

# STUDENT WORKBOOK

# Gyro/Stable Platform Experiment for LabVIEW™ Users

Standardized for ABET\* Evaluation Criteria

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# 1 INTRODUCTION

The objective of this experiment is to design a controller that maintains the direction of the gyroscope module while the top base plate is rotated relative to the bottom base plate. While the disk spins, the SRV02 is used to apply the correct amount of counter torque and maintain the gyroscope heading in the event of disturbances (i.e., rotation of the bottom support plate).

Gyroscopes are used in many different devices, e.g., airplanes, large marine ships, submarines, and satellites.

### **Topics Covered**

- · Modeling the system from first principles.
- · Design a PID-based controller.
- Implement the designed controller on the device. Test if the gyroscope module maintains its headings when a
  disturbance is added.

### **Prerequisites**

In order to successfully carry out this laboratory, the user should be familiar with the following:

- · Transfer function fundamentals.
- Basics of LabVIEW™.
- · LabVIEW Integration lab detailed in Appendix A in the SRV02 Workbook [2].

# 2 BACKGROUND

### 2.1 Modeling

### 2.1.1 Servo Model

The Servo Base Unit (SRV02) open-loop transfer function is given by

$$P(s) = \frac{\Theta_l(s)}{V_m(s)} = \frac{K}{s(\tau s + 1)}$$
 (2.1)

where  $\Theta_l(s) = \mathcal{L}[\theta_l(t)]$  is the load gear position and  $V_m(s) = \mathcal{L}[v_m(t)]$  is the applied motor voltage. The system steady-state gain and time constant are given by:

$$K = 1.53 \text{ rad/s/V},$$

and

$$\tau = 0.0486$$
 s.

**Note**: The model parameters, K and  $\tau$ , were computed for the SRV02 with the GYRO-E module mounted. If desired, you can conduct an experiment to find more precise values of K and  $\tau$  for your particular servo. See *SRV02 Modeling* laboratory in [2] for more information.

### 2.1.2 Gyroscope Gain

In order to derive a model of the system, an understanding of gyroscopic principles is required. For a detailed derivation of the dynamic equations, see the textbook references [1], [4], [5] given in the *References* section.

Consider the simplified model shown in Figure 2.1. The inertial disc, or flywheel, spins at a relatively constant velocity,  $\omega_f$ . When the base rotates at a speed of  $\omega_b$ , the resulting gyroscopic torque about the sensitive axis is

$$\tau_g = \omega_b L_f \tag{2.2}$$

where

$$L_f = J_f \omega_f$$

is is the angular momentum of the flywheel and  $J_f$  is its moment of inertia. The springs mounted on the gyroscope counteract the gyroscopic torque,  $\tau_q$ , by the following amount

$$\tau_s = K_r \alpha \tag{2.3}$$

where  $K_r$  is the *rotational* stiffness of the springs.

Given that the spring torque equals the gyroscopic torque,  $\tau_s = \tau_g$ , we can equate equations 2.2 and 2.3 to obtain the expression

$$K_r \alpha = \omega_b J_f \omega_f. \tag{2.4}$$

The base speed is proportional to the deflection angle through the gain  $G_a$ ,

$$\omega_b = G_q \alpha. \tag{2.5}$$

By examining 2.4 and 2.5, we find that the gyroscopic sensitivity gain is given by

$$G_g = \frac{\omega_b}{\alpha} = \frac{K_r}{J_f \omega_f}.$$
 (2.6)



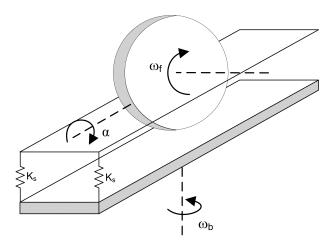


Figure 2.1: Simplified rotary gyroscope model.

Thus the deflection at the gyroscope sensitive axis is directly proportional to the speed of rotational speed of the base (in the steady state). This means that the deflection angle,  $\alpha$ , can be used to measure the rotation of the platform relative to the base *without* a direct measurement. **Note:** the dynamics in the sensitive axis are ignored and a more complete model would include these dynamics as  $\alpha(s)/\omega_b(s)$ .

#### 2.1.3 Joint Stiffness

The two springs are attached as shown in Figure 2.2. The stiffness at the axis of rotation is derived in the following fashion. Assume the springs have a spring constant  $K_s$  and an un-stretched length  $L_u$ . The length of the springs at the normal position, i.e.,  $\alpha=0$ , is given by L. If the axis is rotated by an angle  $\alpha$ , then the two forces about the sensitive axis are given by (for small  $\alpha$ )

$$F_1 = K_s \Delta L_1 = K_s (L - L_u - \alpha R)$$

and

$$F_2 = K_s \Delta L_2 = (L - L_u + \alpha R).$$

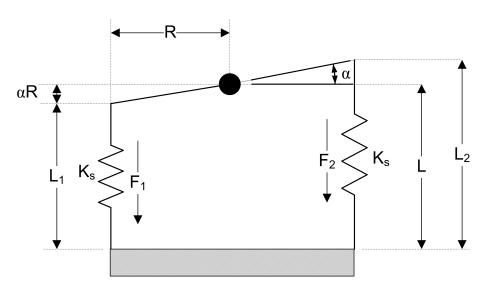


Figure 2.2: Forces acting on springs.

The spring torque about the pivot due to the two forces is

$$\tau_s = R(F_2 - F_1) = 2R^2 K_s \alpha.$$

$$K_r = \frac{\tau_s}{\alpha} = 2R^2 K_s. \tag{2.7}$$

### 2.2 Control Design

### 2.2.1 Desired Position Control Response

The block diagram shown in Figure 2.3 is a general unity feedback system with compensator (controller) C(s) and a transfer function representing the plant, P(s). The measured output, Y(s), is supposed to track the reference signal R(s) and the tracking has to match to certain desired specifications.

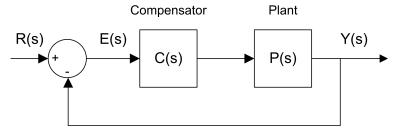


Figure 2.3: Unity feedback system.

The output of this system can be written as:

$$Y(s) = C(s) P(s) (R(s) - Y(s))$$

By solving for Y(s), we can find the closed-loop transfer function:

$$\frac{Y(s)}{R(s)} = \frac{C(s) P(s)}{1 + C(s) P(s)}$$

When a second order system is placed in series with a proportional compensator in the feedback loop as in Figure 2.3, the resulting closed-loop transfer function can be expressed as:

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta \,\omega_n \, s + \omega_n^2} \tag{2.8}$$

where  $\omega_n$  is the natural frequency and  $\zeta$  is the damping ratio. This is called the *standard second-order* transfer function. Its response properties depends on the values of  $\omega_n$  and  $\zeta$ .

### 2.2.2 Control Specifications

The desired time-domain specifications for stabilizing the gyroscope are:

$$\omega_n = 6\pi \text{ rad/s}$$
 (2.9)

or 3 Hz, and

$$\zeta = 0.7. \tag{2.10}$$

#### 2.2.3 GYRO PD Controller

To stabilize the heading of the gyroscope, we will develop a Proportional-Derivative (PD) controller depicted in Figure 2.4.



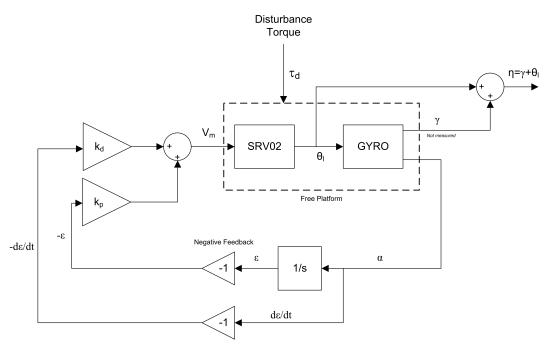


Figure 2.4: Gyroscope PD control block diagram

Assume that the support plate (and servo) rotate relative to the base plate by the angle  $\gamma$  (not measured) and that the gyro module rotates relative to the servo module by the angle  $\theta_l$  (measured), the total rotation of the gyro module relative to the base plate can be expressed by

$$\eta = \gamma + \theta_l. \tag{2.11}$$

We want to design a controller that maintains the gyro heading, i.e., keeps  $\eta=0$ , independent of  $\gamma$  and we can only use the measurement from the gyro sensor,  $\alpha$ . In other terms, we want to stabilize the system such that  $\dot{\eta}\to 0$ . Differentiating Equation 2.11 gives

$$\dot{\eta} = \dot{\gamma} + \dot{\theta}_{I}$$
.

Given that  $\dot{\eta} = \omega_b$  and the gyro gain definition in Equation 2.5, this becomes

$$G_q \alpha = \dot{\gamma} + \dot{\theta}_l.$$

Taking the Laplace and solving for  $\alpha(s)/s$  we have

$$\frac{\alpha(s)}{s} = \frac{1}{G_a}(\gamma(s) + \Theta_l(s)).$$

Introducing the new variable

$$\epsilon(s) = \frac{\alpha(s)}{s},$$

which is the integral of the deflection angle, the gyro transfer function can be changed to the following

$$\epsilon(s) = \frac{1}{G_a} (\gamma(s) + \Theta_l(s)).$$

Add the SRV02 dynamics given in Section 2.1.1 into  $\Theta_l(s)$  to introduce our control variable  $V_m(s)$ 

$$\epsilon(s) = \frac{1}{G_g} \left( \gamma(s) + \frac{K}{s(\tau s + 1)} V_m(s) \right). \tag{2.12}$$

### Adding the PD control

$$V_m(s) = -(k_p + k_d s)\epsilon(s)$$

to 2.12 and solving for  $\epsilon(s)/\gamma(s)$  we obtain the closed-loop transfer function

$$\frac{\epsilon(s)}{\gamma(s)} = \frac{s(\tau s + 1)}{G_g \tau s^2 + (Kk_d + G_g)s + Kk_p}.$$
(2.13)



# 3 PRE-LAB QUESTIONS

1. Find the steady-state speed of the flywheel,  $\omega_f$ , given the motor equation

$$v_{g,m} = i_{g,m}R_{g,m} + k_{g,m}\omega_f \tag{3.1}$$

where  $i_{g,m}$  is the nominal current,  $v_{g,m}$  is the nominal voltage,  $R_{g,m}$  is the motor resistance, and  $k_{g,m}$  is the back-emf constant. The motor parameter values are given in the Gyroscope User Manual [3].

2. Find the value of the gyroscope sensitivy gain,  $G_q$ . The flywheel moment of inertia is

$$J_f = \frac{1}{2} \, m_f r_f^2 = 0.00103 \; ext{N-m-s}^2 / ext{rad}.$$

Note that the inertia unit N-m-s<sup>2</sup>/rad is equivalent to kg-m<sup>2</sup>. Refer to the Gyroscope User Manual for parameter values.

- 3. The closed-loop transfer function was found in 2.13. Find the PD control gains,  $k_p$  and  $k_d$ , in terms of  $\omega_n$  and  $\zeta$ . **Hint:** Remember the standard second order system equation.
- 4. Based on the nominal SRV02 model parameters, K and  $\tau$  given in Section 2.1.1, calculate the control gains needed to satisfy the time-domain response requirements given in Section 2.2.2.

## 4 LAB EXPERIMENTS

### 4.1 Control Implementation

The gyroscopic control developed in Section 2.2 is implemented on the actual system. The goal is to see if the gyro module can maintain its heading when a disturbance is added by the user, i.e., the base plate is rotated.

The *GYRO Control* VI shown in Figure 4.1 is used to run the PD control on the Quanser Rotary Gyroscope system. The VI interfaces with the DC motor and sensors of the system.

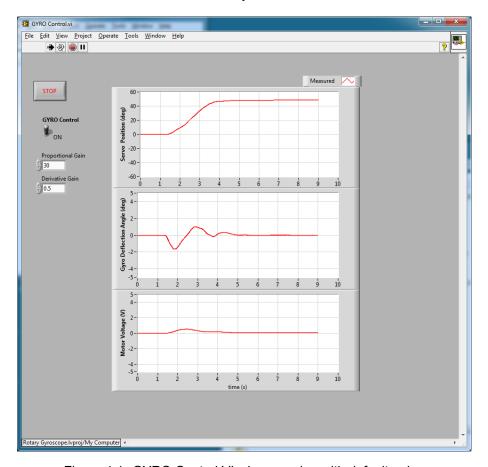


Figure 4.1: GYRO Control VI when running with default gains

**IMPORTANT:** Before you can conduct this experiment, you need to make sure your hardware is setup and that the lab files are configured properly. If they have not been configured already, then go to Section 5 to configure the lab files first.

Follow these steps to run the gyroscope control:

- 1. The amplifier should be turned ON and the disc should be rotating, as discussed in Section 5.
- 2. Make sure the switch on the front panel of the VI is in **upward ON position** to enable the PD control.
- 3. Run the VI by clicking on the white arrow in the top left corner.
- 4. Manually rotate the bottom base plate about 45 degrees (or any other set angle). The GYRO module should be maintaining its heading. Sample scope response are shown in Figure 4.2.



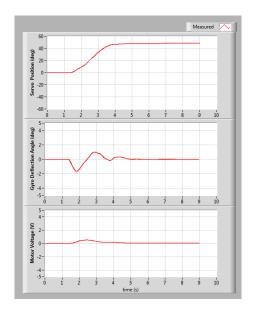


Figure 4.2: Typical Rotary Gyroscope response when PD control is ON

- 5. Click on the *Stop* button on the front panel of the VI to stop the controller once you have obtained a representative response.
- 6. Plot the response of the servo angle, gyro deflection angle, and servo voltage.

**Exporting scopes**: After running the VI, right-click on the scope and go to *Export | Export Simplified Image*. Choose the graphic file type and export it to the clipboard. The response can then be pasted.

- 7. Return the base plate to its original location (i.e., before you rotated it).
- 8. Run the VI again.
- 9. Turn off the PD control by setting the switch to the downward OFF position, i.e., 0 V is applied to the motor.
- 10. Rotate the bottom base plate by the same amount as previously done, e.g., 45 degrees clockwise. Plot the response.
- 11. Examine how the GYRO module responds when you rotate the base plate. Explain the resulting responses when the PD control is ON and OFF. Based on your observations, explain what the PD control is actually doing and how it relates to gyroscopes.

# 5 SYSTEM REQUIREMENTS

#### **Required Software**

Make sure LabVIEW™ is installed with the following required add-ons:

- LabVIEW™
- 2. NI-DAQmx
- 3. NI LabVIEW™ Control Design and Simulation Module
- 4. Quanser Rapid Control Prototyping Toolkit®

**Note**: Make sure the Quanser Rapid Control Prototyping (RCP) Toolkit is installed after LabVIEW. See the RCP Toolkit Quick Start Guide for more information.

### **Required Hardware**

- Data acquisition (DAQ) device that is compatible with Quanser Rapid Control Prototyping Toolkit<sup>®</sup>. This includes Quanser DAQ boards such as Q2-USB, Q8-USB, QPID, and QPIDe and some National Instruments DAQ devices.
- · Quanser SRV02-ET rotary servo.
- · Quanser Rotary Gyroscope (attached to SRV02).
- Quanser VoltPAQ-X1 power amplifier, or equivalent.

#### **Before Starting Lab**

Before you begin this laboratory make sure:

- LabVIEW™ is installed on your PC.
- DAQ device has been successfully tested (e.g., using the test software in the Quick Start Guide).
- Rotary Gyroscope and amplifier are connected to your DAQ board as described Reference [3].

### 5.1 Overview of Files

File Name	Description
Gyroscope User Manual.pdf	This manual describes the hardware of the GYRO-E sys-
	tem and explains how to setup and wire the system for the
	experiments.
Gyroscope Workbook (Student).pdf	This laboratory guide contains pre-lab questions and lab
	experiments demonstrating how to design and implement
	controllers for both the joint space and work space on the
	GYRO-E plant using LabVIEW™ .
Rotary Gyroscope.lvproj	LabVIEW Project file that contains all the VIs necessary to
	run the system.
GYRO Control.vi	VI that implements the PD controller on the GYRO-E sys-
	tem using LabVIEW™ .

Table 5.1: Files supplied with the Rotary Gyroscope



### 5.2 Experiment Setup

Follow these steps before beginning the in-lab procedure outlined in Section 4:

- 1. Setup the Rotary Servo Base Unit, i.e., SRV02, with the Gyroscope module as detailed in the Gyroscope User Manual ([3]).
- Load the LabVIEW™ software.
- 3. Open the LabVIEW Project Rotary Gyroscope.lvproj.
- 4. Open the GYRO Control.vi, shown in Figure 4.1.
- 5. Enter the PD controller gains,  $k_p$  and  $k_d$ , you found in Section 3 in the *Proportional Gain* and *Derivative Gain* controls on the VI front panel.
- Configure DAQ: Ensure the HIL Initialize block is configured for the DAQ device that is installed in your system.
  To do this, go to the block diagram (CTRL-E) and double click on the HIL Initialize Express VI shown in Figure 5.1.

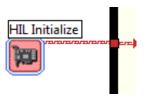


Figure 5.1: HIL Initialize Express VI

7. Under the Main tab, select the data acquisition device that is installed on your system in the *Board type* section. For example, in Figure 5.2 the Q2-USB is chosen.

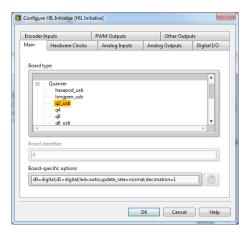


Figure 5.2: Select DAQ board that will be used to control system

8. Turn ON the amplifier (e.g., VoltPAQ-X1). The flywheel on the GYRO-E module should begin spinning. Wait till it reaches its steady-state speed.

## 6 LAB REPORT

For the gyroscope experiment, follow the outline corresponding to that experiment to build the *content* of your report. Also, in Section 6.2 you can find some basic tips for the *format* of your report.

### 6.1 Template for Content (Gyroscope)

#### I. PROCEDURE

- 1. Briefly describe the main goal of the experiment.
- 2. Briefly describe the experiment procedure in Step 6 in Section 4.1.
- 3. Briefly describe the experiment procedure in Step 10 in Section 4.1.

#### II. RESULTS

Do not interpret or analyze the data in this section. Just provide the results.

- 1. Gyroscope control ON response, Step 6 in Section 4.1.
- 2. Gyroscope control OFF response, Step 10 in Section 4.1.

#### III. ANALYSIS

Provide details of your calculations (methods used) for analysis for each of the following:

1. Effect of having the PD control on and off, Step 11 in Section 4.1.

#### IV. CONCLUSIONS

Interpret your results to arrive at logical conclusions for the following:

1. How does this relate to an actual gyroscope system, Step 11 in Section 4.1.



### 6.2 Tips for Report Format

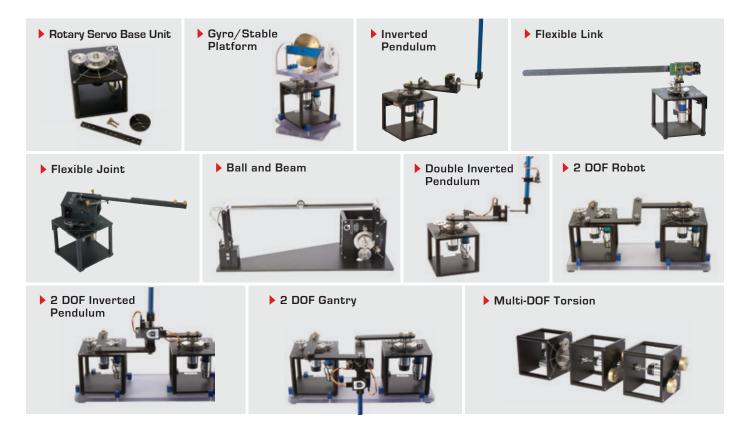
#### **PROFESSIONAL APPEARANCE**

- Has cover page with all necessary details (title, course, student name(s), etc.)
- Each of the required sections is completed (Procedure, Results, Analysis and Conclusions).
- · Typed.
- · All grammar/spelling correct.
- · Report layout is neat.
- · Does not exceed specified maximum page limit, if any.
- · Pages are numbered.
- · Equations are consecutively numbered.
- Figures are numbered, axes have labels, each figure has a descriptive caption.
- Tables are numbered, they include labels, each table has a descriptive caption.
- Data are presented in a useful format (graphs, numerical, table, charts, diagrams).
- · No hand drawn sketches/diagrams.
- · References are cited using correct format.

# REFERENCES

- [1] Robert H. Cannon. Dynamics of Physical Systems. McGraw Hill Book Company, 1967.
- [2] Quanser Inc. SRV02 lab manual. 2011.
- [3] Quanser Inc. SRV02 Gyroscope User Manual, 2012.
- [4] Carl Machover. Basics of Gyroscopes. John F. Rider, 1960.
- [5] Paul H. Savet. Gyroscopes: Theory and Design. McGraw Hill Book Company, 1961.

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