopter specialty aerospace plant. It contains information about the system modeling, control design, as well as the experimental procedure used to simulate and implement the controller.
3 DOF Heli Equations.mws	Maple worksheet used to analytically derive the state-space model involved in the experiment. Waterloo Maple 9, or a later release, is required to open, modify, and execute this file.
3 DOF Heli Equations.html	HTML presentation of the Maple Worksheet. It allows users to view the content of the Maple file without having Maple 9 installed. No modifications to the equations can be performed when in this format.
3D HELI LAB.lvproj	LabVIEW project file that contains all of the LabVIEW files needed to run the 3 DOF Helicopter experiment.
3D HELI Modeling.vi	Used to generate a state-space model for the 3 DOF Helicopter .
3D HELI Control Design.vi	Used with the model generated in the *3D HELI Modeling.vi* to design a state feedback controller for the 3 DOF Helicopter .
3D HELI Control Simulation.vi	Used with the model and state feedback controller gains to simulate the 3 DOF Helicopter response.
3 DOF HELI Control.vi	Uses the Quanser Rapid Control Prototyping (RCP) Toolkit to control the 3 DOF Helicopter with a joystick input device.
3 DOF HELI Control (cRIO).vi	Uses the Quanser Rapid Control Prototyping (RCP) Toolkit to control the 3 DOF Helicopter for users of the National Instruments cRIO. Used with *Joystick_Read.vi* to allow joystick control of the 3 DOF Helicopter .
Joystick_Read.vi	Acts as a joystick host VI when the 3 DOF Helicopter is connected to the National Instruments cRIO and the user wishes to command the vehicle with a joystick. Joystick data is transmitted from the desktop PC to the cRIO using the RCP stream functionality.
Joystick Test.vi	Used to test the USB joystick.

Table 2.1: Files supplied with the 3 DOF Helicopter experiment.

3 MODELING

The mathematical model developed for the 3 DOF Helicopter system is summarized in Section 3.1. In Section 4, the feedback system used to control the position of the helicopter is described.

3.1 Dynamics

The free-body diagram of the 3 DOF Helicopter is illustrated in Figure 3.1 and accompanies the Maple worksheet named *3 DOF Helicopter Equations.mws* or its HTML equivalent *3 DOF Helicopter Equations.html*. The equations can be edited and re-calculated by executing the worksheet using Maple.

The 3 DOF Helicopter modeling conventions used are:

1. The helicopter is horizontal when the elevation angle equals $\epsilon = 0$.

2. The travel angle increases positively, $\dot{\lambda}(t) > 0$, when the body rotates in the counter-clockwise (CCW) direction.

3. The pitch angle is positive, $\rho(t) > 0$, when the front motor is higher than the back motor.

The 3 DOF Helicopter model that is used in this laboratory is analogous to a tandem rotor helicopter such as the Boeing HC-1B Chinook illustrated in Figure 3.2. As described in the FBD shown in Figure 3.1, the pitch of the helicopter, ρ, is the rotation of the helicopter about a line perpendicular to the length of the body located at the centre of gravity. For example, the illustration in Figure 3.2 would have a positive pitch given that the nose of the helicopter is above the horizon. The elevation axis is defined as a line parallel to the length of the body, at the base coordinate frame. Therefore, a change in the elevation angle, ϵ, translates into a change in the "altitude" of the helicopter as it rotates about the base frame. For example, if the helicopter shown in Figure 3.2 were rotating about an imaginary elevation axis, it might have a slightly negative elevation since the base of the helicopter is visible. Finally, the travel axis is defined as a vertical line at the base coordinate frame perpendicular to the elevation axis. A change in the travel angle, λ, translates into forward flight about the travel axis. For example, if the helicopter shown in Figure 3.2 were attached to an imaginary travel axis limiting its mobility, forward flight would result in a circular trajectory about the base frame.

The worksheet goes through the kinematics of the system. Thus describing the front motor, back motor, helicopter body, and counterweight relative to the base coordinate system shown in Figure 3.1. These resulting equations are used to find the potential energy and translational kinetic energy of the front motor, back motor, and counterweight of the system. The thrust forces acting on the elevation, pitch, and travel axes from the front and back motors are defined and made relative to the quiescent voltage or operating point.

$$V_{op} = \frac{1}{2} \frac{g(L_w m_w - L_a m_f - L_a m_b)}{L_a K_f} \tag{3.1}$$
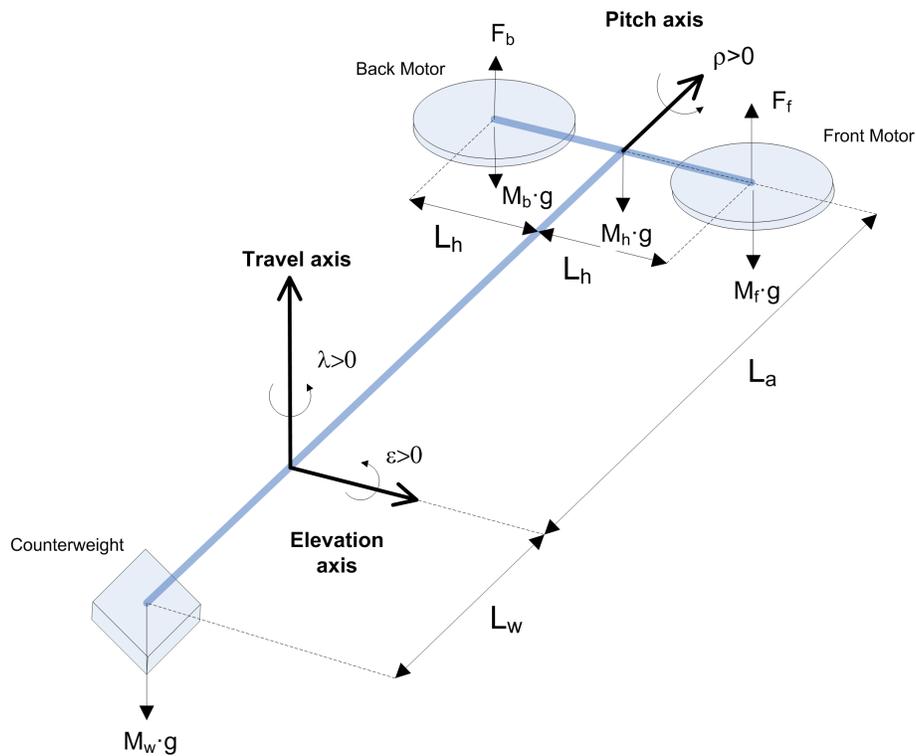
where the parameters are defined in [1].

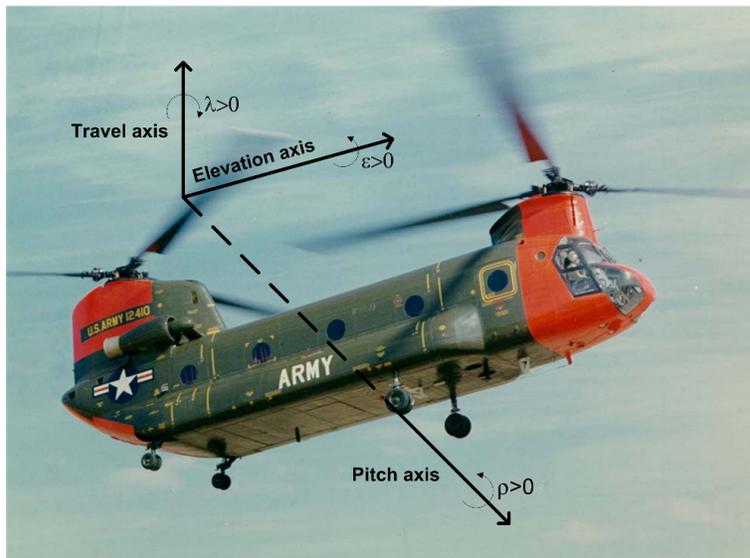Figure 3.1: Free-body diagram of 3 DOF Helicopter .

Figure 3.2: Translation of the free-body diagram onto a Boeing HC-1B Chinook.

3.2 State-Space Model

Using the Euler-Lagrange formula, the nonlinear equations of motion of the 3 DOF Helicopter system are derived. These equations are linearized about zero and the linear state-space model (A,B,C,D) describing the voltage-to-angular joint position dynamics of the system is found. Given the state-space representation

$$\frac{d}{dt}x = Ax + Bu$$

and

$$\frac{d}{dt}y = Cy + Du$$

the state vetor for the 3 DOF Helicopter is defined

$$x^T = \left[\epsilon, \rho_d, \lambda, \frac{d}{dt}\epsilon, \frac{d}{dt}p, \frac{d}{dt}\lambda \right] \tag{3.2}$$

where the output vector is

$$x^T = [\epsilon, \rho_d, \lambda]$$

where the variables ϵ, ρ, and λ are the elevation, pitch, and travel angles. The corresponding helicopter state-space matrices (as derived in the Maple worksheet) are

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{(L_w m_w - 2L_a m_f)g}{m_w L_w^2 + 2m_f L_h^2 + 2m_f L_a^2} & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{3.3}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{L_a K_f}{2m_f L_a^2 + m_w L_w^2} & \frac{L_a K_f}{2m_f L_a^2 + m_w L_w^2} \\ \frac{1}{2}\frac{K_f}{m_f L_h} & -\frac{1}{2}\frac{K_f}{m_f L_h} \\ 0 & 0 \end{bmatrix}, \tag{3.4}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \text{ and}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

The model parameters used in the (A,B) matrices in Equation 3.3 and Equation 3.4 are defined in [1].

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4 CONTROL DESIGN

4.1 State-Feedback

In this section a linear proportional-integral-derivative (PID) controller is designed to regulate the elevation and travel angles of the 3 DOF Helicopter at set positions. The PID control gains are computed using the Linear-Quadratic Regulator algorithm. The state-feedback controller entering the front motor, V_f, and the back motor, V_b, is defined

$$\begin{bmatrix} V_f \\ V_b \end{bmatrix} = K_{PD}(x_d - x) + V_i + \begin{bmatrix} V_{op} \\ V_{op} \end{bmatrix},$$

where

$$K_{PD} = \begin{bmatrix} K_{1,1} & K_{1,2} & K_{1,3} & K_{1,4} & K_{1,5} & K_{1,6} \\ K_{2,1} & K_{2,2} & K_{2,3} & K_{2,4} & K_{2,5} & K_{2,6} \end{bmatrix}$$

is the proportional-derivative control gain,

$$x_d^T = \begin{bmatrix} \epsilon_d & p_d & \lambda_d & 0 & 0 & 0 \end{bmatrix}$$

is the desired state, x is the state defined in Equation 3.2,

$$V_i = \begin{bmatrix} \int k_{1,7}(x_{d,1} - X_1)\,dt + \int k_{1,8}(x_{d,3} - X_3)\,dt \\ \int k_{2,7}(x_{d,1} - X_1)\,dt + \int k_{2,8}(x_{d,3} - X_3)\,dt \end{bmatrix}$$

is the integral control, and V_{op} is the operating point voltage defined in Equation 3.1. The variables ϵ_d, ρ_d, and λ_d, are the elevation, pitch, and travel setpoints (the desired angles of the helicopter). In the control the pitch command is set to zero, thus ρ_d = 0. The gains $k_{1,1}$ through $k_{1,3}$ are the front motor control proportional gains and the gains $k_{2,1}$ through $k_{2,3}$ are the back motor control proportional gains. Similarly, $k_{1,4}$ through $k_{1,6}$ are the front motor control derivative gains and $k_{2,4}$ through $k_{2,6}$ are the back motor control derivative gains. The integral control gains used in the front motor control are $k_{1,7}$ and $k_{1,8}$ and the integral gains $k_{2,7}$ and $k_{2,8}$ are used in the back motor regulator.

4.2 Linear Quadratic Regulator

The PID control gains are computed using the Linear-Quadratic Regular scheme. The system state is first augmented to include the integrals of the elevation and travel states,

$$x_i^T = \begin{bmatrix} \epsilon, \rho, \lambda, \dfrac{d}{dt}\epsilon, \dfrac{d}{dt}\rho, \dfrac{d}{dt}\lambda, \int \epsilon\,dt, \int \lambda\,dt \end{bmatrix}$$

Usign the feedback law

$$u = -Kx_i,$$

the weighting matrices

$$Q = \begin{bmatrix} 100 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.1 \end{bmatrix}$$

and

$$R = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix}$$

and the state-space matrices (A,B) found previously, the control gain

$$K = \begin{bmatrix} 37.67 & 13.21 & -11.50 & 20.95 & 4.769 & -16.10 & 10.00 & -1.000 \\ 37.67 & -13.21 & 11.50 & 20.95 & -4.769 & 16.10 & 10.00 & 1.000 \end{bmatrix}$$

is calculated by minimizing the cost function

$$J = \int_0^\infty x_i^T Q x_i + u^T R u \, dt.$$

In terms of the PID control gains described earlier, the full control gain is expressed

$$K = \begin{bmatrix} K_{1,1} & K_{1,2} & K_{1,3} & K_{1,4} & K_{1,5} & K_{1,6} & K_{1,7} & K_{1,8} \\ K_{2,1} & K_{2,2} & K_{2,3} & K_{2,4} & K_{2,5} & K_{2,6} & K_{1,7} & K_{1,8} \end{bmatrix}$$

5 IN-LAB PROCEDURE

5.1 3 DOF Helicopter LabVIEW Files

The LabVIEW files supplied with the 3 DOF Helicopter contain various controls that implement the model and controllers presented previously. The *3 DOF HELI Control* VI implements the LQR PID position controller discussed in Section 4.

Note: For National Instruments cRIO users, the *3 DOF HELI Control (cRIO)* VI must be used instead of *3 DOF HELI Control*.

The block diagram of the *3 DOF HELI Control* VI is displayed in Figure 5.1. As discussed in Section 4, the position and velocity states are multiplied by the corresponding elements of control gain K. The state includes the integral of the elevation and travel angles and those are multiplied by the integral gains in K.
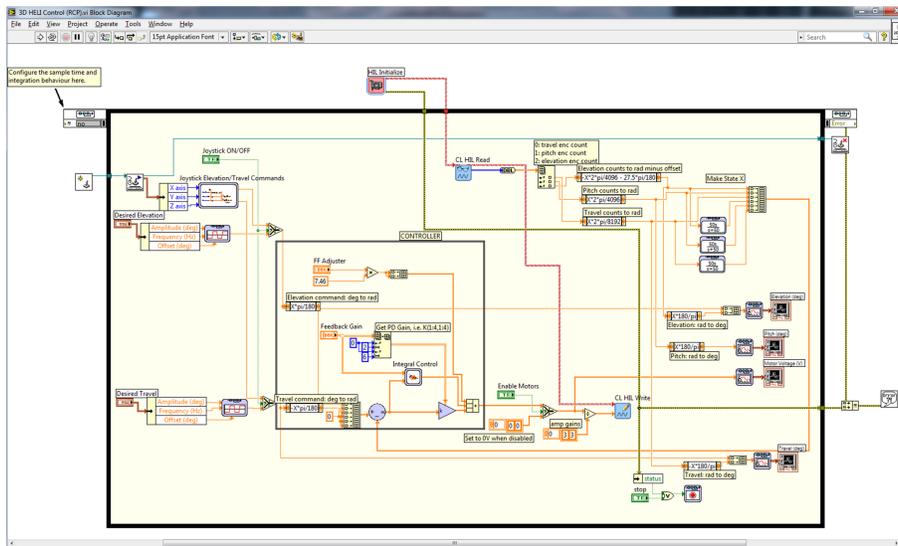
Figure 5.1: Block diagram used to run the closed-loop controller on the 3 DOF Helicopter .

5.2 Modeling, Control Design and Simulation

The following sections describe how to utilize LabVIEW™ and the Quanser Rapid Control Prototyping Toolkit® to develop the model, closed-loop controller, and simulation of the 3 DOF Helicopter experiment.

5.2.1 Objectives

- Generate a linear state-space model from the 3 DOF Helicopter system parameters

- Save the state-space model to a file that will be used for LQR control

- Design a LQR feedback controller to stabilize the 3 DOF Helicopter plant

- Simulate the performance of the LQR controller using the 3 DOF Helicopter nonlinear model

5.2.2 Modeling Procedure

Follow these steps to generate the state-space model of the 3 DOF Helicopter:

1. Load the LabVIEW™ software.

2. Open the LabVIEW project called *3D HELI LAB.lvproj*, shown in Figure 5.2.
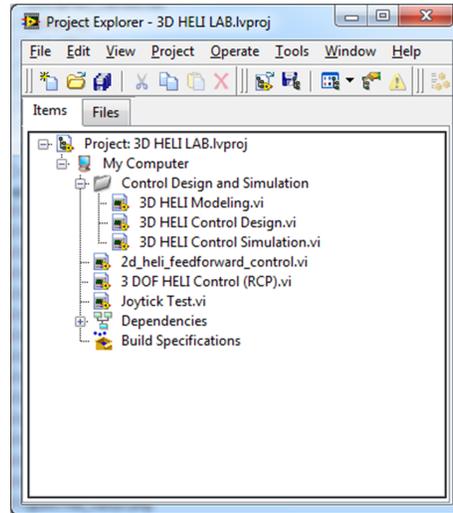

Figure 5.2: LabVIEW project used for the 3 DOF Helicopter system.

3. Under the *Control Design and Simulation* directory in the project explorer, open the *3D HELI Modeling* VI.

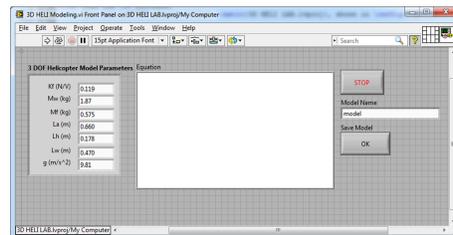4. the front panel of the *3D HELI Modeling* VI is shown in Figure 5.3.

Figure 5.3: Model of the 3 DOF Helicopter.

5. Ensure all of the 3 DOF Helicopter model parameters are set in the VI list of parameters.

6. Run the VI. The resulting state-space model is shown in the Equation display on the front panel, as shown in Figure 5.4.

Figure 5.4: State-space model equations for the 3 DOF Helicopter .

7. While the VI is running, click the OK button on the front panel to save the model to a file. This model file is used throughout the rest of the control design and simulation VIs.

5.3 Controller Design Procedure

Follow these steps to design a LQR controller for the 3 DOF Helicopter linear model:

1. From the project explorer, open the *3D HELI Control Design* VI.

2. Run the *3D HELI Control Design* VI and when prompted select the model file saved from the *3D HELI Modeling* VI.

3. Verify that the Controllability tab shows that the system has 6 states and that the controllability matrix rank is 6 and thus controllable, as shown in Figure 5.5.
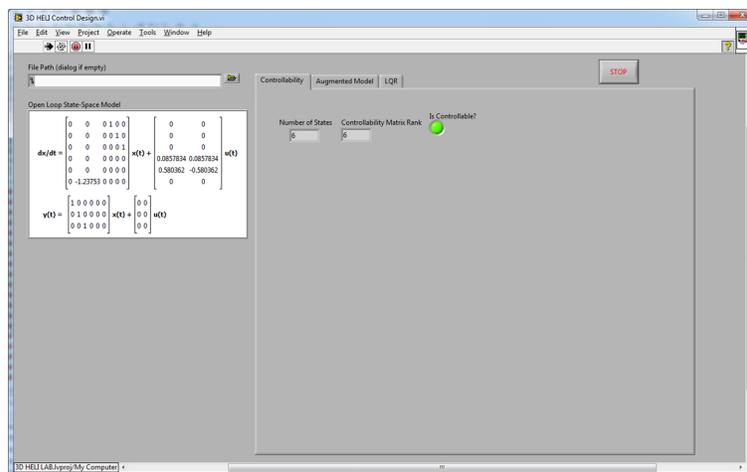

Figure 5.5: State-space model controllability.

4. Click on the Augmented Model tab to generate the linear 3 DOF Helicopter model augmented with integrated states $\int \epsilon dt$ and $\int \lambda dt$, as shown in Figure 5.6.
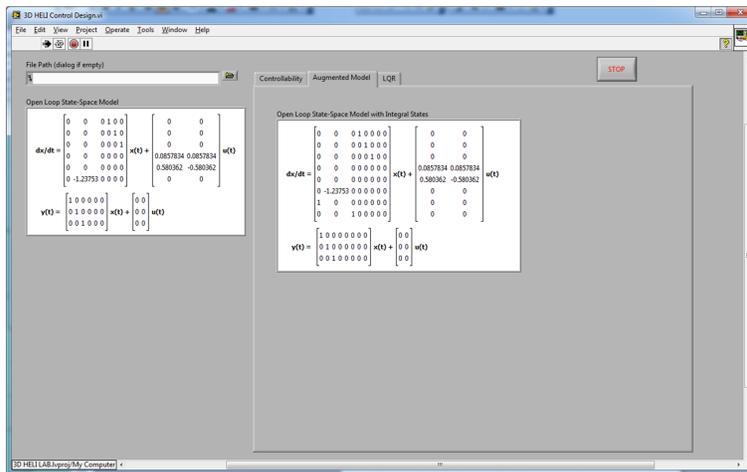
Figure 5.6: Linear model augmented with integrated states.

5. Click on the LQR tab and verify that the generated feedback gain K will stabilize the system, as shown in Figure 5.7. You can change the Q and R matrices to adjust the weighting parameters of the LQR controller design.
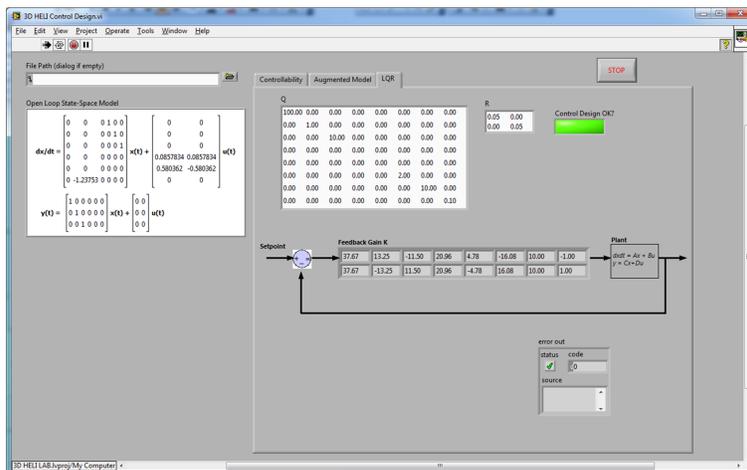

Figure 5.7: LQR controller design.

6. Record the LQR feedback gain K for use in the simulation and implementation of the controller in the next sections. You can return to this VI to regenerate new feedback gains after testing the performance of the closed-loop controller.

5.3.1 Closed-Loop Simulation Procedure

Follow these steps to simulate the performance of the closed-loop LQR controller with the 3 DOF Helicopter nonlinear model:

1. Open the *3D HELI Control Simulation* VI (shown in Figure 5.8), which is used to simulate the closed-loop controller with the full 3 DOF Helicopter nonlinear model.
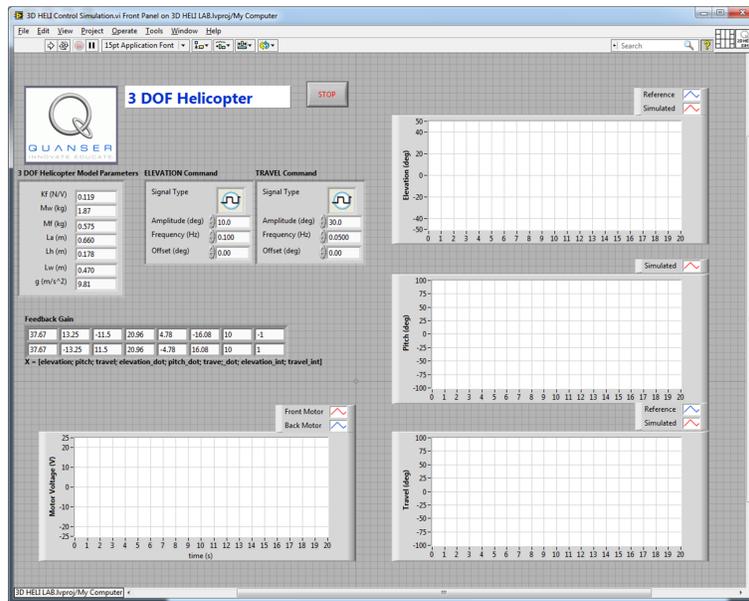
Figure 5.8: Simulation of the feedback controller using the 3 DOF Helicopter nonlinear model.

2. Input the LQR feedback gain K into the front panel of the *3D HELI Control Simulation* VI shown in Figure 5.9.
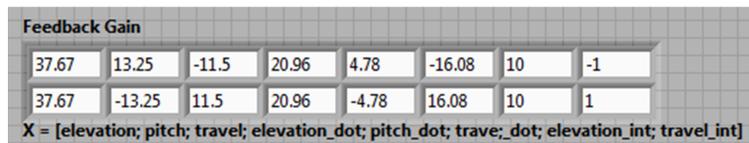

Figure 5.9: Input the feedback gain into the *3D HELI Control Simulation* VI.

3. Check that the 3 DOF Helicopter model parameters are entered correctly on the VI front panel.

4. Run the *3D HELI Control Simulation* VI.

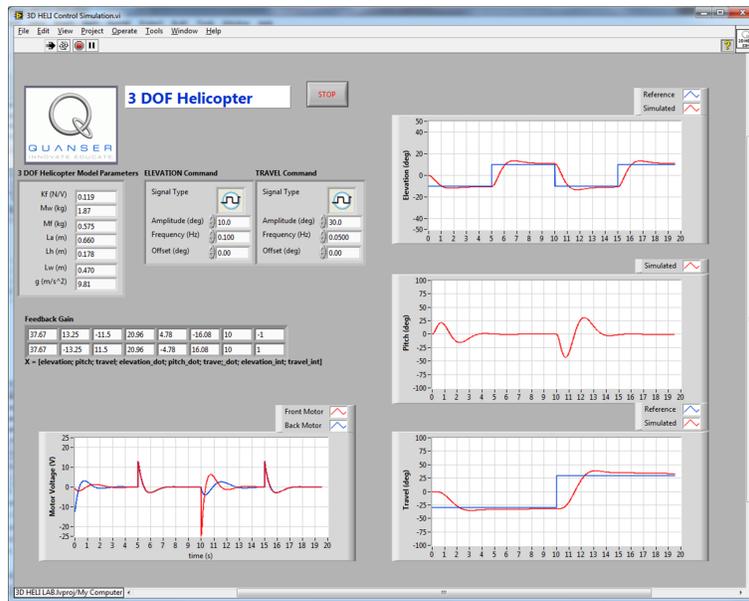5. Observe the closed-loop simulation response as shown in Figure 5.10.

Figure 5.10: Simulation response for the 3 DOF Helicopter .

6. The command set points for the elevation and travel angles can be changed with the ELEVATION Command and TRAVEL Command front panel controls. Adjust the commands and observe the tracking performance of the simulated response.

5.4 Closed-Loop Position Control Implementation

5.4.1 Objectives

The objectives of running the 3 DOF Helicopter in closed-loop are to:

• Investigate the closed-loop performance of the LQR+I controller running on the actual 3 DOF Helicopter plant.

5.4.2 Procedure

Follow this procedure to run the LQR+I controller on the actual helicopter plant:

1. Load LabVIEW™.

2. Open the LabVIEW project called *3D HELI LAB.lvproj*.

3. Open the *3D HELI Control* VI that is used to run the 3 DOF Helicopter experiment, shown in Figure 5.11.
   Note: National Instruments cRIO users should open the *3D HELI Control (cRIO)* VI instead of *3D HELI Control*.
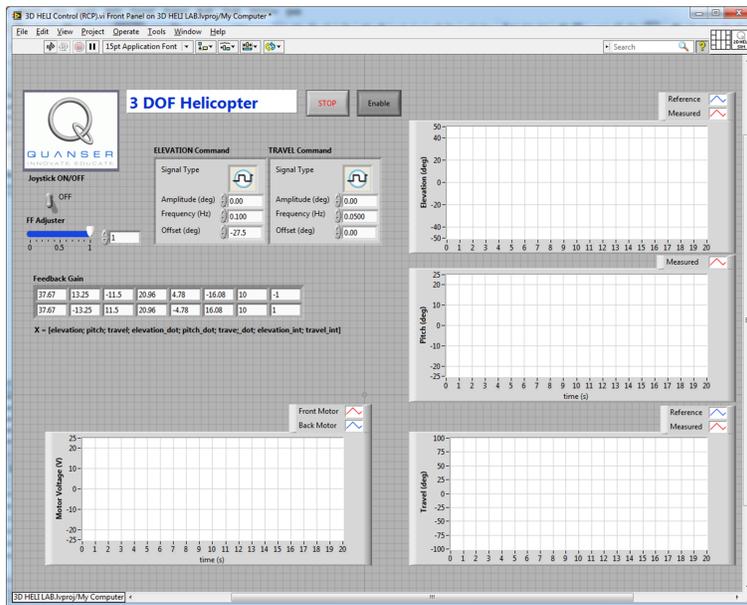
Figure 5.11: LabVIEW VI used to run the closed-loop controller on the 3 DOF Helicopter .

4. Using the feedback gain K generated from Section 4.2, input the feedback gain K into the front panel of the VI.

5. Open the *3D HELI Control* VI (or the *3D HELI Control (cRIO)* VI for cRIO users) block diagram (shortcut CTRL+E) shown in Figure 5.12. It contains the RCP controls that interface with the hardware of the actual plant. The CL HIL Write control outputs the voltages computed by the controller to the DAQ board and the CL HIL Read control reads the encoder measurements.
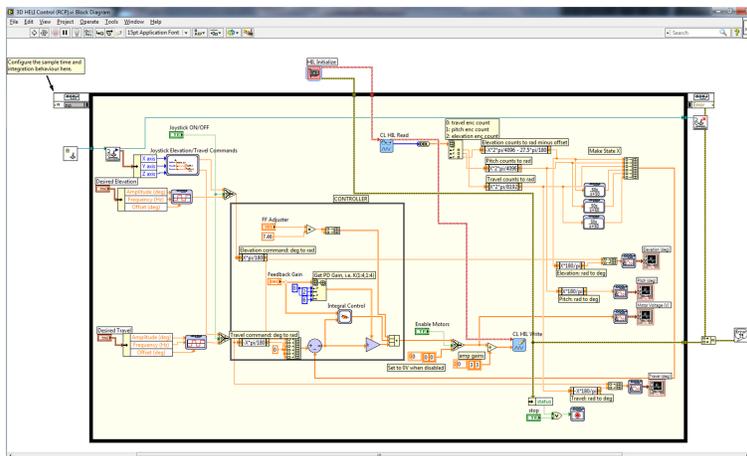

Figure 5.12: Block diagram used to run the closed-loop controller on the 3 DOF Helicopter .

6. **Configure DAQ:** Double-click on the HIL Initialize control and ensure it is configured for the DAQ device that is installed in your system. In this VI, the default is setup for the Quanser Q8-USB hardware-in-the-loop board.

7. Ensure the helicopter has been setup and all the connections have been made as instructed in the 3 DOF Helicopter user manual.

8. Turn ON the amplifier. For the VoltPAQ-X2, the green LED on the amplifier should be lit.

9. In the VI front panel make sure the Joystick ON/OFF switch is set to the OFF position.

10. Run the *3D HELI Control* VI (or the *3D HELI Control (cRIO)* VI for cRIO users). You should now hear the propellers running.

11. In the *ELEVATION Command* section, slowly ramp up the *Offset* value to 0 degrees to bring the helicopter up to horizontal.

12. Set the *ELEVATION Command Amplitude* to 10 degrees and see the response in Figure 5.13, which depicts the typical measured response under a step elevation angle. The measured response is the red line and the reference is the blue line.
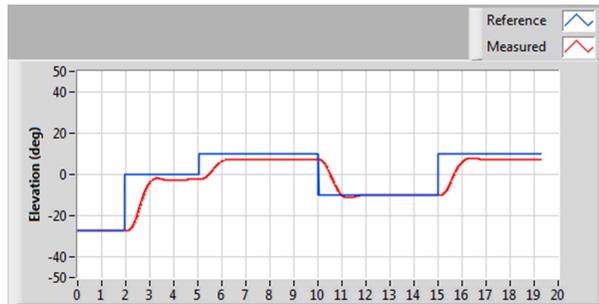

Figure 5.13: Closed-loop LQR response under elevation reference step.

13. Inside the *ELEVATION Command* front panel control set the *Amplitude (deg)* to 0 and inside the *TRAVEL Command* front panel control set the *Amplitude (deg)* to 20. The helicopter should track the desired travel angle.

14. Figure 5.14 depicts the typical measured travel response given a desired step travel angle.
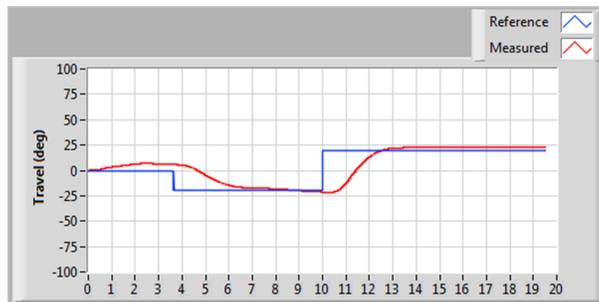

Figure 5.14: Closed-loop LQR response under travel reference step.

15. Alternatively, the desired angle can be generated using the joystick. To use the joystick, set the *Joystick ON/OFF* switch shown in Figure 5.11 to the ON position. The rate at which the desired angle increases or decreases given a joystick position can be changed using the *Rate Command* knob. When starting, set the *Rate Command* knob on the joystick to the midpoint position.

Note: For users of the National Instruments cRIO, the *Joystick_Read* VI must be used in conjunction with the *3D HELI Control (cRIO)* VI. The cRIO does not provide joystick support, so to overcome this the *Joystick_Read* VI is executed on the desktop PC. Upon opening the *Joystick_Read* VI, the user must enter the IP address of the cRIO in the *cRIO IP Address* field of the front panel. The VI must then be started prior to starting *3D HELI Control (cRIO)*.

⚠ **Caution: Do not switch the joystick ON when the controller is running.** Set the joystick switch before starting the VI if the joystick is to be used.

16. Gradually bring the helicopter back to its starting position.

17. Click on the *STOP* button on the VI front panel to stop running the system.

18. Power off the amplifier.

6 TECHNICAL SUPPORT

To obtain support from Quanser, go to `http://www.quanser.com/` and click on the Tech Support link. Fill in the form with all the requested software and hardware information as well as a description of the problem encountered. Also, make sure your e-mail address and telephone number are included. Submit the form and a technical support person will contact you.

REFERENCES

[1] Quanser Inc. *3 DOF Helicopter User Manual*, 2011.

Aerospace plants for teaching and research

▶ **3 DOF Helicopter**

▶ **2 DOF Helicopter**
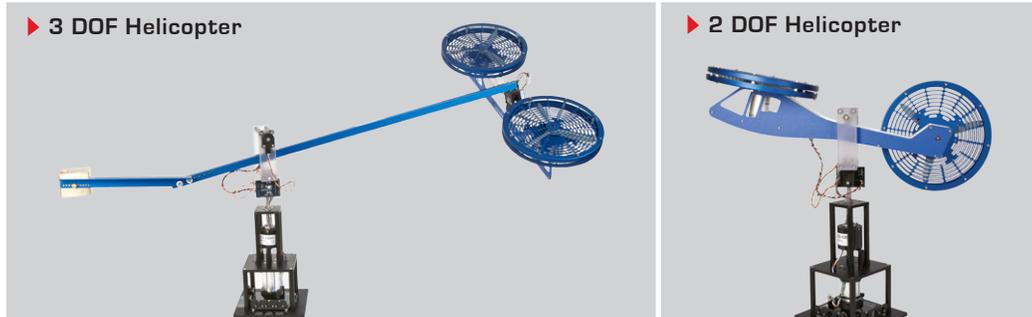
These plants are ideal for intermediate or advanced level teaching. They are also suitable for research relating to traditional or modern control applications of aerospace engineering. For more information please contact info@quanser.com.

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