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STUDENT WORKBOOK

Linear Flexible Joint Experiment for MATLAB®/Simulink® Users

Standardized for ABET* Evaluation Criteria

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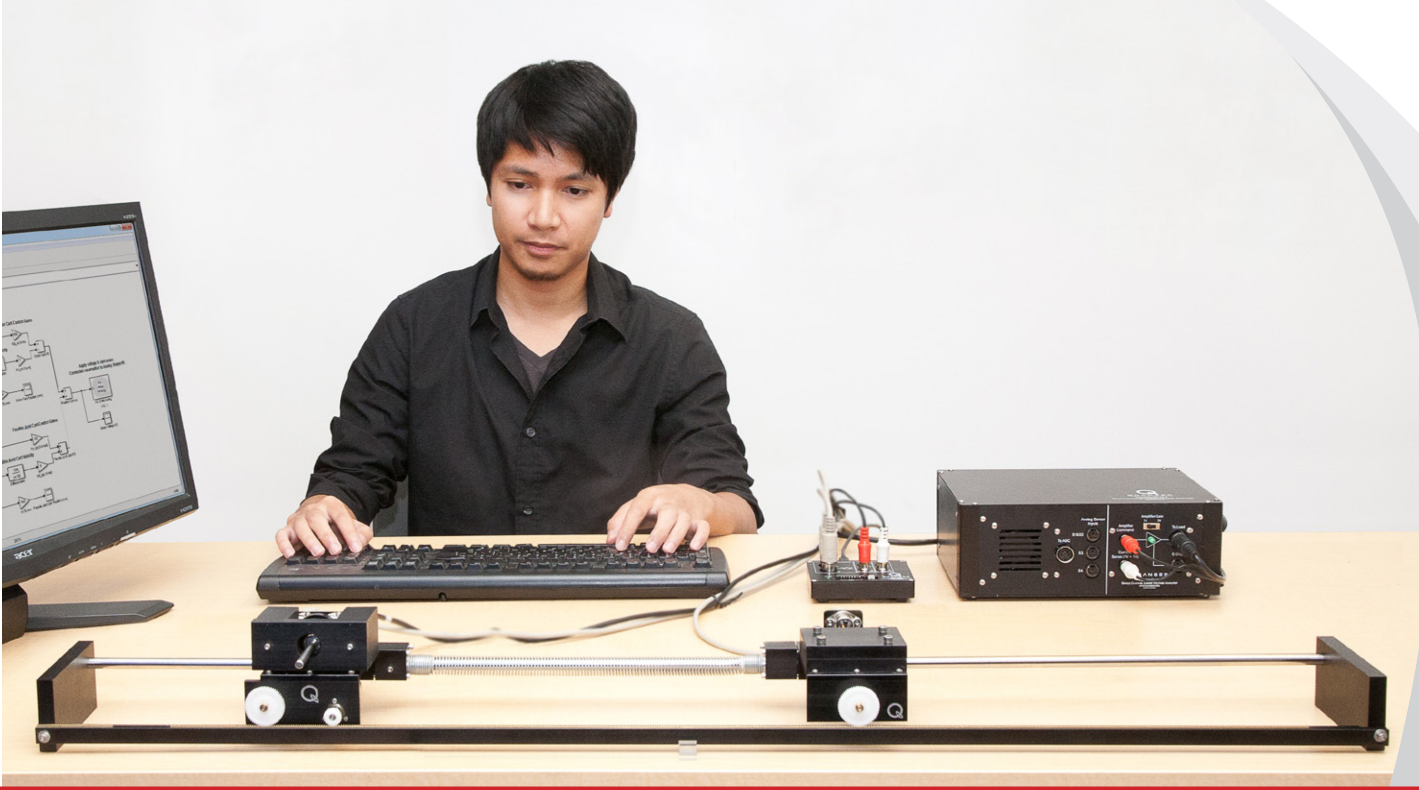

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1 INTRODUCTION

The objective of this laboratory is to develop a feedback system to track the spring-driven cart to a desired position as quickly as possible while minimizing the overshoot and residual vibration. A full-state-feedback controller is designed using the Linear Quadratic Regulator (LQR) methodology.

Topics Covered

- Developing of a mathematical model of the linear flexible joint system using Lagrangian mechanics.
- Obtaining the linear state-space representation of the open-loop system
- Designing a state-feedback controller using the LQR algorithm.
- Simulating the closed-loop system to ensure that the specifications are met
- Implementing the controller on the LFJC-E plant and evaluating its performance

Prerequisites

In order to successfully carry out this laboratory, the user should be familiar with the following:

- The required software and hardware outlined in Section 4.
- State-space modeling fundamentals.
- Some knowledge of state-feedback.
- Basics of QUARC®.
- Laboratory described in the QUARC Integration [2] in order to be familiar using QUARC®with the IP02 Base Unit.

2 MODELING

2.1 Background

2.1.1 Model Convention

The linear flexible joint cart system schematic is shown in Figure 2.1. The positive direction of linear displacement is to the right when facing the cart.

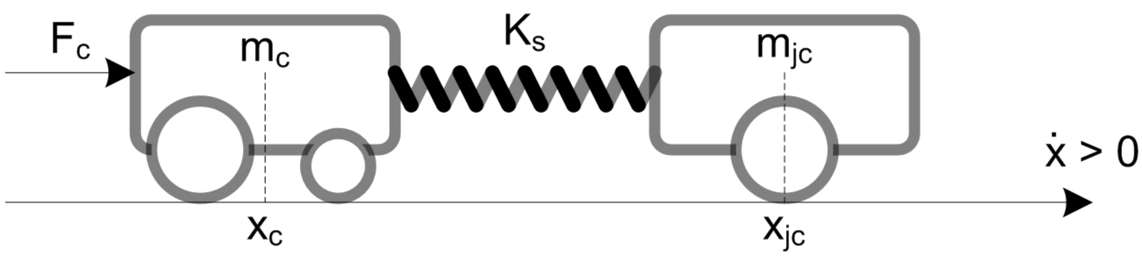


Figure 2.1: Linear Flexible Joint Cart schematic

2.1.2 Equations of Motion

In this section the Lagrange method is used to find the equations of motion of the system. This systematic method is often used for more complicated systems such as robotic manipulators with multiple joints. The equations of motion (EOM) of the system could also be determined using a free body diagram and force analysis.

More specifically, the dynamics equations that describe the motion of the linear cart and flexible joint cart with respect to the motor voltage will be obtained. By neglecting the Coulomb (static) friction of the LFJC-E system, we can formulate a linear representation of the pure spring-mass-damper system using the Euler-Lagrange equation:

$$\frac{\partial^2 L}{\partial t \partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i$$

The variables q_i are called *generalized coordinates*. For this system let

$$q(t)^\top = [x_c(t)\ x_{jc}(t)] \tag{2.1}$$

where $x_c(t)$ is the cart position and $x_{jc}(t)$ is the LFJC-E cart position. The corresponding velocities are

$$\dot{q}(t)^\top = \left[\frac{\partial x_c(t)}{\partial t}\ \frac{\partial x_{jc}(t)}{\partial t}\right]$$

Note: The dot convention for the time derivative will be used throughout this document, i.e., $\dot{x_c} = \frac{dx_c}{dt}$. The time variable t will also be dropped for x_c and x_{jc}, i.e., $x_c = x_c(t)$ and $x_{jc} = x_{jc}(t)$.

With the generalized coordinates defined, the Euler-Lagrange equations for the linear pendulum gantry are:

$$\begin{aligned}
\frac{\partial^2 L}{\partial t \partial \dot{x_c}} - \frac{\partial L}{\partial x_c} &= Q_{x_c} \\
\frac{\partial^2 L}{\partial t \partial \dot{x_{jc}}} - \frac{\partial L}{\partial x_{jc}} &= Q_{x_{jc}}
\end{aligned}$$

The Lagrangian of the system, L, is described:

$$L = T - V$$

where T is the total kinetic energy of the system and V is the total potential energy of the system. Therefore, the Lagrangian represents the difference between the kinetic and potential energy of the system.

The potential energy of a system is fundamentally a measure of the energy that a system has due to some kind of work that was performed to move it from a lower energy configuration. In the case of the LFJC-E since the vertical motions of the carts are fixed, the system's total potential energy is due to the elastic potential energy of the linear spring. Therefore, the total potential energy of system is:

$$V_T = \frac{1}{2} K_s (x_{jc} - x_c)^2 \tag{2.2}$$

where K_s is the joint spring constant.

The total kinetic energy of the system is the sum of the translational and rotational kinetic energies arising from the motion of the IP02 cart, and the translational kinetic energy of the LFJC-E cart. The translational kinetic energy of the IP02 cart, T_{ct}, can be expressed as:

$$T_{ct} = \frac{1}{2} m_c \dot{x_c}^2$$

and the rotational energy due to the DC motor, T_{cr}, is:

$$T_{cr} = \frac{1}{2} \frac{\eta_g J_m K_g^2 \dot{x_c}^2}{r_{mp}^2}$$

where the corresponding IP02 parameters are defined in the IP02 User Manual [3]. Finally, the translational kinetic energy of the LFJC-E is:

$$T_{jct} = \frac{1}{2} m_{jc} \dot{x_{jc}}^2$$

If we combine the kinetic energy equations together, the resultant total kinetic energy of the system is:

$$T_T = \frac{1}{2} J_{eq} \dot{x_c}^2 + \frac{1}{2} m_{jc} \dot{x}_{jc}^2 \tag{2.3}$$

where

$$J_{eq} = M + \frac{\eta_g K_g^2 J_m}{r_{mp}^2}$$

The generalized forces, Q_{x_c} and $Q_{x_{jc}}$, are used to describe the non-conservative forces applied to a system with respect to the generalized coordinates. The generalized forces acting on the linear carts are:

$$Q_{x_c} = F_c - B_{eq} \dot{x_c} \tag{2.4}$$

and acting on the pendulum is

$$Q_{x_{jc}} = -B_{eq_{jc}} \dot{x}_{jc}. \tag{2.5}$$

where B_{eq} and $B_{eq_{jc}}$ are the equivalent viscous damping of the IP02 and LFJC-E carts respectively.

Note: The nonlinear Coulomb friction applied to the linear carts have been neglected in the dynamic model.

By substituting the total kinetic and potential energy of the system shown in Equation 2.2 and Equation 2.3, and the generalized forces into the Euler-Lagrange formulation, the equations of motion are:

$$J_{eq} \ddot{x_c} + K_s x_c - K_s x_2 = F_c - B_{eq} \dot{x_c} \tag{2.6}$$

and

$$m_{jc}\ddot{x}_{jc} - K_s x_c + K_s x_{jc} = -B_{eq_{jc}}\dot{x}_{jc}. \tag{2.7}$$

Solving for the acceleration terms in the equations of motion yields:

$$\ddot{x}_c = \frac{1}{J_{eq}}\Big(- K_s x_c + K_s x_{jc} - B_{eq}\dot{x}_c + F_c\Big) \tag{2.8}$$

and

$$\ddot{x}_{jc} = \frac{1}{m_{jc}}\Big(K_s x_c - K_s x_{jc} - B_{eq_{jc}}\dot{x}_{jc}\Big). \tag{2.9}$$

The linear force applied to the cart, F_c, is generated by the servo motor and described by the equation:

$$F_c = \frac{\eta_g K_g K_t}{R_m r_{mp}}\left(-\frac{K_g K_m \dot{x}_c}{r_{mp}} + \eta_m V_m\right) \tag{2.10}$$

where the servo motor parameters are defined in the IP02 User Manual [3].

The equations of motion for the IP02 and LFJC-E derived in Equation 2.8 and Equation 2.9 match the typical form of an EOM for a single body:

$$J\ddot{x} + b\dot{x} + g(x) = F_1 \tag{2.11}$$

where x is a linear position, J is the moment of inertia, b is the damping, $g(x)$ is the gravitational function, and F_1 is the applied force (scalar value).

For a generalized coordinate vector q, this can be generalized into the matrix form

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = F \tag{2.12}$$

where D is the inertial matrix, C is the damping matrix, $g(q)$ is the gravitational vector, and F is the applied force vector.

The equations of motion given in 2.6 and 2.7 can be placed into this matrix format.

2.1.3 State-Space Model

The linear state-space equations are

$$\dot{x} = Ax + Bu \tag{2.13}$$

and

$$y = Cx + Du$$

where x is the state, u is the control input, A, B, C, and D are state-space matrices. For the linear pendulum gantry system, the state and output are defined

$$x^\top = [x_c \; x_{jc} \; \dot{x}_c \; \dot{x}_{jc}] \tag{2.14}$$

and

$$y^\top = [x_c \; x_{jc}]. \tag{2.15}$$

In the output equation, only the position of the IP02 and LFJC-E carts are being measured. Based on this, the C and D matrices in the output equation are

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{2.16}$$

and

$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (2.17)$$

The velocities of the carts can be computed in the digital controller by taking the derivative and filtering the result though a high-pass filter.

2.1.4 Free-Oscillation of a Second Order System

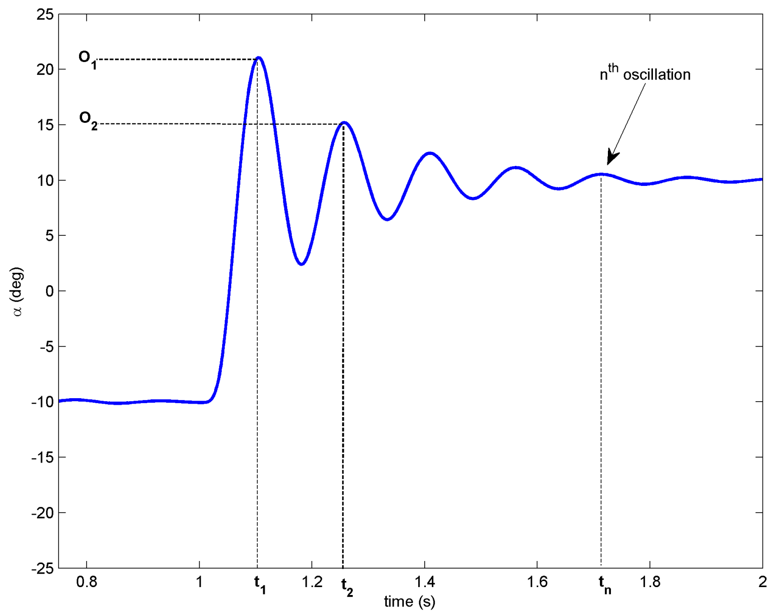


Figure 2.2: Free Oscillation Response

Finding the Natural Frequency

The period of the oscillations in a system response can be found using the equation

$$T_{osc} = \frac{t_{n+1} - t_1}{n} \quad (2.18)$$

where t_n is the time of the n^{th} oscillation, t_1 is the time of the first peak, and n is the number of oscillations considered. From this, the damped natural frequency (in radians per second) is

$$\omega_d = \frac{2\pi}{T_{osc}} \quad (2.19)$$

and the undamped natural frequency is

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}}. \quad (2.20)$$

Finding the Damping Ratio

The damping ratio of a second-order system can be found from its response. For a typical second-order underdamped system, the subsidence ratio (i.e., decrement ratio) is defined as

$$\delta = \frac{1}{n} \ln \frac{O_1}{O_{n+1}} \quad (2.21)$$

where O_1 is the peak of the first oscillation and O_n is the peak of the n^{th} oscillation. Note that $O_1 > O_n$, as this is a decaying response.

The damping ratio is defined

$$\zeta = \frac{1}{\sqrt{1 + \frac{2\pi}{\delta}^2}} \tag{2.22}$$

2.2 Pre-Lab Questions

1. Find the linear state-space model of the flexible joint cart system.

2. Consider the linear spring-damper-mass portion of the system having parameters K_s, $B_{eq_{jc}}$, and m_{jc} respectively. If the displacement of the mass center of gravity is x_{jc}, determine the ordinary differential equation (ODE) that describes the free-oscillatory motion of the mass. **Hint:** Refer to the generalized second order single body EOM shown in Equation 2.11.

3. Taking the Laplace transform of the ODE describing the free motion of the mass found in Question 2, express the coefficients of the ODE in terms of the generic damping and natural frequency parameters, ζ and ω_n, found in the second-order characteristic equation:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 \tag{2.23}$$

Note: For an underdamped system where $\zeta < 1$, the roots of the characteristic equation are the complex conjugate roots:

$$s_1 = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$$

and

$$s_2 = -\zeta\omega_n - j\omega_n\sqrt{1-\zeta^2}$$

2.3 In-Lab Exercises

The goal of this laboratory is to explore the state-space model of the linear flexible joint cart. You will begin by determining the damping and spring constant of your particular LFJC-E. You will then conduct an experiment to validate the model by comparing the response of the model to the response of the actual system.

2.3.1 Impulse Response Test

Experimental Setup

The *q_mdl_params.mdl* Simulink diagram shown in Figure 2.3 is used to measure the impulse response of the LFJC-E load cart. The QUARC blocks are used to interface with the load cart encoder. For more information about QUARC, see Reference [2].

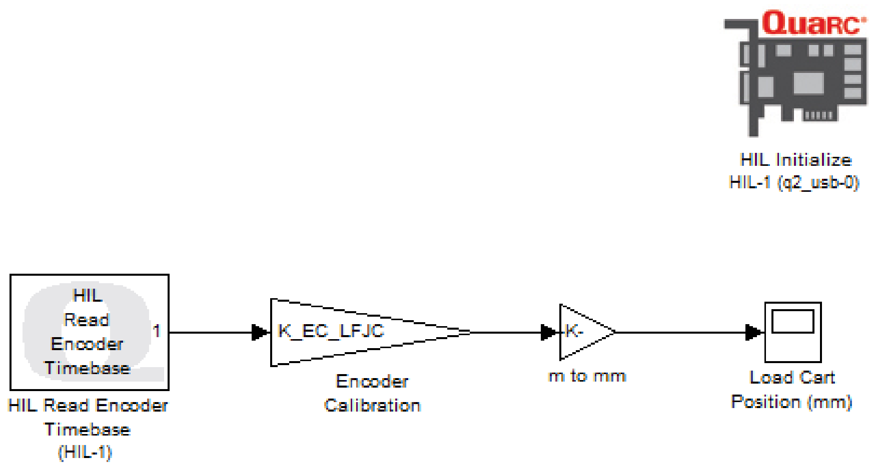


Figure 2.3: q_mdl_params Simulink diagram used to measure encoder response

Note: Before you can conduct these experiments, you need to make sure that the lab files are configured according to your IP02 and LFJC-E setup. If they have not been configured already, then go to Section 4.2 to configure the lab files before you begin.

1. Clamp the IP02 cart to the track end-plate so that it cannot move.
2. Open the *q_mdl_params.mdl* Simulink model, shown in Figure 2.3.
3. Go to QUARC | Build to build the controller.
4. Select QUARC | Start to begin running the
5. With the IP02 cart immobile, you should be able to fully compress the linear spring by pushing on the LFJC-E load cart. When the cart is released, the spring restoring force will apply an impulse force and initiate free oscillations of the LFJC-E spring-damper-mass system.
6. Release the load cart and observe the oscillations on the *Load Cart Position (mm)* scope. When the oscillations have decayed significantly, click on the Stop button in the Simulink diagram toolbar (or select QUARC | Stop from the menu) to stop running the code.
7. Your response should resemble the scope shown in Figure 2.4. Plot the response of the load cart in a MATLAB figure and attach it to your report.

Note: When the simulation stops, the last 10 seconds of data is automatically saved in the MATLAB workspace to the variables *data_load_cart*. The time is stored in the *data_load_cart(:,1)* vector, the measured position is saved in the *data_load_cart(;,2)* arrays.

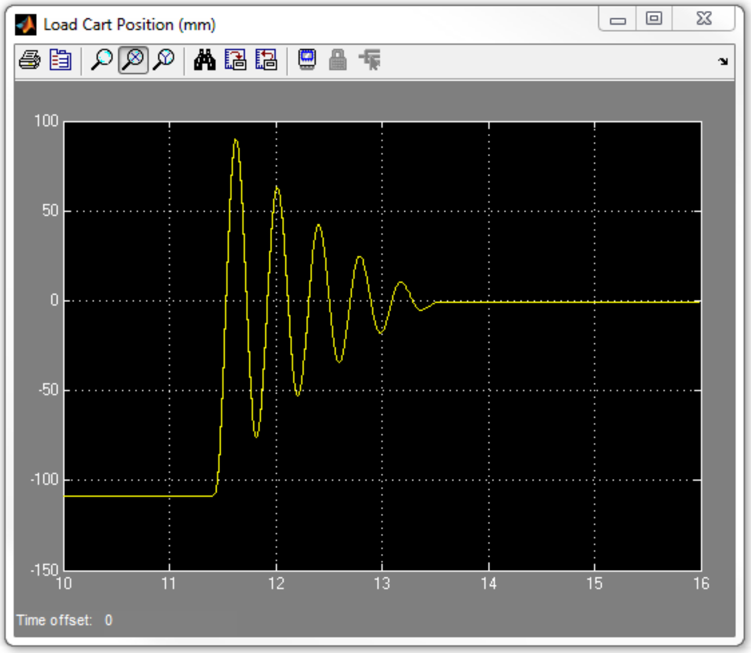


Figure 2.4: LFJC-E load cart impulse response

8. Using the response from Question 7 and the subsidence ratio Equation 2.21, calculate the experimental damping ratio, ζ.

9. Using the measured damping ratio, we can now calculate the experimental natural frequency of the system. Using the response from Question 7 and the period of the oscillations expressed in Equation 2.18, calculate the experimental natural frequency, ω_n.

10. Using the measured values of ζ and ω_n, and the relationship you derived in Section 2.2, determine the exact values of the equivalent spring stiffness, K_s, and equivalent damping coefficient, $B_{eq_{jc}}$, that correspond to your particular LFJC-E system. How do your values compare to the specification values of $K_s = 142$ N/m and $B_{eq_{jc}} = 2.2$ N-s/m, given in the LFJC-E User Manual [5]?

    Note: The mass of the LFJC-E load cart with the spring assembly is 0.293 kg, and each additional weight is 0.125 kg. Therefore, the total mass of the LFJC-E load cart with two weights, m_{jc}, is 0.543 kg.

2.3.2 Model Analysis

1. Ensure that you have configured the lab files for your IP02 and LFJC-E setup. If they have not been configured already, then go to Section 4.2 to configure the lab files before you begin.

2. Run the *setup_lfjc* script. The LFJC-E and IP02 model parameters are automatically loaded using the *config_ip02.m* and *config_lfjc.m* functions. It then calls the *LFJC_ABCD_eqns_student.m* script to load the model in the MATLAB workspace.

3. Open the *LFJC_ABCD_eqns_student.m* script. The script should contain the following code:

```
A = eye(4,4);
B = [0;0;1;0];
C = eye(2,4);
D = zeros(2,1);
```

```
%Actuator Dynamics
A(3,3) = A(3,3) - B(3)*eta_g*Kg^2*eta_m*Kt*Km/r_mp^2/Rm;
A(4,3) = A(4,3) - B(4)*eta_g*Kg^2*eta_m*Kt*Km/r_mp^2/Rm;
B = eta_g*Kg*eta_m*Kt/r_mp/Rm*B;
```

The representative C and D matrices have already been included. You need to enter the state-space matrices A and B that you found in Section 2.2. The actuator dynamics have been added to convert your state-space matrices to be in terms of voltage. Recall that the input of the state-space model you found in Section 2.2 is the force acting on the IP02 cart. However, *we do not control force directly - we control the servo input voltage*. The above code uses the voltage-torque relationship given in Equation 2.10 in Section 2.1.2 to transform torque to voltage.

Note: In MATLAB, the model parameters are denoted as Jeq, M_jc, Beq_jc, Ks, and Beq.

4. Run the *LFJC_ABCD_eqns_student.m* script to load the state-space matrices in the MATLAB workspace. Show the numerical matrices that are displayed in the MATLAB prompt.

5. Using MATLAB, find the open-loop poles of the system.

Before ending this lab... To do the pre-lab questions in Section 3.3, you need the A and B matrices (numerical representation) and the open-loop poles. Make sure you record these.

2.3.3 Model Validation

Experimental Setup

The *q_mdl_lfjce.mdl* Simulink diagram shown in Figure 2.5 is used to confirm that the actual system hardware matches the state-space model derived in Section 2.1.3. The QUARC blocks are used to interface with encoders and DC motor of the system. For more information about QUARC, see Reference [2]. This model applies a voltage to the DC motor, and outputs the IP02 and LFJC-E cart positions, as well as the spring deflection.

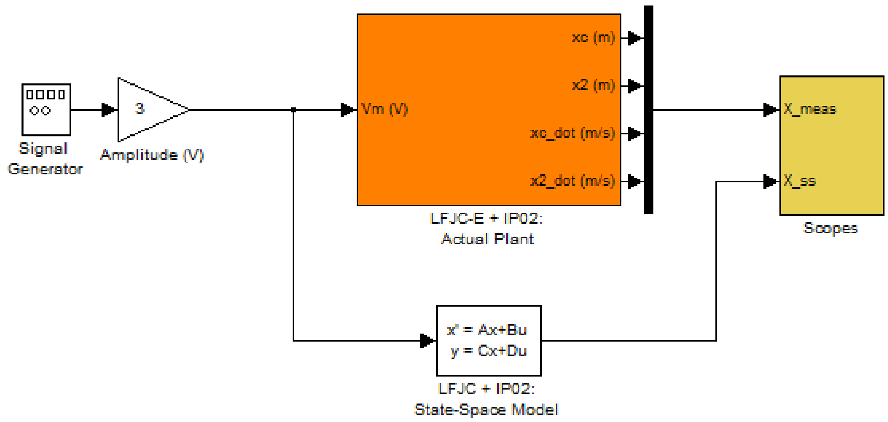
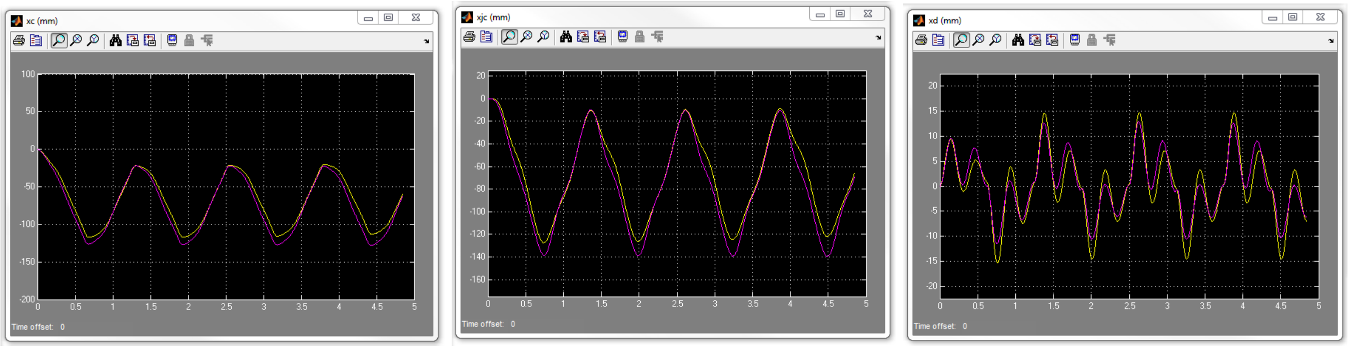


Figure 2.5: q_mdl_lfjce Simulink diagram used to confirm system model

Note: Before you can conduct these experiments, you need to make sure that the lab files are configured according to your IP02 and LFJC-E setup. If they have not been configured already, then go to Section 4.2 to configure the lab files before you begin.

1. Make sure that you have completed the procedure in Section 2.3.2 and the state-space model parameters are present in the MATLAB workspace.
2. Double-click on the *Signal Generator* block and ensure the following parameters are set:
   - Wave form: square
   - Amplitude: 1.0
   - Frequency: 1.0
   - Units: Hertz
3. Set the *Amplitude (V)* slider to 3 V.
4. Open the LFJC-E load cart position, IP02 cart position, and spring deflection scopes, *xjc (mm)*, *xc (mm)* and *dx (mm)*.
5. Click on QUARC | Build to compile the Simulink diagram.
6. Ensure that the LFJC-E is in the middle of the track, and that the motor and encoder cables are able to move freely.
7. Select QUARC | Start to run the model. The IP02 cart, and LFJC-E load cart should begin moving back and forth along the track, and the scopes should be as shown in Figure 2.6. The yellow trace is the measured position and the purple trace is the simulated position.



(a) IP02 Cart Position (mm) (b) LFJC-E Cart Position (mm) (c) Spring Deflection (mm)

Figure 2.6: Force impulse response of the LFJC-E

8. Open the *setup_lfjc* script and update the values of the equivalent spring stiffness, K_s, and equivalent LFJC-E damping, B_eq_{jc}, with the experimental values you found in Section 2.3.1. The values can be entered in the following location:

```
% ##### USER-DEFINED MODEL PARAMETERS #####
% Specify perticular equivalent spring stiffness, Ks, and equivalent
% damping, Beq_jc, parameters:

%Ks = 0;
%Beq_jc = 0;
```

9. Re-run the model and observe any changes in the response. Does the updated simulation match the measured response better than the default? Why might this be the case?

10. Create a MATLAB figure that compares the simulated and experimental response of the cart positions and spring deflection. Comment on why there are discrepancies in the response of the updated model and the actual system.

    Note: When the model is stopped, the *xc (mm)* scope saves the last five seconds of response data to the MATLAB workspace in the *data_xc* parameter. The LFJC-E cart position data is saved in the *data_xjc* parameter, and the spring deflection data is saved in *data_xd*. The variables have the following structure: *data_x(:,1)* is the time vector, *data_x(:,2)* is the measured position, and *data_x(:,3)* is the simulated position.

3 POSITION CONTROL

3.1 Specifications

The response of the linear flexible joint cart should satisfy the following requirements:

- **Percent Overshoot**: $PO \leq 10$ %
- **Maximum settling time (5 %)**: $t_s \leq 0.6$ s
- **Steady-state error**: $e_{ss} = 0$.
  The control effort should be minimized as much as possible within the specified hard limit.

Note: The previous specifications are given in response to a ±20 mm square wave cart position setpoint.

3.2 Background

3.2.1 Linear Quadratic Regular (LQR)

If (A,B) are controllable, then the Linear Quadratic Regular optimization method can be used to find a feedback control gain. Given the plant model in Equation 2.13, the LQR algorithm finds the control input u that minimizes the cost function

$$J = \int_0^\infty x(t)'Qx(t) + u(t)'Ru(t)\,dt, \tag{3.1}$$

where Q and R are the weighting matrices. The weighting matrices affect how LQR minimizes the function and are, essentially, tuning variables.

Given the control law $u = -Kx$, the state-space becomes

$$\begin{aligned} \dot{x} &= Ax + B(-Kx) \\ &= (A - BK)x \end{aligned}$$

Designing a controller with the Linear Quadratic Regular (LQR) technique is an iterative process. In software, you have to select the Q and R matrices, generate the gain K using the LQR algorithm, and then simulate the system or implement the control to access the control performance. The relationship between changing Q and R and the closed-loop response is not evident. However, we can gain insight into how changing the different elements in Q and R will effect the response. We will only be changing the diagonal elements in Q, thus let

$$Q = \begin{bmatrix} q_1 & 0 & 0 & 0 \\ 0 & q_2 & 0 & 0 \\ 0 & 0 & q_3 & 0 \\ 0 & 0 & 0 & q_4 \end{bmatrix}. \tag{3.2}$$

Since we are dealing with a single-input system, R is a scalar value. Using the Q and R defined, the cost function given in Equation 3.1 becomes:

$$J = \int_0^\infty q_1 x_1^2 + q_2 x_2^2 + q_3 x_3^2 + q_4 x_4^2 + Ru^2\,dt. \tag{3.3}$$

3.2.2 Feedback Control

The feedback control loop that controls the position of the mass cart is illustrated in Figure 3.1. The reference state is defined

$$x_d = [x_{jcd}\ 0\ 0\ 0]$$

where x_{jcd} is the desired LFJC-E load cart position. The controller is

$$u = K(x_{jcd} - x). \tag{3.4}$$

Note that when $x_{jcd} = 0$ then $u = -Kx$, which is the control used in the LQR algorithm.

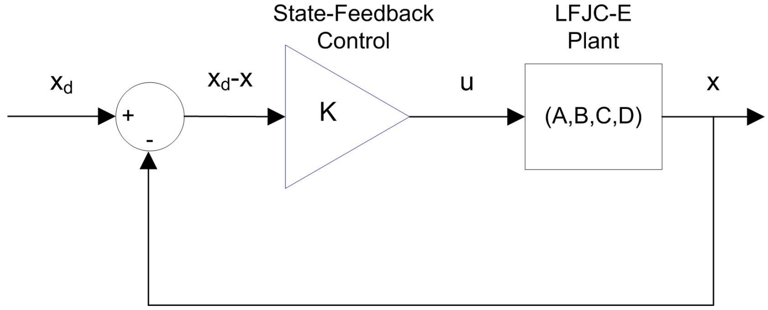


Figure 3.1: State-feedback control loop

3.3 Pre-Lab Questions

1. Based on the location of the open-loop poles determined in Section 2.1.1, what can you infer about the stability and dynamic behaviour of the open-loop system?
2. For the feedback control $u = -Kx$, the Linear-Quadratic Regular algorithm finds a gain K that minimizes the cost function J. Matrix Q sets the weight on the states and determines how u will minimize J (and hence how it generates gain K). From the relationship between the cost function and Q and R parameters expressed in Equation 3.3, explain how increasing the diagonal elements, q_i, effects the generated gain $K = [k_1\ k_2\ k_3\ k_4]$.
3. Explain the effect of increasing R has on the generated gain, K.
4. Based on your results in Question 1 and Question 2, predict which elements of the Q matrix will require the largest weight in order to achieve the system specifications outlined in Section 3.1.

3.4 In-Lab Exercises

The goal of this laboratory is to design a state-feedback controller for the linear flexible joint cart system using the Linear-Quadratic-Regulator (LQR) algorithm. The controller is then implemented on the system, and an experiment is conducted to access the performance of the controller.

3.4.1 Control Simulation

Experiment Setup

The *s_lqr_lfjce* Simulink diagram shown in Figure 3.2 is used to simulate the closed-loop response of the linear flexible joint cart using the state-feedback control described in Section 3.2.2 with the control gain K.

The *Amplitude (m)* gain block is used to change the desired cart position. The state-feedback gain K is read from the MATLAB workspace. The Simulink *State-Space* block reads the A, B, C, and D state-space matrices that are loaded in the MATLAB workspace. The *Find State X* block contains high-pass filters to find the velocity of the IP02 and LFJC-E carts.

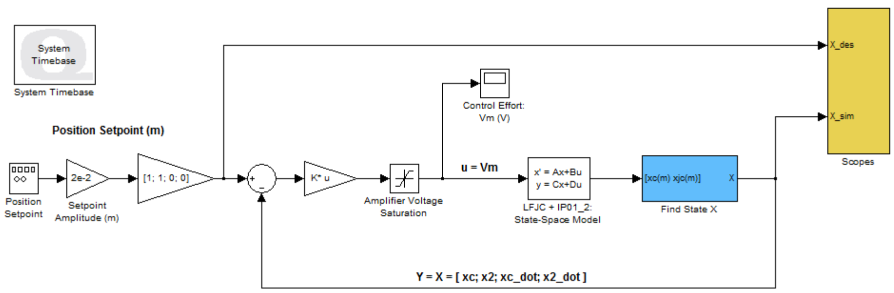


Figure 3.2: s_lqr_lfjce Simulink diagram used to simulate the state-feedback control

Note: Before you can conduct this experiment, you need to make sure that the lab files are configured according to your system setup. If they have not been configured already, go to Section 4.2 to configure the lab files first.

1. Open *setup_ip02_lfjc.m* and go down to the *LQR Control* section shown here:

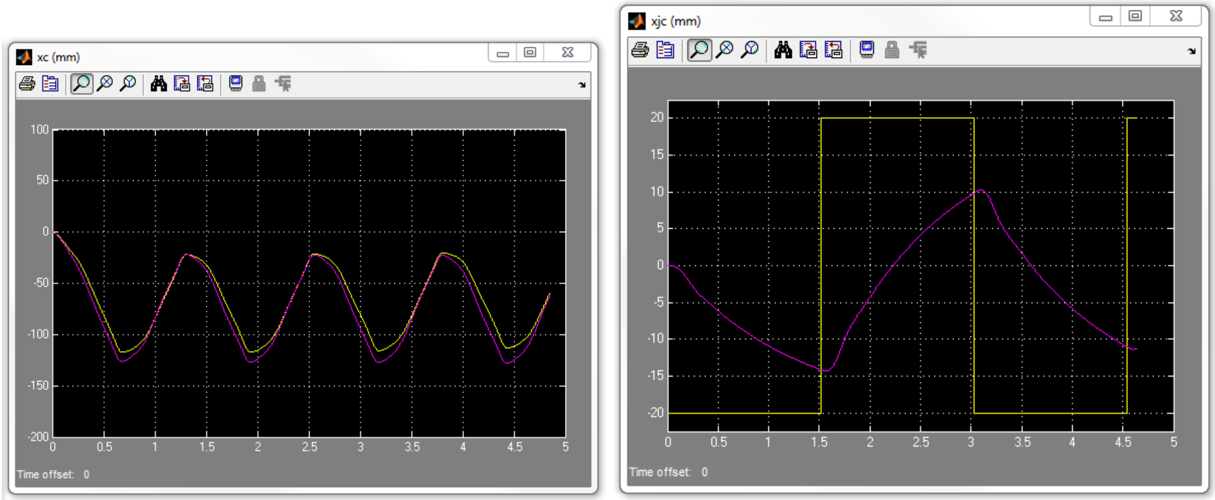
```
% ############### LQR CONTROL ###############
if strcmp ( CONTROLLER_TYPE, 'MANUAL' )
     Q = diag([1 1 1 1]);
     R = 0.01;

     [ K, S, EIG_CL ] = lqr( A, B, Q, R );
```

   The Q and R are initially set to the default values of:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } R = 0.01.$$

2. These will not give you the desired response, but run the script to generate the default gain K.

3. Run *s_lqr_lfjce* to simulate the closed-loop response with this gain. See figures 3.3a and 3.3b for the typical response.



(a) IP02 Cart Position (b) LFJC-E Cart Position

Figure 3.3: Default Simulated Closed-Loop Response

4. Vary the value of the R parameter and observe its effect on the response of the system.

5. If $Q = diag[q_1, q_2, q_3, q_4]$, vary each q_i independently and examine its effect on the gain and the closed-loop response. For example, when increasing q_3, what happens to x_c and x_{jc}? Vary each q_i by the same order of magnitude and compare how the new gain K changes compared to the original gain. Keep $R = 0.01$ throughout your testing. Summarize your results.

   Note: Recall your analysis in pre-lab Question 2 and 4 where the effect of adjusting Q on the generated K was assessed generally by inspecting the cost function equation. You may find some discrepancies in this exercise and the pre-lab questions.

6. Find a Q and R that will satisfy the specifications given in Section 3.1. When doing this, don't forget to keep the DC motor voltage within ± 10 V. This control will later be implemented on actual hardware. Enter the weighting matrices, Q and R, used and the resulting gain, K.

7. Plot the responses from the *xc (mm)*, *xjc (mm)*, *xd (mm)* and *vm (V)* scopes in a MATLAB figure. When the QUARC controller is stopped, these scopes automatically save the last 5 seconds of their response data to the variables *data_xc*, *data_xjc*, *data_xd* and *data_vm*. For the position data arrays, the time is in the first column (e.g. *data_xc(:,1)*, the setpoint is in the second column (e.g. *data_xc(:,2)*), and the simulated data is in the third column (e.g. *data_xc(:,3)*). The spring deflection and control effort data arrays are composed of the time vector in the first column, and the data in the second column.

8. Measure the settling time and overshoot of the simulated LFJC-E load cart position response. Does the response satisfy the specifications given in Section 3.1?

3.4.2 Control Implementation

In this section, the LQR based state-feedback controller that was designed and simulated in the previous sections is run on the actual linear flexible joint cart.

Experiment Setup

The *q_lqr_lfjce* Simulink diagram shown in Figure 3.4 is used to run the state-feedback control on the linear inverted pendulum.

The *Amplitude (m)* gain block is used to change the desired cart position. The state-feedback gain K is read from the MATLAB workspace. The *LFJC-E + IP02:Actual Plant* block interfaces with the IP02 and LFJC-E cart motor and sensors. The *LFJC-E + IP02:State-Space Model* and *Find State X* blocks are used to simulate the response of the IP02 and LFJC-E using the linear state-space model presented in Section 2.1.1.

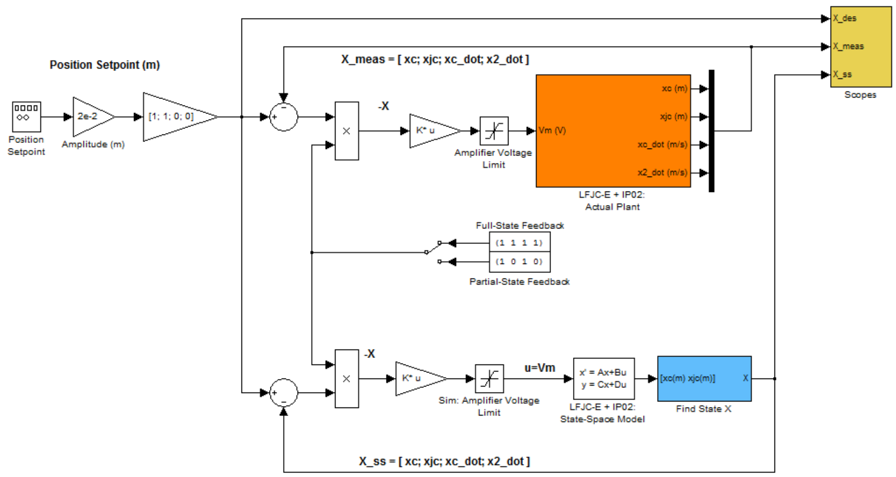


Figure 3.4: q_lqr_lfjce Simulink diagram used to implement the state-feedback control

Note: Before you can conduct this experiment, you need to make sure that the lab files are configured according to your system setup. If they have not been configured already, go to Section 4.2 to configure the lab files first.

1. Run the *setup_ip02_lfjc* script.
2. Make sure the gain K you found in Section 3.4.1 is loaded.
3. Open the *q_lqr_lfjce* Simulink diagram.
4. Turn ON the power amplifier.
5. Go to QUARC | Build to build the controller.
6. Ensure that the LFJC-E system is in the centre of the track. Go to QUARC | Run to start the controller.
7. The response should look similar to your simulation. Once you have obtained a suitable response, go to QUARC | Stop to stop the controller. Similar to the simulation, the response data will be saved to the workspace. Use the response data to plot the IP02 cart, LFJC-E load cart, and motor voltage responses in a MATLAB figure.
8. Measure the settling time, overshoot, and steady-state error of the LFJC-E load cart position. Are the specifications given in Section 3.1 satisfied?
9. How could you alter the controller to compensate for the steady-state error?

3.4.3 Partial-State Feedback

1. Run *setup_ip02_lfjc*. Ensure the gain K you found in Section 3.4.1 is loaded.
2. Open the q_lqr_lfjce.mdl Simulink diagram.
3. Turn ON the power amplifier.
4. To generate an appropriate step reference, ensure the *Signal Generator* is set to the following:
   - Signal type = *square*
   - Amplitude = 1
   - Frequency = 0.33 Hz
5. In the Simulink diagram, set the *Amplitude (m)* gain block to 0.02 to generate a step with an amplitude of 20 millimeters (i.e., square wave goes between ± 0.02 m which results in a step amplitude of 0.04 m).
6. Go to QUARC | Build to build the controller.
7. Ensure that the cart is located in the centre of the track.
8. Set the *Manual Switch* to the *Partial-State Feedback* (downward) position. Select QUARC | Start to begin running the controller.
9. Stop the controller once you have obtained a representative response.
10. Create a MATLAB figure representing the IP02 cart and LFJC-E cart position response.
11. Examine the difference between the partial-state feedback (PSF) response and the full-state feedback (FSF) response. Explain why PSF control behaves this way by looking at the *q_lqr_lfjce* Simulink diagram.
12. Click the Stop button on the Simulink diagram toolbar (or select QUARC | Stop from the menu) to stop the experiment.
13. Turn off the power to the amplifier if no more experiments will be performed in this session.

4 SYSTEM REQUIREMENTS

Before you begin this laboratory make sure:

- QUARC®is installed on your PC, as described in Reference [4].
- You have a QUARC compatible data-acquisition (DAQ) card installed in your PC. For a listing of compliant DAQ cards, see Reference [1].
- IP02, LFJC-E and amplifier are connected to your DAQ board as described Reference [5].

4.1 Overview of Files

File Name	Description
Linear Flexible Joint Cart Workbook (Student).pdf	This laboratory guide contains a modeling and position control experiment demonstrating state-space modeling and feedback control of the linear flexible joint cart. The in-lab exercises are explained using the QUARC software.
setup_ip02_lfjc.m	The main Matlab®script that sets the IP02 and LFJC-E model and control parameters. **Run this file only to setup the laboratory.**
config_ip02.m	Returns the configuration-based IP02 model specifications *Rm, Jm, Kt, Eff_m, Km, Kg, Eff_g, M, r_mp,* and *Beq*.
config_lfjc.m	Returns the configuration-based LFJC-E model specifications *Jeq, Mc_jc, Beq_jc, and Ks* .
d_ip0_lfjc_lqr	Determines the control gain *K*.
LFJC_ABCD_eqns_student.m	Creates the student state space model of the linear flexible joint cart system.
s_lqr_lfjce	Simulink file that simulates the closed-loop flexible joint load cart position control step response.
q_mdl_params	Simulink file that measures the force impulse response of the LFJC-E load cart.
q_mdl_lfjce	Simulink file that simulates and compares the state-space model to the linear flexible joint cart system.
q_lqr_lfjce	Simulink file that implements the state-feedback position controller

Table 4.1: Files supplied with the Linear Flexible Joint Cart Laboratory.

4.2 Configuring the IP02 and the Lab Files

Before beginning the lab exercises the IP02 device the *q_mdl_lfjce*, *q_mdl_params*, and *q_lqr_lfjce* Simulink diagram and the *setup_ip02_lfjc.m* script must be configured.

Follow these steps to get the system ready for this lab:

1. Load the Matlab®software.
2. Browse through the Current Directory window in Matlab®and find the folder that contains the linear pendulum gantry files, e.g. q_mdl_lfjce.

3. Double-click on the q_mdl_lfjce.mdl file to open the Simulink diagram shown in Figure Figure 2.5.

4. **Configure DAQ:** Double-click on the HIL Initialize block in the Simulink diagram and ensure it is configured for the DAQ device that is installed in your system. For instance, the model shown in Figure 2.5 is setup for the Quanser Q2-USB hardware-in-the-loop board. See the QUARC Installation Guide [4] for more information on configuring the HIL Initialize block.

5. Repeat the HIL Initialize configuration for *q_mdl_params* and *q_lqr_lfjce*.

6. Go to the *Current Directory* window and double-click on the setup_ip02_lfjc.m file to open the setup script.

7. **Configure setup script:** The beginning of the setup script is shown below. Ensure the script is setup to match the configuration of your actual IP02 and LFJC-E devices. For example, the script given below is setup for an IP02 plant with the additional weight, an LFJC-E with two additional weights, and it is actuated using the Quanser VoltPAQ device with a gain of 1. See the IP02 User Manual [3], and LFJC-E User Manual [5] for more information on IP02 plant options and corresponding accessories.
   Finally, make sure MODELING_TYPE is set to 'MANUAL'.

```
% ##### USER-DEFINED LFJC-E CONFIGURATION #####
%Type of IP02 Cart Load: set to 'NO_WEIGHT', 'WEIGHT'
%IP02_WEIGHT_TYPE = 'NO_WEIGHT';
IP02_WEIGHT_TYPE = 'WEIGHT';
% Type of LFJC Load: set to 'NO_WEIGHT', 'ONE_WEIGHT', 'TWO_WEIGHT'
%LFJC_WEIGHT_TYPE = 'NO_WEIGHT';
%LFJC_WEIGHT_TYPE = 'ONE_WEIGHT';
LFJC_WEIGHT_TYPE = 'TWO_WEIGHT';
% Turn on or off the safety watchdog on the cart position: set it to 1 , or 0
X_LIM_ENABLE = 1;       % safety watchdog turned ON
%X_LIM_ENABLE = 0;      % safety watchdog turned OFF
% Safety Limits on the IP01 or IP02 cart displacement (m)
X_MAX = 0.3;            % cart displacement maximum safety position (m)
X_MIN = - X_MAX;        % cart displacement minimum safety position (m)
% Amplifier Gain: set to VoltPAQ to 1
K_AMP = 1;
% Amplifier Type: set to 'VoltPAQ' or 'Q3'
AMP_TYPE = 'VoltPAQ';
% AMP_TYPE = 'Q3';
% Digital-to-Analog Maximum Voltage (V); for MultiQ cards set to 10
VMAX_DAC = 10;

% ##### USER-DEFINED CONTROLLER DESIGN #####
% Type of Controller: set it to 'LQR_AUTO', 'LQR_GUI_TUNING', 'MANUAL'
%CONTROLLER_TYPE = 'LQR_AUTO';    % LQR controller design: automatic mode
CONTROLLER_TYPE = 'MANUAL';    % controller design: manual mode
```

5 LAB REPORT

When you prepare your lab report, you can follow the outline given in Section 5.1 to build the *content* of your report. Also, in Section 5.2 you can find some basic tips for the *format* of your report.

5.1 Template for Content

I. PROCEDURE

I.1. Modeling Experiment

1. Briefly describe the main goal of this experiment and the procedure.
    - Briefly describe the experimental procedure (Section 2.3.1), *Impulse Response Test*
    - Briefly describe the experimental procedure in Step 7 in Section 2.3.1.
    - Briefly describe the experimental procedure (Section 2.3.2), *Model Analysis*
    - Briefly describe the experimental procedure in Step 3 in Section 2.3.2.
    - Briefly describe the experimental procedure in Step 4 in Section 2.3.2.
    - Briefly describe the experimental procedure (Section 2.3.3), *Model Validation*
    - Briefly describe the experimental procedure in Step 9 in Section 2.3.3.

I.2. Position Control Experiment

1. Briefly describe the main goal of this experiment and the procedure.
    - Briefly describe the experimental procedure (Section 3.4.1), *Control Simulation*
    - Briefly describe the experimental procedure in Step 4 in Section 3.4.1.
    - Briefly describe the experimental procedure in Step 5 in Section 3.4.1.
    - Briefly describe the experimental procedure in Step 7 in Section 3.4.1.
    - Briefly describe the experimental procedure (Section 3.4.2), *Control Implementation*
    - Briefly describe the experimental procedure in Step 9 in Section 3.4.2.
    - Briefly describe the experimental procedure (Section 3.4.3), *Partial-State Feedback*
    - Briefly describe the experimental procedure in Step 10 in Section 3.4.3.

II. RESULTS
Do not interpret or analyze the data in this section. Just provide the results.

II.1. Modeling Experiment

1. Impulse response plot from Step 7 in Section 2.3.1, *Impulse Response Test*
2. Numeric matrices from Step 4 in Section 2.3.2, *Model Analysis*
3. Comparison plot from Step 10 in Section 2.3.3, *Model Validation*

II.2. Position Control Experiment

1. Cart position and motor voltage plots from Step 7 in Section 3.4.1, *Control Simulation*
2. Cart position and motor voltage plots from Step 7 in Section 3.4.2, *Control Implementation*
3. Cart position plots from Step 10 in Section 3.4.3, *Partial-State Feedback*

III. ANALYSIS
Provide details of your calculations (methods used) for analysis for each of the following:

III.1. Modeling Experiment

1. Step 8 in Section 2.3.1, *Impulse Response Test*.
2. Step 9 in Section 2.3.1, *Impulse Response Test*.
3. Step 10 in Section 2.3.1, *Impulse Response Test*.
4. Step 5 in Section 2.3.2, *Model Analysis*.
5. Step 9 in Section 2.3.3, *Model Validation*.

III.2. Control Simulation

1. Step 6 in Section 3.4.1, *Control Simulation*.
2. Step 8 in Section 3.4.1, *Control Simulation*.
3. Step 8 in Section 3.4.2, *Control Implementation*.
4. Step 11 in Section 3.4.3, *Partial-State Feedback*.

IV. CONCLUSIONS
Interpret your results to arrive at logical conclusions.

1. Steps 10, and 10 in Section 2.3.
2. Steps 8, and 8 in Section 3.4.

5.2 Tips for Report Format

PROFESSIONAL APPEARANCE

- Has cover page with all necessary details (title, course, student name(s), etc.)
- Each of the required sections is completed (Procedure, Results, Analysis and Conclusions).
- Typed.
- All grammar/spelling correct.
- Report layout is neat.
- Does not exceed specified maximum page limit, if any.
- Pages are numbered.
- Equations are consecutively numbered.

- Figures are numbered, axes have labels, each figure has a descriptive caption.
- Tables are numbered, they include labels, each table has a descriptive caption.
- Data are presented in a useful format (graphs, numerical, table, charts, diagrams).
- No hand drawn sketches/diagrams.
- References are cited using correct format.

REFERENCES

[1] Quanser Inc. *QUARC User Manual*.

[2] Quanser Inc. *IP02 QUARC Integration*, 2008.

[3] Quanser Inc. *IP02 User Manual*, 2009.

[4] Quanser Inc. *QUARC Installation Guide*, 2009.

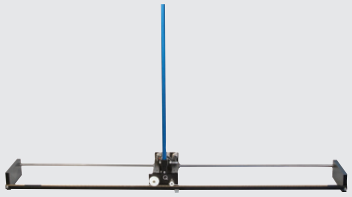
[5] Quanser Inc. *LFJC-E User Manual*, 2012.

Nine linear motion plants for teaching fundamental and advanced controls concepts

IP02 Base Unit

Linear Flexible Joint

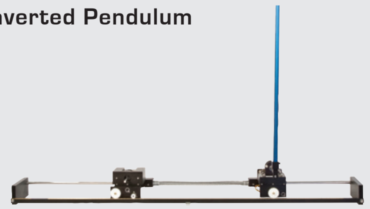
Linear Pendulum



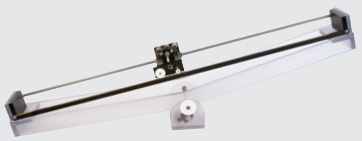
Linear Flexible Inverted Pendulum

Linear Double Inverted Pendulum

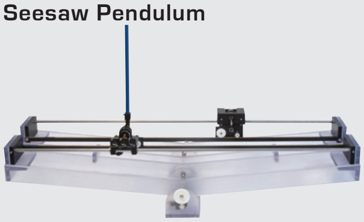
Linear Flexible Joint with Inverted Pendulum



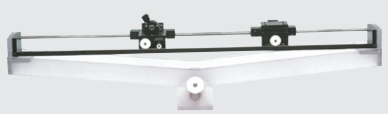
Seesaw



Seesaw Pendulum



Linear Flexible Joint on Seesaw



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