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STUDENT WORKBOOK

1 DOF Torsion Experiment for LabVIEW™ Users

Standardized for ABET* Evaluation Criteria

Developed by:
Jacob Apkarian, Ph.D., Quanser
Hervé Lacheray, M.A.SC., Quanser
Michel Lévis, M.A.SC., Quanser

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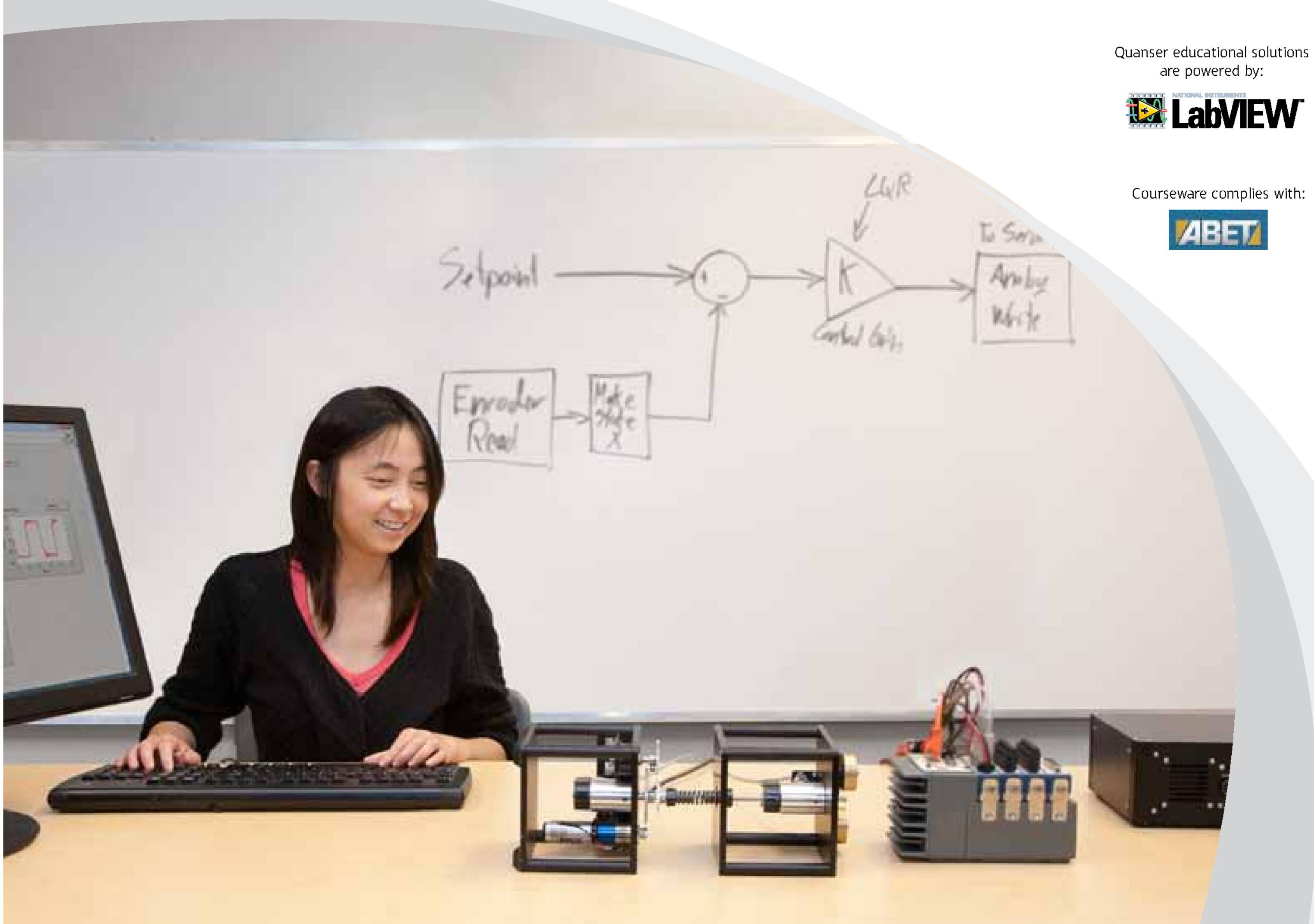

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Quanser Inc.
119 Spy Court
Markham, Ontario
L3R 5H6
Canada
info@quanser.com
Phone: 1-905-940-3575
Fax: 1-905-940-3576

Printed in Markham, Ontario.

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1 INTRODUCTION

The challenge of this experiment is to design a feedback controller to position the output of a rotational system with one torsional flexible coupling as fast as possible with minimum vibration. Such a system emulates torsional compliance and joint flexibility, which are common characteristics in mechanical systems such as high-gear-ratio harmonic drives or lightweight transmission shafts.

The laboratory objective is to design a state-feedback controller for the rotary torsion module which allows you to command a desired load angle position via a flexible coupling. The controller should eliminate the load's vibrations while maintaining a fast response. Full-state and partial-state feedback control strategies are compared. The Linear Quadratic Regulator (LQR) tuning algorithm is used. Frequency tests are carried out to examine the system's resonance.

Topics Covered

- Model the rotary 1 DOF torsional system from first-principles.
- Identify the stiffness and the damping of the system.
- Using a nominal model parameters, design a state-feedback control using Linear Quadratic Regulator (LQR).
- Tune the LQR control in simulation.
- Implement the state-feedback control on the actual 1 DOF Torsion device using LabVIEW™ .
- Evaluate the different between partial-state feedback and full-state feedback.

Prerequisites

In order to successfully carry out this laboratory, the user should be familiar with the following:

- Transfer function fundamentals, e.g. obtaining a transfer function from a differential equation.
- Basics of LabVIEW™ .
- LabVIEW Integration Laboratory in [2] in order to be familiar using LabVIEW™ with the SRV02.

2 MODELING

2.1 Background

2.1.1 Equations of Motion

The Rotary 1 DOF Torsion schematic is depicted in Figure 2.1. Notice that the positive rotation is in counter-clockwise direction. Thus when a positive voltage is applied to the motor, a positive torque is generated and both the servo and the torsion load move counter-clockwise.

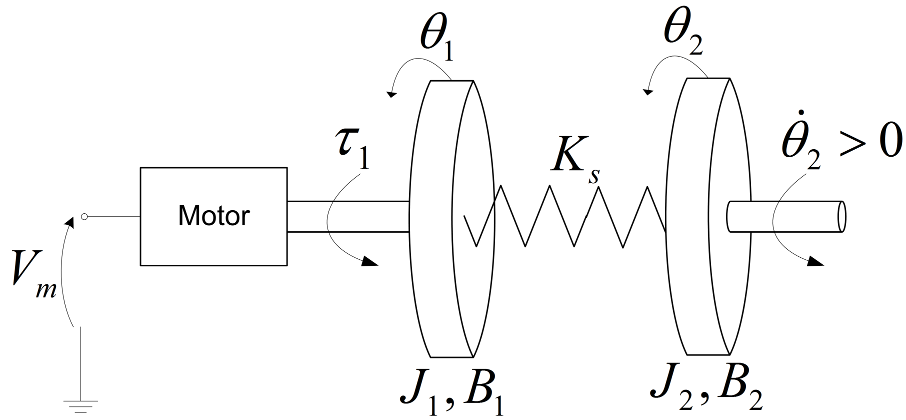

Figure 2.1: Rotary 1 DOF Torsion System

The symbols used for the schematic and the modeling are listed in Table 2.1.

Symbol	Description
V_m	Servo dc motor input voltage.
τ_1	Torque generated by servo.
θ_1	Angular position of servo load shaft.
J_1	Equivalent moment of inertia of servo.
B_1	Equivalent viscous damping acting at servo load shaft.
J_2	Equivalent moment of inertia of torsion load shaft.
B_2	Equivalent viscous damping acting at torsion load shaft.
θ_2	Angular position of torsion load shaft.
K_s	Stiffness of flexible coupling.

Table 2.1: 1 DOF Torsion Model Parameters

The equations of motion (EOMs) can be represented by a set of linear equations of the form

$$J\ddot{q}(t) + B\dot{q}(t) + Kq(t) = \tau \quad (2.1)$$

where J is the system inertia, B is the damping, K is the stiffness, τ is the applied torque, and q are generalized coordinates. For multi-axis systems, the terms J, B, and K would be matrices and q a vector. For the 1 DOF Torsion we have two equations in the form:

$$\ddot{\theta}_1 = g_1(\theta_1, \theta_2, \dot{\theta}_1, \tau_1) \quad (2.2)$$

and

$$\ddot{\theta}_2 = g_2(\theta_1, \theta_2, \dot{\theta}_2) \quad (2.3)$$

where g_1 and g_2 are functions that represent the angular acceleration of the SRV02 load shaft and the 1 DOF Torsion load shaft, respectively. The acceleration of the SRV02, Equation 2.2, depends on the torque being applied, τ_1, the angle of the servo, θ_1, the angle of the torsion module, θ_2, and the velocity of the SRV02 gear (due to damping). The torsion acceleration depends on the angle of the servo, θ_1, as well as the angle of the torsion module, θ_2, and its velocity, $\dot{\theta}_2$.

The torque applied at the base of the rotary arm (i.e., at the load gear) is generated by the servo motor as described by the equation

$$\tau = \frac{\eta_g K_g \eta_m k_t (V_m - K_g k_m \dot{\theta}_1)}{R_m}. \tag{2.4}$$

See the SRV02 User Manual [1] for a description of the corresponding SRV02 parameters (e.g., such as the back-emf constant, k_m).

2.1.2 Linear State-Space Model

The linear state-space equations are

$$\dot{x} = Ax + Bu \tag{2.5}$$

and

$$y = Cx + Du \tag{2.6}$$

where x is the state, u is the control input, A, B, C, and D are state-space matrices. For the 1 DOF Torsion system, the state and output are defined

$$x^\top = [\theta_1 \quad \theta_2 \quad \dot{\theta}_1 \quad \dot{\theta}_2] \tag{2.7}$$

and

$$y^\top = [x_1 \quad x_2]. \tag{2.8}$$

In the output equation, only the position of the servo and torsion angles are being measured. Based on this, the C and D matrices in the output equation are

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{2.9}$$

and

$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{2.10}$$

The velocities of the servo and torsion angles can be computed in the digital controller, e.g., by taking the derivative and filtering the result though a high-pass filter.

2.1.3 Free-Oscillation of a Second Order System

The free-oscillatory equation of motion of a second-order system described by

$$J\ddot{x} + B\dot{x} + Kx = 0 \tag{2.11}$$

is shown in Figure 2.2. Assuming the initial conditions $x(0^-) = x_0$ and $\dot{x}(0^-) = 0$, the Laplace transform of Equation 2.11 is

$$X(s) = \frac{\frac{x_0}{J}}{s^2 + \frac{B}{J}s + \frac{K}{J}} \tag{2.12}$$

The prototype second-order equation is defined

$$s^2 + 2\zeta\omega_n s + \omega_n^2,$$

where ζ is the damping ratio and ω_n is the natural frequency. Equating the characteristic equation in 2.12 to this gives

$$\omega_n^2 = \frac{K}{J}$$

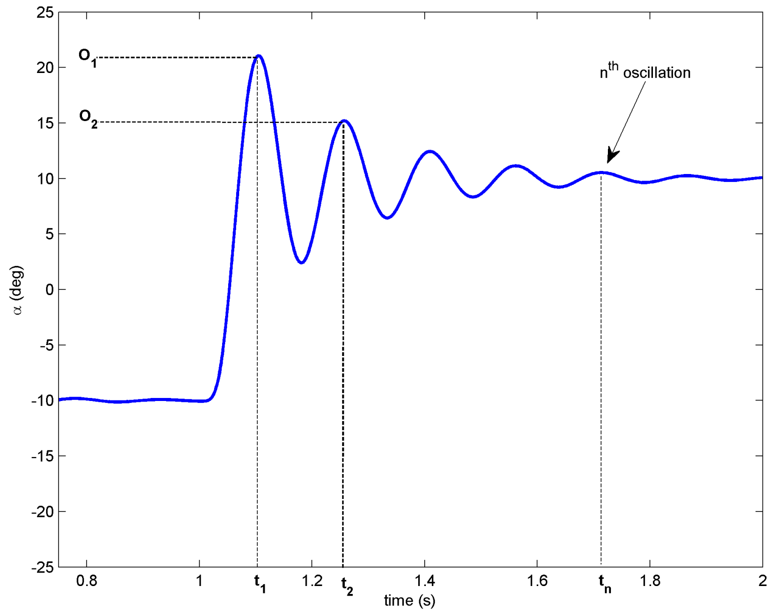

Figure 2.2: Free Oscillation Response

and

$$2\zeta\omega_n = \frac{B}{J}$$

Based on the measured damping ratio and natural frequency, the friction (or stiffness) of the system is

$$K = J\omega_n^2 \quad (2.13)$$

and the viscous damping is

$$B = 2\zeta\omega_n J. \quad (2.14)$$

Finding the Natural Frequency

The period of the oscillations in a system response can be found using the equation

$$T_{osc} = \frac{t_n - t_1}{n-1} \quad (2.15)$$

where t_n is the time of the n^{th} oscillation, t_1 is the time of the first peak, and n is the number of oscillations considered. From this, the damped natural frequency (in radians per second) is

$$\omega_d = \frac{2\pi}{T_{osc}} \quad (2.16)$$

and the undamped natural frequency is

$$\omega_n = \frac{\omega_d}{\sqrt{1-\zeta^2}}. \quad (2.17)$$

Finding the Damping Ratio

The damping ratio of a second-order system can be found from its response. For a typical second-order underdamped system, the subsidence ratio (i.e., decrement ratio) is defined as

$$\delta = \frac{1}{n}\ln\frac{O_1}{O_n} \quad (2.18)$$

where O_1 is the peak of the first oscillation and O_n is the peak of the n^{th} oscillation. Note that $O_1 > O_n$, as this is a decaying response.

The damping ratio is defined

$$\zeta = \frac{1}{\sqrt{1 + \frac{2\pi}{\delta}^2}} \tag{2.19}$$

2.2 Pre-Lab Exercises

1. Find the equation of motion of the servo by completing Equation 2.2 and Equation 2.3.
2. Given state x defined in Equation 2.7, find the linear state-space matrices A and B.
3. Find the the natural frequency of the response shown in Figure 2.2 if the peak values for the first and fifth oscillation are: $t_1 = 1.12$ and $t_5 = 1.71$ seconds. Because the damping is low, assume the damped and undamped natural frequency are equivalent.

2.3 Lab Experiments

In the first part of this laboratory, the stiffness of the torsion is determined by measuring its natural frequency. In the second part, the state-space model is finalized and validated against actual measurements.

2.3.1 Finding Stiffness

In Section 2.1.3 we found an equation describing the free-oscillation response of a second-order system. This can also be used to describe the response of the torsional unit when initially perturbed and left to decay.

Physical Parameters for the Lab

In order to do some of the laboratory exercises, you will need these values:

$$\begin{aligned} B_1 &= 0.015 \text{ N-m/(rad/s)} \\ J_1 &= 2.18 \times 10^{-3} \text{ kg-m}^2 \\ J_2 &= 5.45 \times 10^{-4} \text{ kg-m}^2 \end{aligned}$$

Note: The equivalent viscous damping, B_1, and moment of inertia, J_1, parameters are for the SRV02 when there is no load (i.e., the parameter found in the SRV02 Modeling Laboratory was for servo with the disc load).

Experimental Setup

The *TORSION Stiffness ID* LabVIEW Virtual Instrument (VI) shown in Figure 2.3 is used to find the natural frequency of the torsion unit. This VI feed a short 2V step input to the Rotary Servo and outputs the measured torsion module angle.

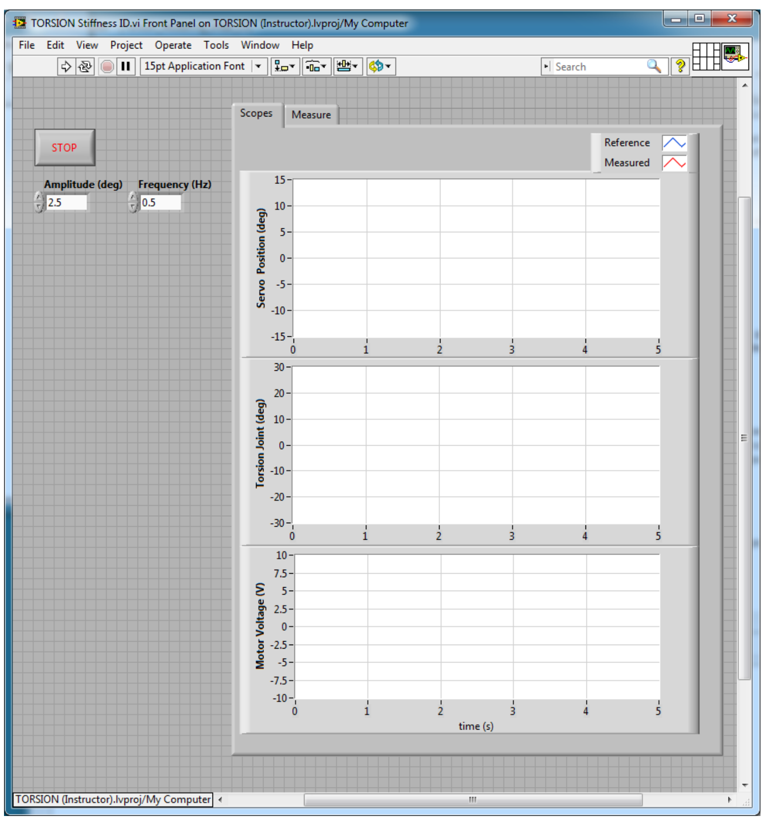

Figure 2.3: VI used to find stiffness

IMPORTANT: Before you can conduct this experiment, you need to make sure that the lab files are configured according to your system setup. If they have not been configured already, then you need to go to Section 4.2 to configure the lab files first.

1. In *TORSION (Student).lvproj*, open the *TORSION Stiffness ID* VI. The VI front panel is shown in Figure 2.3. Make sure the VI is configured for your data acquisition device, as explained in Section 4.2.
2. Run the VI. The servo should begin tracking the reference angle. The *theta1 (deg)* and *theta2 (deg)* scopes should be reading a response similarly as shown in Figure 2.4. Note that the controller is set to run for 5 seconds.

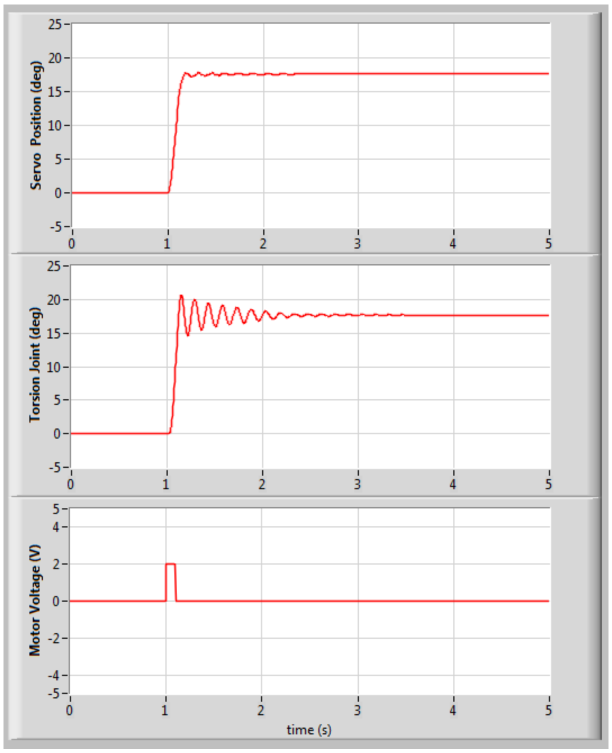

Figure 2.4: Typical Torsion Free Oscillation Response

3. Attach a plot of the free-oscillation response you obtained. One option is to right-click on the Chart and use the *Export* | *Export Simplified Image* command.
4. Find the natural frequency of the flexible coupling. As in Pre-Lab Question 3, the damping in low. Therefore assume the damped natural frequency (which is being measured) is equivalent to the undamped natural frequency.
5. Calculate the stiffness of the flexible coupling, K_s.

2.3.2 State-Space Model

Create the state-space model representing the Quanser 1 DOF Torsion system. This model can then be loaded and used for model validation and control design.

Experimental Setup
The *TORSION Modeling (Student)* VI is shown running with default parameters in Figure 2.5. It is used to generate the state-space model of the 1 DOF Torsion based on the entered model parameters.

1. In the *TORSION (Student).lvproj*, open the *TORSION Model (Student).vi* found in the *Control Design and Simulation* folder. The front panel of the VI is depicted in Figure 2.5.

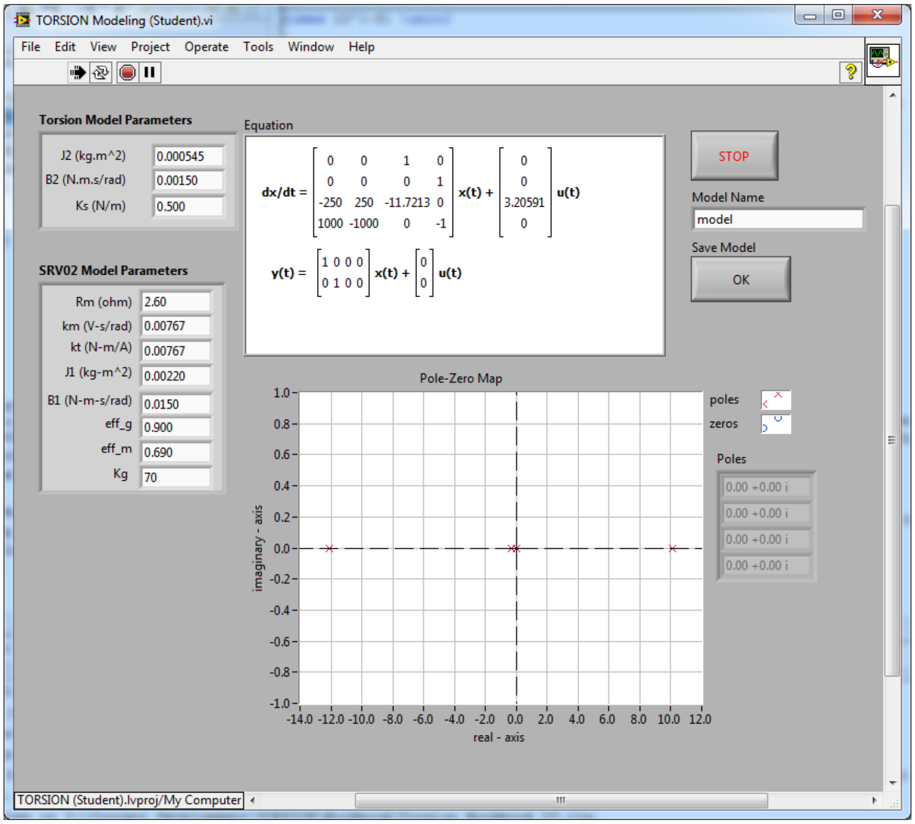

Figure 2.5: Modeling VI used to generate state-space model

2. Enter the stiffness you found in Section 2.3.1 in the control box *Ks* on the VI front panel. Otherwise, it will use a default value of the stiffness.

3. In the VI block diagram. The MathScript RT node has the following state-space matrices entered:

```
A = [0 0 1 0;
      0 0 0 1;
      -250 250 0 0;
      1000 -1000 0 0];

B = [0; 0; 25; 0];

C = zeros(2,4);

D = zeros(2,1);
```

4. Enter the state-space matrices you found in Section 2.2 for A, B, C, and D. In LabVIEW, the SRV02 moment of inertia and damping are denoted *J1* and *B1*, the TORSION moment of inertia and damping are defined as *J2* and *B2*, and the stiffness is *Ks*.

5. Run the *TORSION Modeling* VI to load the state-space matrices. Show the numerical matrices that are displayed in the VI.

   Note: The input of the state-space model you found in Section 2.2 is the torque acting at the servo load gear. However, *we do not control torque directly - we control the servo input voltage*. Recall the voltage-torque relationship given in Equation 2.4 in Section 2.1.1. In MathScript Node, the actuator dynamics are added to your state-space matrices with the code:

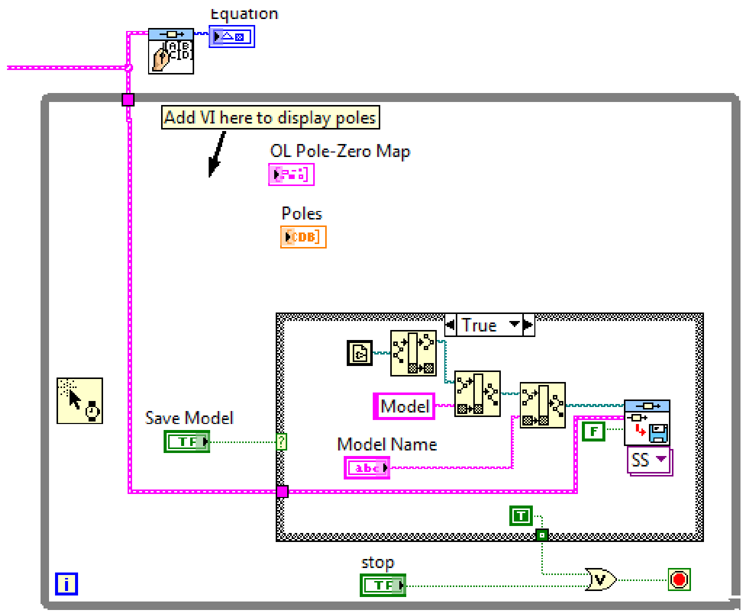

Figure 2.6: Complete block diagram to display poles.

```
% Add actuator dynamics
B = Kg * eta_g * kt * eta_m * B / Ra;
A(3,3) = A(3,3) - Kg*kb*B(3);
A(4,3) = A(4,3) - Kg*kb*B(4);
```

6. Click on the OK button to save your model. This model can be loaded later for analysis or control design.
7. Complete the block diagram shown in Figure 2.6 to display the open-loop poles of the system in the *OL Pole-Zero Map* and *Poles* indicators (using the state-space model). Look through the *Control Design* palette and then run the VI to display the poles.

   Note: These will be required for a pre-lab question in Section 3.3.
8. Click on the *Stop* button to stop the VI.
9. Now that your model is saved and the open-loop poles are recorded, you can close this VI.

2.3.3 Model Validation

By running a simulation and the actual device in parallel, we can verify whether the dynamic model (which drives the simulation) accurately represents our system.

Experimental Setup

The *TORSION Model Validation* VI shown in Figure 2.7 applies a step input to both the Quanser Rotary 1 DOF Torsion hardware and its dynamic model and reads the measured and simulated servo and torsion angles, θ_1 and θ_2. The *1 DOF Torsion* subsystem contains the Quanser Rapid Control Prototyping Toolkit® blocks that interface to the actual hardware. The *State-Space* block reads the A, B, C, and D state-space matrices that are loaded in the MathScript node in LabVIEW. This model outputs the servo and torsion angles.

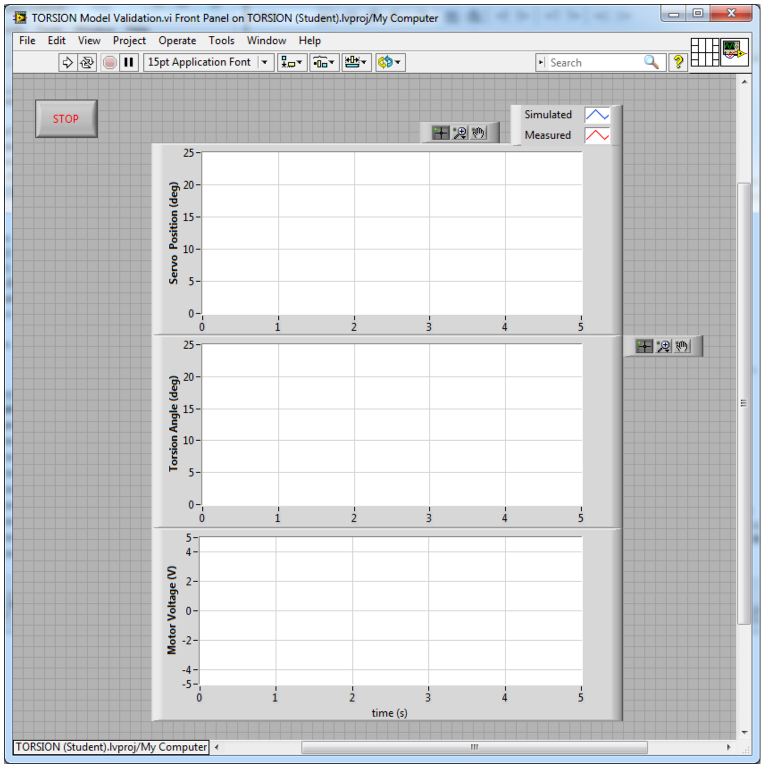

Figure 2.7: VI used validate the model

IMPORTANT: Before you can conduct this experiment, you need to make sure that the lab files are configured according to your system setup. If they have not been configured already, then go to Section 4.3 to configure the lab files first.

1. In the *TORSION (Student).lvproj*, open the *TORSION Model Validation* VI. Make sure the VI is configured for your data acquisition device, as explained in Section 4.3.
2. Go into the VI block diagram (CTRL-E), double-click on the *Rotary Torsion Model* State-Space VI and click on the *Open* icon to load the model you saved previously from Section 2.3.2.
3. Run the VI. The VI is shown running with the default model given in Figure 2.8. The *Servo Position (deg)* scope displays the simulated servo angle in blue and the measured angle in red. Similarly, the *Torsion (deg)* scope shows the simulated torsion angle in blue and the measured angle in red.
4. If your simulation and measured response match, go to the next step. If they do not, then there is an issue with your model. Here are some issues to investigate:
   - Ensure the state-space model was entered properly in the script.
   - The stiffness, K_s, found in Section 2.3.1 is not correct. Review your calculations or redo the experiment.
   - Review your model derivation in Section 2.2, e.g., might be a mistake in solving the EOMs.
5. Plot the servo angle, torsion angle, and input voltage responses and attach them to your report. To save the plots, you can right-click on the chart and go to the *Export* | *Export Simplified Image* with *BMP* and *Clipboard* options.
6. How well does your model represent the actual system? We want a model that is fairly representative but, having said that, keep in mind that no model is perfect. This is just a quick test to see how well your model represents the actual device. As shown in 2.8, the simulation does not match the measured response perfectly.
7. Give at least one reason why the model does not represent the system accurately.

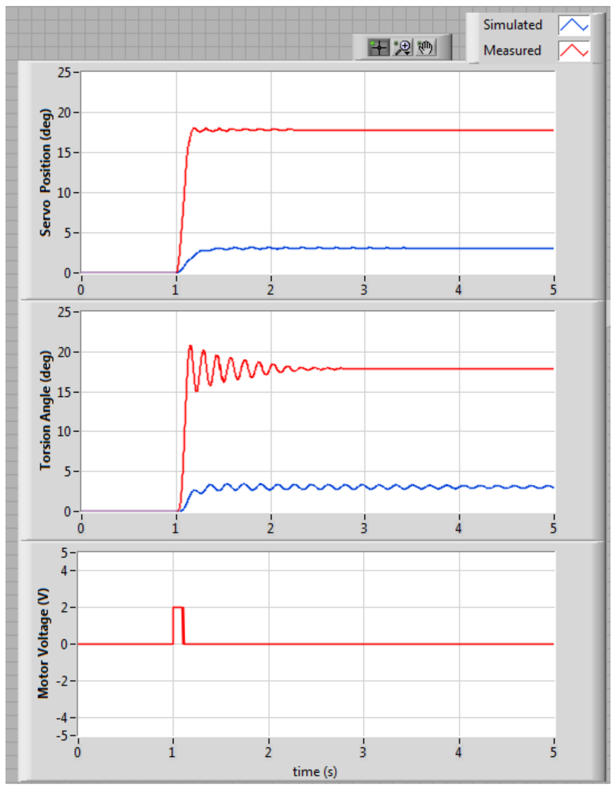

Figure 2.8: Model Validation Response using Default Model (does not accurately represent the system)

2.4 Results

Fill out Table 2.2 with your answers from your modeling lab results - both simulation and implementation.

Description	Symbol	Value	Unit
Finding Stiffness			
Natural frequency	ω_n		rad/s
Stiffness	K_s		N m/rad
Model Validation			
State-Space Matrix	A		
State-Space Matrix	B		
State-Space Matrix	C		
State-Space Matrix	D		
Open-loop poles	OL		

Table 2.2: Results

3 CONTROL DESIGN

3.1 Specifications

The time-domain requirements are:

Specification 1: Torsion angle 2% settling time: $t_s \leq 0.4$ s.

Specification 2: Torsion angle percentage overshoot: $PO \leq 5$ %.

Specification 3: Torsion angle steady-state error: $e_{ss} \leq 0.5$ deg.

Specification 4: Maximum control effort / voltage: $|V_m| \leq 10$ V.

These specifications are to be satisfied when the servo is tracking a ± 10 degree angle square wave.

3.2 Background

In Section 2.2, we found a linear state-state space model that represents the 1 DOF Torsion system. This model is used to investigate the stability properties of the 1 DOF Torsion system in Section 3.2.1. In Section 3.2.2, the notion of controllability is introduced. Using the Linear Quadratic Regular algorithm, or LQR, is a common way to find the control gain and is discussed in Section 3.2.3. Lastly, Section 3.2.4 describes the state-feedback control used to control the torsion load position while minimizing the effect from the flexible coupling.

3.2.1 Stability

The stability of a system can be determined from its poles ([4]):

- Stable systems have poles only in the left-hand plane.
- Unstable systems have at least one pole in the right-hand plane and/or poles of multiplicity greater than 1 on the imaginary axis.
- Marginally stable systems have one pole on the imaginary axis and the other poles in the left-hand plane.

The poles are the roots of the system's characteristic equation. From the state-space, the characteristic equation of the system can be found using

$$\text{det}\,(sI - A) = 0 \tag{3.1}$$

where *det()* is the determinant function, s is the Laplace operator, and I the identity matrix. These are the *eigenvalues* of the state-space matrix A.

3.2.2 Controllability

If the control input, u, of a system can take each state variable, x_i where $i = 1 \ldots n$, from an initial state to a final state then the system is controllable, otherwise it is uncontrollable ([4]).

Rank Test The system is controllable if the rank of its controllability matrix

$$T = \left[B \; AB \; A^2B \ldots A^nB\right] \tag{3.2}$$

equals the number of states in the system,

$$\text{rank}(T) = n. \tag{3.3}$$

3.2.3 Linear Quadratic Regular (LQR)

If (A,B) are controllable, then the Linear Quadratic Regular optimization method can be used to find a feedback control gain. Given the plant model in Equation 2.5, find a control input u that minimizes the cost function

$$J = \int_0^\infty x(t)'Qx(t) + u(t)'Ru(t)\,dt, \tag{3.4}$$

where Q and R are the weighting matrices. The weighting matrices affect how LQR minimizes the function and are, essentially, tuning variables.

Given the control law $u = -Kx$, the state-space in Equation 2.5 becomes

$$\begin{aligned}\dot{x} &= Ax + B(-Kx) \\ &= (A - BK)x\end{aligned}$$

3.2.4 Feedback Control

The feedback control loop that in Figure 3.1 is designed to stabilize the servo and torsion to a desired angular position, θ_d, while minimizing the vibration introduced by the flexible coupling.

The reference state is defined

$$x_d = \begin{bmatrix}\theta_d & \theta_d & 0 & 0\end{bmatrix}$$

and the controller is

$$u = K(x_d - x).$$

Note that if $x_d = 0$ then $u = -Kx$, which is the control used in the LQR algorithm.

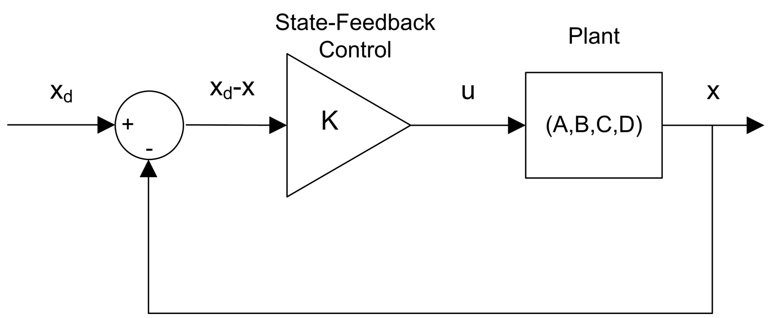

Figure 3.1: State-feedback control loop

3.3 Pre-Lab Questions

1. Based on your analysis of the system in the Modeling Laboratory (Section 2.3), is the system stable, marginally stable, or unstable? From your experience in Section 2.3, does the stability you determined analyically match how the actual system behaves?

2. Designing a controller with the Linear Quadratic Regular (LQR) technique is an iterative process. In software, you have to tune the Q and R matrices, generate the gain K using LQR, and either simulate the system or implement the control to see if you have the desired response. The relationship between changing Q and R and the closed-loop response is not evident. However, we can have a better idea on how changing the different elements in Q and R will effect the response. We will only be changing the diagonal elements in Q, thus let

$$Q = \begin{bmatrix} q_1 & 0 & 0 & 0 \\ 0 & q_2 & 0 & 0 \\ 0 & 0 & q_3 & 0 \\ 0 & 0 & 0 & q_4 \end{bmatrix}. \tag{3.5}$$

   Since we are dealing with a single-input system, R is a scalar value. Using the Q and R defined, expand the cost function given in Equation 3.4.

3. For the feedback control $u = -Kx$, the Linear-Quadratic Regular algorithm finds a gain K that minimizes the cost function J. Matrix Q sets the weight on the states and determines how u will minimize J (and hence how it generates gain K). From your solution in Question 2, explain how increasing the diagonal elements, q_i, effects the generated gain $K = [k_1\ k_2\ k_3\ k_4]$.

4. Explain the effect of increasing R has on the generated gain, K.

3.4 Lab Experiments

The control gain is designed using LQR through simulation first. Once a gain that satisfies the requirements is found, it is implemented on the actual Quanser 1 DOF Torsion system.

3.4.1 Control Simulation

Using the linear state-space model of the system and the designed control gain, the closed-loop response can be simulated. This way, we can test the controller and see if it satisfies the given specifications before running it on the hardware platform.

Experiment Setup

The *TORSION Control Simulation* VI shown in Figure 3.2 is used to simulate the closed-loop response of the 1 DOF Torsion and find a control gain using LQR.

The block diagram includes a *Signal Generator* block that generates a $\pm$ 10 degree 0.5 Hz square wave. The state-feedback gain K is set in the Gain block and set using the control on the front panel. The *State-Space* block in the VI reads the A, B, C, and D state-space matrices that are loaded from the model.

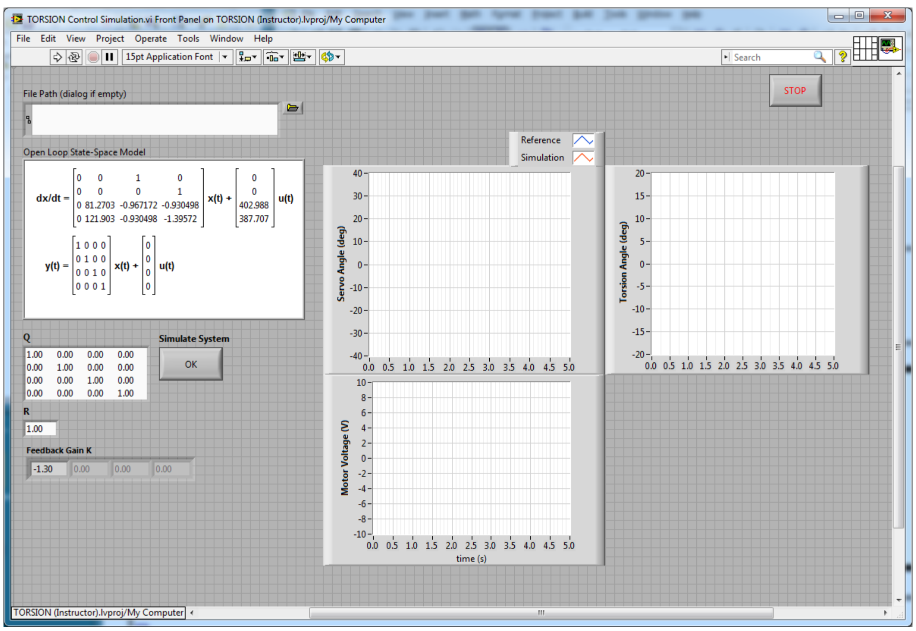

Figure 3.2: VI used to simulate the state-feedback control

IMPORTANT: Before you can conduct this experiment, you need to make sure that the lab files are configured according to your system setup. If they have not been configured already, go to Section 4.4 to configure the lab files first. **Make sure the model you found in Section 2.3.3 is loaded.**

1. In *TORSION (Student).lvproj*, open the *TORSION Control Simulation* VI in the *Control Design and Simulation* folder.
2. On the VI front panel, click on the *File Path* control and select the model you saved in Section 2.3.2.
3. Run the VI. The Q and R are initially set to the default values of:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } R = 1.$$

Using a default model, this will generate a feedback gain to:

$$K = \begin{bmatrix} 5.97 & -4.56 & 0.60 & -0.21 \end{bmatrix}$$

These will not give you the desired response.

4. The closed-loop response with this gain and the default model. See Figure 3.3 for the typical response.

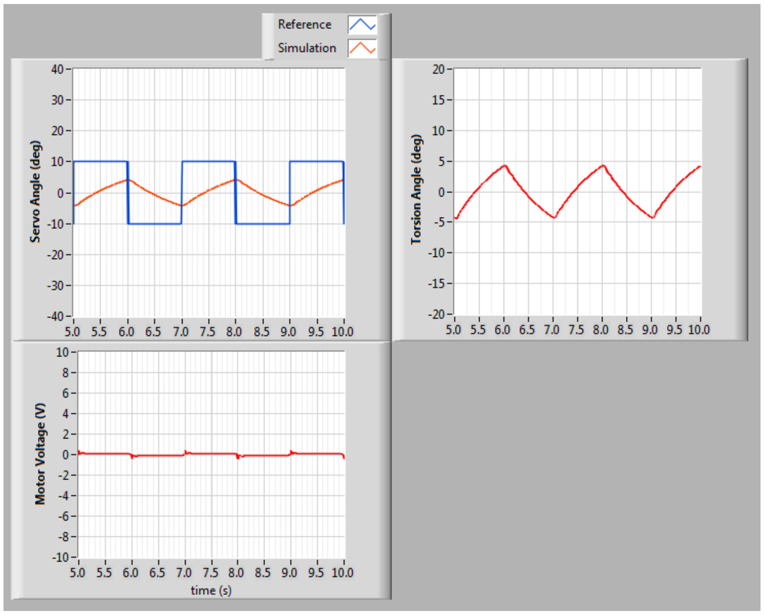

Figure 3.3: Default Simulated Closed-Loop Response

5. If $Q = diag[q_1, q_2, q_3, q_4]$, vary each q_i independently and examine its effect on the gain and the closed-loop response. For example, when increasing q_3, what happens to θ and α? Vary each q_i by the same order of magnitude and compare how the new gain K changes compared to the original gain. Keep $R = 1$ throughout your testing. Summarize your results.

   Note: Recall your analysis in pre-lab Question 3 where the effect of adjusting Q on the generated K was assessed generally by inspecting the cost function equation. You may find some discrepancies in this exercise and the pre-lab questions.

6. Find a Q and R that will satisfy the specifications given in Section 3.1. When doing this, don't forget to keep the dc motor voltage within ± 10 V. This control will later be implemented on actual hardware. Therefore, make sure the actuator is not being saturated. Enter the weighting matrices, Q and R, used and the resulting gain, K.

7. Plot the responses from the *theta (deg)*, *alpha (deg)*, and *Vm (V)* obtained.

8. Measure the settling time, percent overshoot, and steady-state error of the simulated torsion response. Does the response satisfy the specifications given in Section 3.1? Use the cursors in the graphs found in the *Measure* tab to take your measurements.

9. Briefly explain the procedure you used to find Q and R.

3.4.2 Control Implementation

In this experiment, the servo position is controlled while minimizing the vibration of the torsion load using the LQR-based control found in Section 3.4.1. Measurements will then be taken to ensure that the specifications are satisfied.

Experiment Setup

The *TORSION Control* VI shown in Figure 3.4 is used to run the state-feedback control on the Quanser Rotary 1 DOF Torsion system. The VI uses Quanser Rapid Control Prototyping Toolkit® to interface with the DC motor and sensors of the system. The feedback developed in Section 3.4.1 is implemented in the VI using a *Gain* block.

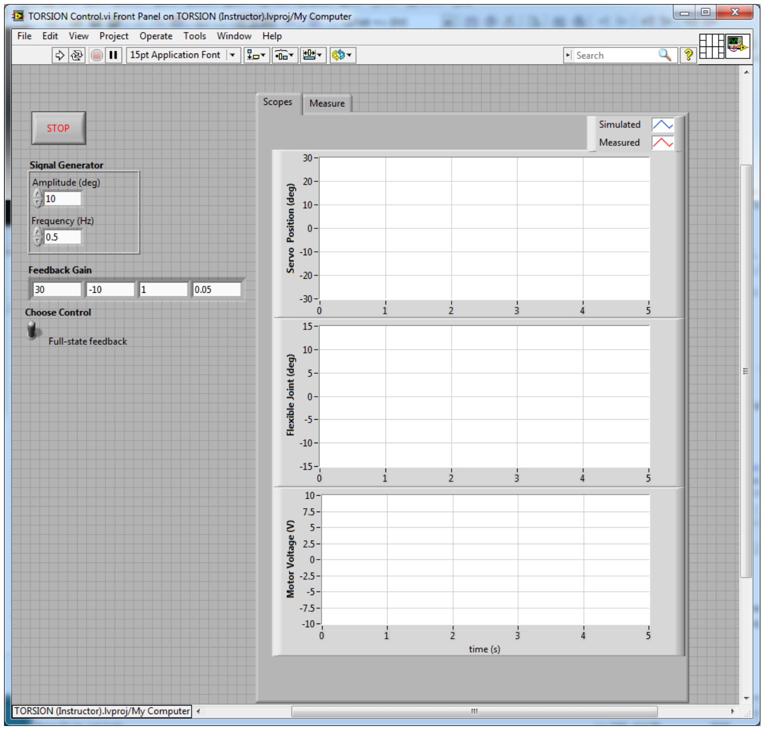

Figure 3.4: VI used the run the LQR control on 1 DOF Torsion system

IMPORTANT: Before you can conduct this experiment, you need to make sure that the lab files are configured according to your system setup. If they have not been configured already, then go to Section 4.5 to configure the lab files first.

1. In *TORSION (Student).lvproj*, open the *TORSION Control* VI. Make sure the VI is configured for your data acquisition device, as explained in Section 4.5.
2. Enter the gain K you found in Section 3.4.1 in *Feedback Gain* control of the VI.
3. Make sure the *Choose Control* switch is set to the upward position for *Full-State Feedback* control.
4. Set the *Amplitude* to 10 and the *Frequency* to 0.5 to command a $\pm$ 10 degrees 0.5 Hz square wave position command to the base servo and torsion module.
5. Run the VI. The torsion shaft should start going between $\pm$ 10 degree signal.
6. Stop the controller once you have obtained a representative response.
7. Plot the responses from the *theta (deg)*, *alpha (deg)*, and *Vm (V)* scopes.
8. Measure the settling time, percent overshoot, and steady-state error of the measured torsion response. Does the response satisfy the specifications given in Section 3.1? As with the simulation, use the cursors in the graphs in the *Measure* tab to take your measurements.

3.4.3 Implementing Partial-State Feedback Control

In this section, the partial-state feedback response of the system is assessed and compared with the full-state feedback control in Section 3.4.2.

1. As in Section 3.4.2, open the *TORSION Control* VI (for your DAQ configuration).
2. Ensure the control gain you settled on in Section 3.4.2 is entered in the *Feedback Gain* control.
3. Make sure the *Choose Control* switch is set to the downward position for *Partial-State Feedback* control.
4. Run the VI. The servo should be tracking the default ± 10 degree square wave.
5. Stop the controller once you have obtained a representative response.
6. As in Section 3.4.2, attach a plot representing the servo and torsion angle response, as well as the input voltage.
7. Examine the difference between the partial-state feedback (PSF) response and the full-state feedback (FSF) response. Explain why PSF control behaves this way by looking at the q_torsion Simulink diagram.

3.5 Results

Fill out Table 3.1 with your answers from your control lab results - both simulation and implementation.

Description	Symbol	Value	Unit
LQR Simulation			
LQR Weighting Matrices	Q		
	R		
LQR Gain	K		
Settling time	t_s		s
Percentage overshoot	PO		%
Steady-state error	e_{ss}		deg
LQR Implementation			
LQR Weighting Matrices	Q		
	R		
LQR Gain	K		
Settling time	t_s		s
Percentage overshoot	PO		%
Steady-state error	e_{ss}		deg

Table 3.1: Results

4 SYSTEM REQUIREMENTS

Required Software

Make sure LabVIEW™ is installed with the following required add-ons:

1. LabVIEW™
2. NI-DAQmx
3. NI LabVIEW™ Control Design and Simulation Module
4. NI LabVIEW™ MathScript RT Module
5. Quanser Rapid Control Prototyping Toolkit®

Note: Make sure the Quanser Rapid Control Prototyping (RCP) Toolkit is installed after LabVIEW. See the RCP Toolkit Quick Start Guide for more information.

Required Hardware

- Data acquisition (DAQ) device that is compatible with Quanser Rapid Control Prototyping Toolkit®. This includes Quanser DAQ boards such as Q2-USB, Q8-USB, QPID, and QPIDe and some National Instruments DAQ devices.
- Quanser Rotary 1 DOF Torsion.
- Quanser VoltPAQ-X1 power amplifier, or equivalent.

Before Starting Lab

Before you begin this laboratory make sure:

- LabVIEW™ is installed on your PC.
- DAQ device has been successfully tested (e.g., using the test software in the Quick Start Guide).
- Rotary 1 DOF Torsion and amplifier are connected to your DAQ board as described Reference [3].

4.1 Overview of Files

File Name	Description
1 DOF Torsion User Manual.pdf	This manual describes the hardware of the Quanser Rotary 1 DOF Torsion system and explains how to setup and wire the system for the experiments.
1 DOF Torsion Workbook (Student).pdf	This laboratory guide contains pre-lab questions and lab experiments demonstrating how to design and implement a position controller on the Quanser Rotary 1 DOF Torsion plant using LabVIEW™ .
TORSION (Student).lvproj	LabVEW project that contains all the required VIs for the 1 DOF Torsion laboratory.
TORSION Modeling (Student).vi	VI used to generate the state-space model of the 1 DOF Torsion system. Note: VI is incomplete.
TORSION Control Simulation.vi	VI that simulates the 1 DOF Torsion system when using state-feedback control. Also used to design the state-feedback gain K using LQR.
TORSION Stiffness ID.vi	VI applied a short step voltage to the servo and measures the corresponding 1 DOF Torsion angle. The measured response can then be used to find the natural frequency of the torsion module.
TORSION Model Validation.vi	VI used to compare the state-space model with the measured response from the 1 DOF Torsion system.
TORSION Control.vi	VI that implements a closed-loop state-feedback controller on the actual torsion system.

Table 4.1: Files supplied with the Quanser Rotary 1 DOF Torsion.

4.2 Setup for Finding Stiffness

Before beginning in-lab procedure outlined in Section 2.3.1, the q_torsion_id Simulink diagram must be properly configured.

Follow these steps:

1. Setup the SRV02 with the 1 DOF Torsion module as detailed in the 1 DOF Torsion User Manual ([3]).
2. Load LabVIEW™ .
3. Open the LabVIEW project called *TORSION (Student).lvproj* shown in Figure 4.1.
4. Open the *TORSION Stiffness ID.vi* shown in Figure 2.3.
5. **Configure DAQ:** Ensure the HIL Initialize block subsystem is configured for the DAQ device that is installed in your system (e.g., 'q1_cRIO', 'q2_usb', 'q8_usb', 'qpid', or 'qpid_e'). The HIL Initialize block is shown in Figure 4.2.
6. **Quanser CompactRIO Users:** Before running the VI, make sure you can connect to your CompactRIO through the Measurement & Automation software. You will also need to configure the cRIO in the LabVIEW Project.
7. **Channel Configuration:** For any of the DAQ-based VIs, the encoder input and ouput channels are set, by default, to match the wiring in the 1 DOF Torsion User Manual ([3]). If the wiring is different on your system, make sure the VI uses the correct channels.

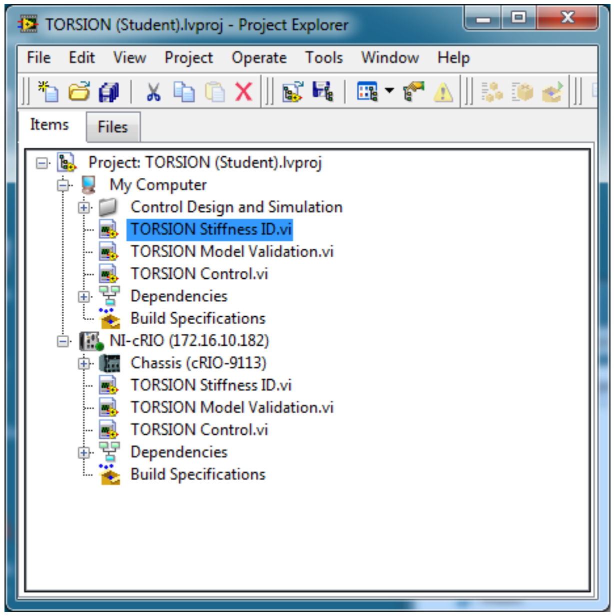

Figure 4.1: TORSION LabVIEW Project (Student version).

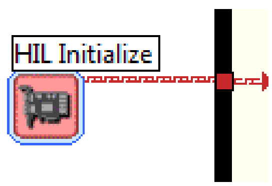

Figure 4.2: HIL Initialize - ensure correct DAQ is configured.

4.3 Setup for Model Validation

Follow these steps to get the system ready for this lab:

1. Setup the SRV02 with the 1 DOF Torsion module as detailed in 1 DOF Torsion User Manual ([3]).
2. Load LabVIEW™ .
3. As described in Section 4.2, open the TORSION LabVIEW Project.
4. In the LabVIEW Project, open the *TORSION Model Validation.vi*.
5. **Configure DAQ:** Ensure the HIL Initialize block in the *SRV02 1 DOF Torsion* subsystem is configured for the DAQ device that is installed in your system.
6. Go into the VI block diagram (CTRL-E), double-click on the *Rotary Torsion Model* State-Space VI and click on the *Open* icon to load the model you saved previously from Section 2.3.2. **Important:** Make sure you go through the State-Space Model lab in Section 2.3.2 prior to doing this lab.

4.4 Setup for 1 DOF Torsion Control Simulation

Follow these steps to configure the lab properly:

1. Load LabVIEW™ .

2. As described in Section 4.2, open the TORSION LabVIEW Project.
3. In the LabVIEW Project, open the *TORSION Control Simulation.vi*.
4. Click on the *File Dialog* control in the VI and load the model you saved previously from Section 2.3.2.

4.5 Setup for 1 DOF Torsion Control Implementation

Follow these steps to get the system ready for this lab:

1. Before performing the control on the actual system, go through the simulation laboratory described in Section 3.4.1 to find the control gain K.
2. Setup the SRV02 with the 1 DOF Torsion module as detailed in 1 DOF Torsion User Manual [3] .
3. Load LabVIEW™ .
4. As described in Section 4.2, open the TORSION LabVIEW Project.
5. In the LabVIEW Project, open the *TORSION Control.vi* shown in Figure 3.4.
6. **Configure DAQ:** Ensure the HIL Initialize block in the *SRV02 1 DOF Torsion* subsystem is configured for the DAQ device that is installed in your system.

5 LAB REPORT

This laboratory contains two groups of experiments, namely,

1. Modeling the Quanser 1 DOF Torsion, and
2. State-feedback control.

For each experiment, follow the outline corresponding to that experiment to build the *content* of your report. Also, in Section 5.3 you can find some basic tips for the *format* of your report.

5.1 Template for Content (Modeling)

I. PROCEDURE

1. *Finding Stiffness*
    - Briefly describe the main goal of the experiment.
    - Briefly describe the experiment procedure in Step 3 in Section 2.3.1.
2. *Model Validation*
    - Briefly describe the main goal of the experiment.
    - Briefly describe loading the model in Step 5 in Section 2.3.3.
    - Briefly describe the model validation experiment in Step 5 in Section 2.3.3.

II. RESULTS

Do not interpret or analyze the data in this section. Just provide the results.

1. Step response plot from step 3 in Section 2.3.1.
2. Open-loop poles found in step 7 in Section 2.3.2.
3. Model validation plot from step 5 in Section 2.3.3.
4. Provide applicable data collected in this laboratory (from Table 2.2).

III. ANALYSIS
Provide details of your calculations (methods used) for analysis for each of the following:

1. Measured torsion module stiffness in step 5 in Section 2.3.1.
2. Model discrepancies given in step 7 in Section 2.3.3.

IV. CONCLUSIONS
Interpret your results to arrive at logical conclusions for the following:

1. How does the model compare with the actual system in step 6 of Section 2.3.3, *State-space model validation*.

5.2 Template for Content (Control)

I. PROCEDURE

1. *Simulation*
   - Briefly describe the main goal of the simulation.
   - Briefly describe the procedure in step 5 of Section 3.4.1 to examine the effect of variables on the gain and closed-loop response.
   - Briefly describe the simulation procedure in step 7 of Section 3.4.1.
   - Briefly explain the procedure used to find Q and R in step 9 of Section 3.4.1.
2. *Full-State Feedback Implementation*
   - Briefly describe the main goal of this experiment.
   - Briefly describe the experimental procedure in step 7 of Section 3.4.2).
3. *Partial-State Feedback Implementation*
   - Briefly describe the main goal of this experiment.
   - Briefly describe the experimental procedure in step 6 of Section 3.4.3.

II. RESULTS
Do not interpret or analyze the data in this section. Just provide the results.

1. Response plot from step 7 in Section 3.4.1, *Full-state feedback LQR controller simulation*.
2. Response plot from step 7 in Section 3.4.2, for *Full-state feedback LQR controller implementation*.
3. Response plot from step 6 in Section 3.4.3, for *Partial-state feedback LQR controller implementation*.
4. Provide applicable data collected in this laboratory (from Table 3.1).

III. ANALYSIS
Provide details of your calculations (methods used) for analysis for each of the following:

1. Settling time and percent overshoot in step 8 in Section 3.4.1, *Full-state feedback LQR controller simulation*.
2. Settling time and percent overshoot in step 8 in Section 3.4.2, for *Full-state feedback LQR controller implementation*.
3. Comparison between partial-state and full-state feedback in step 7 in Section 3.4.3.

IV. CONCLUSIONS
Interpret your results to arrive at logical conclusions for the following:

1. Whether the controller meets the specifications in step 8 in Section 3.4.1, *Full-state feedback LQR controller simulation*.
2. Whether the controller meets the specifications in step 8 in Section 3.4.2, for *Full-state feedback LQR controller implementation*.

5.3 Tips for Report Format

PROFESSIONAL APPEARANCE

- Has cover page with all necessary details (title, course, student name(s), etc.)
- Each of the required sections is completed (Procedure, Results, Analysis and Conclusions).
- Typed.
- All grammar/spelling correct.
- Report layout is neat.
- Does not exceed specified maximum page limit, if any.
- Pages are numbered.
- Equations are consecutively numbered.
- Figures are numbered, axes have labels, each figure has a descriptive caption.
- Tables are numbered, they include labels, each table has a descriptive caption.
- Data are presented in a useful format (graphs, numerical, table, charts, diagrams).
- No hand drawn sketches/diagrams.
- References are cited using correct format.

REFERENCES

[1] Quanser Inc. *SRV02 User Manual*, 2009.

[2] Quanser Inc. SRV02 lab manual. 2011.

[3] Quanser Inc. *1 DOF Torsion User Manual*, 2012.

[4] Norman S. Nise. *Control Systems Engineering*. John Wiley & Sons, Inc., 2008.

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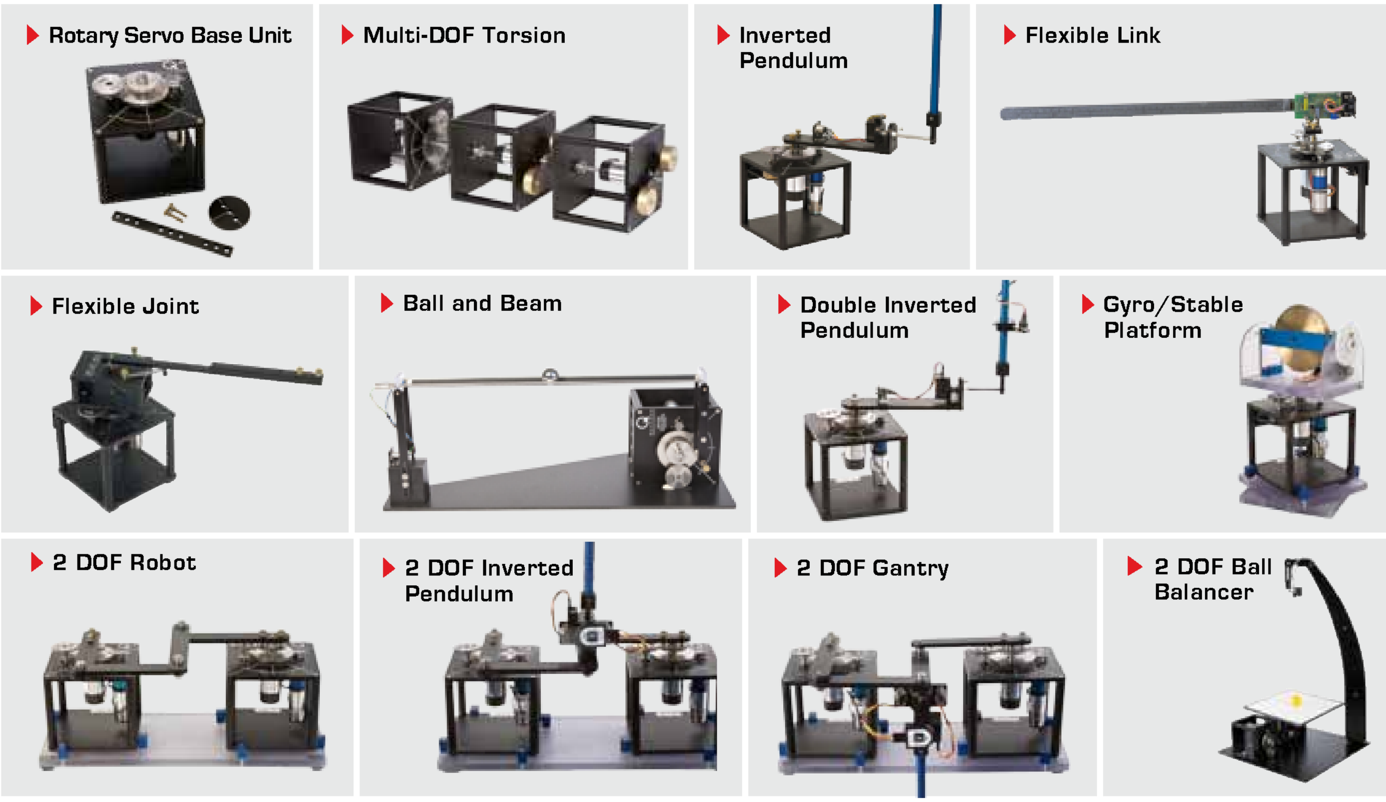

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