

# STUDENT WORKBOOK

## Flexible Joint Experiment for LabVIEW™ Users

Standardized for ABET\* Evaluation Criteria

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# **CONTENTS**

1	Introduction				
2	Modeling				
	2.1	Background	<b>5</b> 5		
	2.2	Pre-Lab Questions	11		
	2.3	Lab Experiments	15		
	2.4	Results	25		
3	Contr	26			
	3.1	Specifications	26		
	3.2	Background	26		
	3.3	Pre-Lab Questions	31		
	3.4	Lab Experiments	33		
	3.5	Results	45		
4	System Requirements		46		
	4.1	Overview of Files	47		
	4.2	Software Setup	47		
5	Lab Report				
	5.1	Template for Content (Modeling)	49		
	5.2	Template for Content (Control)	50		
	5.3	Tips for Report Format	52		
6	Scoring Sheet for Pre-Lab Questions (Modeling)				
7	Scoring Sheet for Lab Report (Modeling)				
8	Scoring Sheet for Pre-Lab Questions (Control)				
9	Scoring Sheet for Lab Report (Control)				
Α	Instructor's Guide				



## 1 INTRODUCTION

The objective of this experiment is to control the position of the servo while minimizing the motions the flexible rotary link.

#### **Topics Covered**

- Modeling the Rotary Flexible Joint using Lagrange.
- Find the linear state-space model of the system.
- · Do some basic model validation.
- Design a state-feedback controller using Pole-Placement (PP).
- · Simulate the closed-loop flexible joint system.
- Implement the designed controller on the device.
- Compare the simulated and measured closed-loop results.
- · Assess the behaviour of implementing a partial-state feedback controller.

#### **Prerequisites**

In order to successfully carry out this laboratory, the user should be familiar with the following:

- Data acquisition card (e.g., Q2-USB), the power amplifier (e.g., VoltPAQ-X1), and the main components of the SRV02 (e.g. actuator, sensors), as described in References [3], [4], and [2], respectively.
- Wiring and operating procedure of the Rotary Flexible Joint plant with the amplifier and DAQ devices, as discussed in Reference [7].
- Transfer function fundamentals, e.g. obtaining a transfer function from a differential equation.
- State-space modeling, e.g., obtaining state equations from a set of differential equations.
- SRV02 LabVIEW Integration lab described in Reference [6] to become familiar with using LabVIEW™ with the SRV02.

## 2 MODELING

## 2.1 Background

#### 2.1.1 Model

The Rotary Flexible Joint model is shown in Figure 2.1. The base of the module is mounted on the load gear of the SRV02 system. The servo angle,  $\theta$ , increases positively when it rotates counter-clockwise (CCW). The servo (and thus the link) turn in the CCW direction when the control voltage is positive, i.e.,  $V_m > 0$ .

The total length of the link can be varied by changing the mounting position of the shorter top arm. The main bottom arm, which is connected to the pivot, has a length of  $L_1$  and a mass of  $m_1$ . The length and mass of the top link is  $L_2$  and  $m_2$ . The distance between the pivot and the middle of the top arm, which can be changed, is denoted by the variable  $d_{12}$ . The moment of inertia of the entire link is specified by  $J_l$  and it changes depending on the position of the top arm. See the *Rotary Flexible Joint User Manual* (in [7]) for the values of these parameters. The deflection angle of the link is denoted as  $\alpha$  and increases positively when rotated CCW.

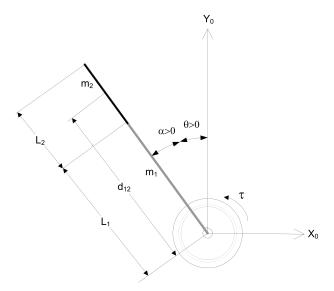


Figure 2.1: Rotary Flexible Joint Angles

The flexible joint system can be represented by the diagram shown in Figure 2.2. Our control variable is the input servo motor voltage,  $V_m$ . This generates a torque,  $\tau$ , at the load gear of the servo that rotates the base of the link. The viscous friction coefficient of the servo is denoted by  $B_{eq}$ . This is the friction that opposes the torque being applied at the servo load gear. The friction acting on the link is represented by the viscous damping coefficient  $B_l$ . Finally, the flexible joint is modeled as a linear spring with the stiffness  $K_s$ .

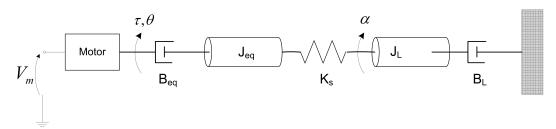


Figure 2.2: Rotary Flexible Joint Model



## 2.1.2 Finding the Equations of Motion

Instead of using classical mechanics, the Lagrange method is used to find the equations of motion of the system. This systematic method is often used for more complicated systems such as robot manipulators with multiple joints.

More specifically, the equations that describe the motions of the servo and the link with respect to the servo motor voltage, i.e. the dynamics, will be obtained using the Euler-Lagrange equation:

$$\frac{\partial^2 L}{\partial t \partial \dot{q_i}} - \frac{\partial L}{\partial q_i} = Q_i \tag{2.1}$$

The variables  $q_i$  are called *generalized coordinates*. For this system let

$$q(t)^{\top} = [\theta(t) \ \alpha(t)] \tag{2.2}$$

where, as shown in Figure 2.2,  $\theta(t)$  is the servo angle and  $\alpha(t)$  is the flexible joint angle. The corresponding velocities are

$$\dot{q}(t)^{\top} = \left[ \frac{\partial \theta(t)}{\partial t} \; \frac{\partial \alpha(t)}{\partial t} \right]$$

**Note:** The dot convention for the time derivative will be used throughout this document, i.e.,  $\dot{\theta} = \frac{d\theta}{dt}$  and  $\dot{\alpha} = \frac{d\alpha}{dt}$ . The time variable t will also be dropped from  $\theta$  and  $\alpha$ , i.e.,  $\theta := \theta(t)$  and  $\alpha := \alpha(t)$ .

With the generalized coordinates defined, the Euler-Lagrange equations for the rotary flexible joint system are

$$\frac{\partial^2 L}{\partial t \partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = Q_1 \tag{2.3}$$

and

$$\frac{\partial^2 L}{\partial t \partial \dot{\alpha}} - \frac{\partial L}{\partial \alpha} = Q_2 \tag{2.4}$$

The Lagrangian of a system is defined

$$L = T - V \tag{2.5}$$

where T is the total kinetic energy of the system and V is the total potential energy of the system. Thus the Lagrangian is the difference between a system's kinetic and potential energies.

The generalized forces  $Q_i$  are used to describe the non-conservative forces (e.g., friction) applied to a system with respect to the generalized coordinates. In this case, the generalized force acting on the rotary arm is

$$Q_1 = \tau - B_{ea}\dot{\theta} \tag{2.6}$$

and acting on the link is

$$Q_2 = -B_l \dot{\alpha}. \tag{2.7}$$

The torque applied at the base of the rotary arm (i.e., at the load gear) is generated by the servo motor as described by the equation

$$\tau = \frac{\eta_g K_g \eta_m k_t (V_m - K_g k_m \dot{\theta})}{R_m}.$$
 (2.8)

See [2] for a description of the corresponding SRV02 parameters (e.g. such as the back-emf constant,  $k_m$ ). The servo damping (i.e. friction),  $B_{eq}$ , opposes the applied torque. The flexible joint is not actuated, the only force acting on the link is the damping,  $B_l$ .

Again, the Euler-Lagrange equations is a systematic method of finding the equations of motion (EOMs) of a system. Once the kinetic and potential energy are obtained and the Lagrangian is found, then the task is to compute various derivatives to get the EOMs.

## 2.1.3 Potential and Kinetic Energy

#### **Kinetic Energy**

Translational kinetic equation is defined as

$$T = \frac{1}{2}mv^2,\tag{2.9}$$

where m is the mass of the object and v is the linear velocity.

Rotational kinetic energy is described as

$$T = \frac{1}{2}J\omega^2 \tag{2.10}$$

where J is the moment of inertia of the object and  $\omega$  is its angular rate.

#### **Potential Energy**

Potential energy comes in different forms. Typically in mechanical system we deal with *gravitational* and *elastic* potential energy. The *relative* gravitational potential energy of an object is

$$V_a = mg\Delta h, (2.11)$$

where m is the object mass and  $\Delta h$  is the change in altitude of the object (from a reference point). The potential energy of an object that rises from the table surface (i.e., the reference) up to 0.25 meter is  $\Delta h = 0.25 - 0 = 0.25$  and the energy stored is  $V_q = 0.25mg$ .

The equation for elastic potential energy, i.e., the energy stored in a spring, is

$$V_e = \frac{1}{2}K\Delta x^2 \tag{2.12}$$

where K is the spring stiffness and  $\Delta x$  is the linear or angular change in position. If an object that is connected to a spring moves from in its initial reference position to 0.1 m, then the change in displacement is  $\Delta x = 0.1 - 0 = 0.1$  and the energy stored equals  $V_e = 0.005K$ .

## 2.1.4 Linear State-Space Model

The linear state-space equations are

$$\dot{x} = Ax + Bu \tag{2.13}$$

and

$$y = Cx + Du (2.14)$$

where x is the state, u is the control input, A, B, C, and D are state-space matrices. For the Rotary Flexible Joint system, the state and output are defined

$$x^{\top} = [\theta \ \alpha \ \dot{\theta} \ \dot{\alpha}] \tag{2.15}$$

and

$$y^{\top} = [x_1 \ x_2]. \tag{2.16}$$

In the output equation, only the position of the servo and link angles are being measured. Based on this, the  ${\cal C}$  and  ${\cal D}$  matrices in the output equation are

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{2.17}$$

and

$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{2.18}$$

The velocities of the servo and link angles can be computed in the digital controller, e.g., by taking the derivative and filtering the result though a high-pass filter.



## 2.1.5 Finding Second Order System Parameters

Consider a second-order system described by

$$J\ddot{x} + B\dot{x} + Kx = 0. \tag{2.19}$$

Assuming the initial conditions  $x(0^-)=x_0$  and  $\dot{x}(0^-)=0$ , the Laplace transform of Equation 2.19 is

$$X(s) = \frac{\frac{x_0}{J}}{s^2 + \frac{B}{I}s + \frac{K}{I}}$$
 (2.20)

The prototype second-order equation is defined

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

where  $\zeta$  is the damping ratio and  $\omega_n$  is the natural frequency. Equating the characteristic equation in 2.20 to this gives

 $\omega_n^2 = \frac{K}{J}$ 

and

$$2\zeta\omega_n = \frac{B}{J}$$

Based on the measured damping ratio and natural frequency, the friction (or stiffness) of the system is

$$K = J\omega_n^2 \tag{2.21}$$

and the viscous damping is

$$B = 2\zeta \omega_n J. \tag{2.22}$$

The natural frequency and damping ratio of a system can be found experimentally, e.g., from a free-oscillation response or frequency response.

#### 2.1.6 Power Spectrum

Fourier is a way to represent a signal in terms of sinusoidals. The Fourier transform, or spectrum, of a signal g(t) is denoted as  $G(\omega)$  and it shows the relative amplitudes and frequencies of the sinusoidals that are used to represent that signal [8].

The Fourier transform contains both the magnitude and phase information. The *power spectrum* of a signal is based on the magnitude of the signal. Figure 2.3, for example, show the power spectrum of the compound sine wave signal

$$g(t) = 3\sin 2\pi t + 2\sin 4\pi t + 0.5\sin 10\pi t.$$

The spectrum shows the peaks occurring at the sine wave frequencies 1, 2, and 5 Hz. Similarly, this can be used to find the resonant frequencies of an actual system.

The power of a signal is the time-average of the its squared value [8] and, for a continuous signal g(t), is defined

$$P_g = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} g(t)^2 dt.$$

We want to define the power of the signal from its transform. First consider the truncated signal of q(t) defined as

$$g_T(t) = \begin{cases} g(t) & |t| \le T \\ 0 & |t| > T \end{cases}$$

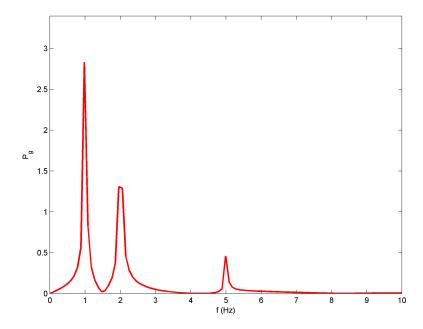


Figure 2.3: Power spectrum of compound sine wave

From Parseval's theorem, the energy of this signal equals [8]

$$E_{g_T} = \int_{-\infty}^{\infty} g_T(t)^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |G_T(\omega)|^2 d\omega$$

The power of a signal can then be expressed as

$$\begin{split} P_g &= \lim_{T \to \infty} \frac{E_{g_T}}{T} \\ &= \frac{1}{2\pi} \lim_{T \to \infty} \int_{-\infty}^{\infty} \frac{|G_T(\omega)|^2}{T} d\omega. \end{split}$$

The power spectral density (PSD) function is

$$S_g(\omega) = \lim_{T \to \infty} \frac{|G_T(\omega)|^2}{T}$$
 (2.23)

Defining the signal power in terms of the PSD and considering only positive frequencies we get

$$P_g = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_g(\omega) d\omega$$
$$= \frac{1}{\pi} \int_{0}^{\infty} S_g(\omega) d\omega$$

Expressing this in term of Hertz we get the expression

$$P_g = 2\int_0^\infty S_g(\omega)df \tag{2.24}$$

In practice, signals are sampled and the algorithm is performed in discrete mode. The Fast-Fourier Transform (FFT) is a computational algorithm used to find the Fourier transform of a signal, i.e., taking the FFT of a signal g(t) generates  $G(\omega)$ . To find the power spectrum, code similar to the following would be used



```
y = fft(g);
Sg = |y|/N;
Pg = 2*Sg(1:N/2);
```

where N is the number of samples, or length, of the signal g(t). If g(t) was sampled at a frequency interval of  $T_s$  and had a duration of  $T_s$ , it would have  $N = T/T_s$  samples. Notice that because this is implemented in discrete mode, we use the amount of samples,  $N_s$ , instead of the signal time duration,  $T_s$ , as used in the PSD expression in Equation 2.23.

The power spectrum can be used to find resonant frequencies of a system. One common technique is feeding a sine sweep (or chirp) signal to the system and measuring the corresponding output response. By taking its FFT and finding the spectrum, the user will see various peaks. These peaks represent the sine wave frequencies that describe the system.

## 2.2 Pre-Lab Questions

- 1. Energy is stored in the flexible joint springs as it rotates by an angle of  $\alpha$  (see Figure 2.1). Find the potential energy of the flexible joint using the parameters described in Section 2.1.
- 2. Find the total kinetic energy of the system contributed by the rotary servo,  $\theta$ , and the rotation of the link,  $\alpha$ . Use the parameters shown in Figure 2.2.
- 3. Compute the Lagrangian of the system.
- 4. Find the first Euler-Lagrange equation given in 2.3. Keep the equations in terms of applied torque,  $\tau$  (i.e., not in terms of DC motor voltage). Also make sure your equations follow the general form:  $J\ddot{x} + B\dot{x} + Kx = u$ .
- 5. Find the second Euler-Lagrange Equation 2.4.
- 6. Find the equations of motion:  $\ddot{\theta} = f_1(\theta, \dot{\theta}, \alpha, \dot{\alpha}, \tau)$  and  $\ddot{\alpha} = f_2(\theta, \dot{\theta}, \alpha, \dot{\alpha}, \tau)$ . Assume the viscous damping of the link is negligible, i.e.,  $B_l = 0$ .
- 7. Given state x defined in Equation 2.15, find the linear state-space matrices A and B.
- 8. Express the moment of inertia of the rotary arm in terms of the length and masses given in Figure 2.1. As already mentioned, the total arm length varies depending on where the short arm is mounted on top of the long arm. The inertia can be computed in the lab once you know how your ROTFLEX is configured.

**Hint:** The total inertia depends on where the top arm is anchored. Use the parallel-axis theorem to determine the inertia of the top link with respect to the pivot.



## 2.3 Lab Experiments

In the first part of this laboratory, the stiffness of the flexible joint is determined by measuring its natural frequency. In the second part, the state-space model is finalized and validated against actual measurements.

#### **Physical Parameters for the Lab**

In order to do some of the laboratory exercises, you will need these values:

 $B_{eq} = 0.004 \,\mathrm{N} \cdot \mathrm{m} \, / \, (\mathrm{rad/s})$   $J_{eq} = 2.08 \times 10^{-3} \,\mathrm{kg} \cdot \mathrm{m}^2$   $m_1 = 0.064 \,\mathrm{kg}$   $L_1 = 0.298 \,\mathrm{m}$   $m_2 = 0.030 \,\mathrm{kg}$   $L_2 = 0.156 \,\mathrm{m}$   $B_1 = 0$ 

**Note:** The equivalent viscous damping,  $B_{eq}$ , and moment of inertia,  $J_{eq}$ , parameters are for the SRV02 when there is no load (i.e., the parameter found in the SRV02 Modeling Laboratory was for servo with the disc load).

## 2.3.1 Finding Stiffness

To find the stiffness we need to find the natural frequency of the flexible joint. This is the frequency where the link attached to the flexible joint begins to oscillate the most. To find this frequency, we use a Sine Sweep signal. The Sine Sweep is a sine wave that goes through a range of frequencies, i.e., from low to high. We can then generate a power spectrum from the measured signal and identify the frequency with the largest amplitude - the natural frequency.

#### **Experimental Setup**

The ROTFLEX Sine Sweep VI shown in Figure 2.4 is used to find the natural frequency of the flexible joint. The VI interfaces with the DC motor and encoders of the system. The LabVIEW Chirp block generates a sine wave that goes from a low frequency to a high frequency for a predetermined duration.

**IMPORTANT:** Before you can conduct this experiment, you need to make sure that the lab files are configured according to your system setup. If they have not been configured already, then you need to go to Section 4.2 to configure the lab files first.

- 1. In *Rotary Flexible Joint.lvproj*, open the *ROTFLEX Sine Sweep* VI. Make sure the VI is configured for your data-acquisition system.
- 2. Run the VI. The SRV02 tracks a sine sweep that goes from 1 Hz to 5 Hz. The VI should be reading a response similarly as shown in Figure 2.4. Note that the VI is set to run for 15 seconds.
- 3. Plot the flexible joint response.
- 4. After the controller stops (i.e., after 15 sec), the data is automatically saved and output by the Collector block, as illustrated in Figure 2.5. Using the sweep response data, complete the MathScript node to generate a plot of the power spectrum. Once the script is complete, you will need to re-run the VI. The background theory and *pseudo code* to find the power spectrum was given in Section 2.1.6. You can also use any available Matlab examples to help you out. Show the commands you used and the plot generated.

**Hint:** When using the discrete Matlab FFT command fft(x,n), the size of n should be base of 2. Use the nextpow2 function to find this size. For example, if your signal size is 250 samples long, this will return 8 which gives  $n = 2^8 = 256$ .

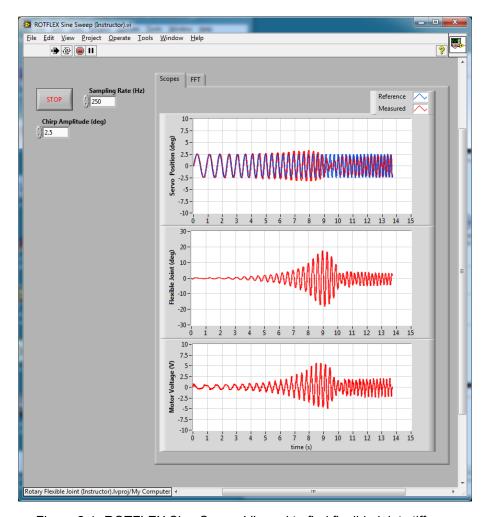


Figure 2.4: ROTFLEX Sine Sweep VI used to find flexible joint stiffness

- 5. Find the natural frequency of the link. Because the damping in relatively low, assume the damped natural frequency (which is being measured) is equivalent to the undamped natural frequency.
- 6. Find the total moment of inertia of the link,  $J_l$ , using the equation you developed in Exercise 8 in Section 2.2. The corresponding value of  $d_{12}$  is given in the Rotary Flexible Joint User Manual [7]. Once the inertia is computed, find the stiffness of the flexible joint,  $K_s$ .

## 2.3.2 State-Space Model

By running a simulation and the actual device in parallel, we can verify whether the dynamic model (which drives the simulation) accurately represents our system.

#### **Experimental Setup**

The ROTFLEX Modeling VI shown running in Figure 2.6 includes a MathScript node to generate the state-space model of the system and displays the open-loop poles.

- 1. In the Rotary Flexible Joint (Student).lvproj, open the ROTFLEX Modeling (Student).vi found in the Control Design and Simulation folder. The front panel of the VI is depicted in Figure 2.6.
- 2. Enter the moment of inertia you calculated in Step 6 of Section 2.3.1 in the *JI* control box on the front panel of the VI.



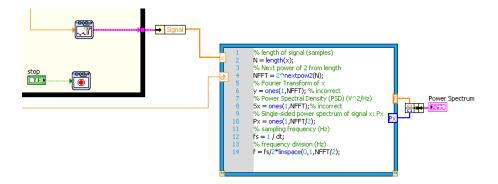


Figure 2.5: Complete the MathScript node to generate the power spectrum of the flexible joint response

- 3. Enter the stiffness you found in Section 2.3.1 in the Ks control box in the front panel.
- 4. Go to the VI block diagram. The MathScript node has the following state-space matrices entered:

```
A = [0 0 1 0;
    0 0 0 1;
    0 500 -5 0;
    0 -750 5 0];

B = [0 0 500 -500]';

C = zeros(2,4);

D = zeros(2,1);
```

5. Enter the state-space matrices you found in Section 2.2 for *A*, *B*, *C*, and *D*. Make sure you use the model parameter inputs that are defined, e.g., the stiffness and link moment of inertia are *Ks* and *Jl* and the SRV02 inertia and damping are denoted *Jeq* and *Beq*.

**Note:** The input of the state-space model you found in Section 2.2 is the torque acting at the servo load gear (i.e., at the pivot of the flexible joint). However, we do not control torque directly - we control the servo input voltage. Recall the voltage-torque relationship given in Equation 2.8 in Section 2.1.2. At the end of MathScript, the actuator dynamics are added to your state-space matrices with the code:

```
Ao = A;

Bo = B;

B = eta_g*Kg*eta_m*kt/Rm*Bo;

A(3,3) = Ao(3,3) - Bo(3)*eta_g*Kg^2*eta_m*kt*km/Rm;

A(4,3) = Ao(4,3) - Bo(4)*eta_g*Kg^2*eta_m*kt*km/Rm;
```

- 6. Run the VI. It should display a different state-space model. Click on the *Save Model* OK button to save the model on your system. Give the numerical matrices that are shown in the *Equation* display.
- 7. Complete the block diagram shown in Figure 2.7 to display the open-loop poles of the system in the *OL Pole-Zero Map* and *Poles* indicators (using the state-space model). Look through the *Control Design* palette and then run the VI to display the poles.

**Note:** These will be required for a pre-lab question in Section 3.3.

8. Now that your model is saved and the open-loop poles are recorded, you can close this VI.

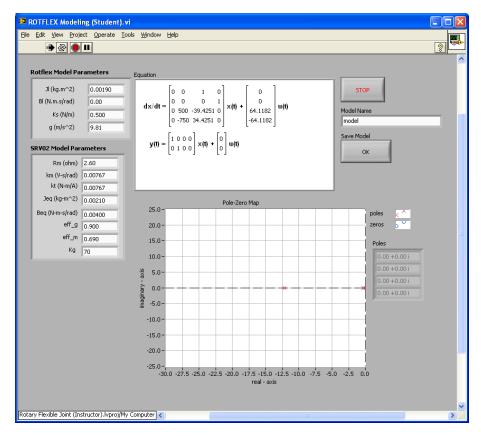


Figure 2.6: ROTFLEX Modeling VI used create state-space model of ROTFLEX system

### 2.3.3 Model Validation

By running a simulation and the actual device in parallel, we can verify whether the dynamic model (which drives the simulation) accurately represents our system.

#### **Experimental Setup**

The ROTFLEX Model Validation VI is shown running in Figure 2.8. It applies a step input to both the Quanser Flexible Joint hardware and to the Flexible Joint model and reads the servo and link angles. The VI contains drivers that interface to the actual hardware. It also reads the A, B, C, and D state-space matrices that is saved from the ROTFLEX Modeling VI.

**IMPORTANT:** Before you can conduct this experiment, you need to make sure that the lab files are configured according to your system setup. If they have not been configured already, then go to Section 4.2 to configure the lab files first.

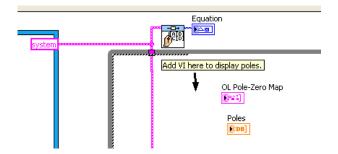


Figure 2.7: Complete block diagram to display poles.



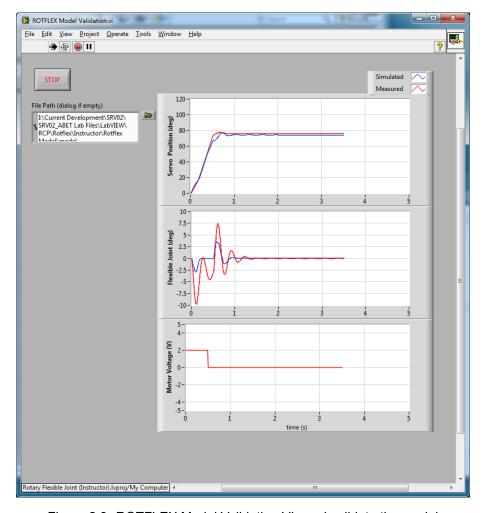


Figure 2.8: ROTFLEX Model Validation VI used validate the model

- 1. In the Rotary Flexible Joint (Student).lvproj, open the ROTFLEX Model Validation (Student) VI. Make sure the VI is configured for your data-acquisition system.
- 2. Make sure the area around the Flexible Joint experiment is clear.
- 3. In the VI block diagram (CTRL-E), double-click on the State-Space block and click on the 'Open' icon to load the model you saved previously from Section 2.3.2.
- 4. Run the VI. The VI is shown running in Figure 2.8. The Servo Position (deg) scope displays the simulated servo angle in blue and the measured angle in red. Similarly, the Flexible Joint (deg) scope shows the simulated flexible joint / link angle in blue and the measured angle in red.
- 5. If your simulation and measured response match, go to the next step. If they do not, then there is an issue with your model. Here are some issues to investigate:
  - Ensure the state-space model was entered properly in the script.
  - The stiffness,  $K_s$ , found in Section 2.3.1 is not correct. Review your calculations or redo the experiment.
  - Review your model derivation in Section 2.2, e.g., might be a mistake in solving the EOMs.
- 6. Plot the servo angle, link angle, and input voltage responses and attach them to your report. To save the plots, you can right-click on the chart and go to the *Export | Export Simplified Image* with *BMP* and *Clipboard* options.
- 7. How well does your model represent the actual system? We want a model that is fairly representative but, having said that, keep in mind that no model is perfect. This is just a quick test to see how well your model represents the actual device.

8.	s. Give at least one reason why the model does not represent the system accurately.		



## 2.4 Results

Fill out Table 2.1 with your answers from your modeling lab results - both simulation and implementation.

Description	Symbol	Value	Unit		
Finding Stiffness					
Natural frequency	$\omega_n$		rad/s		
Stiffness	$K_s$		N m/rad		
State-Space Model					
State-Space Matrix	A				
State-Space Matrix	В				
State-Space Matrix	С				
State-Space Matrix	D				
Open-loop poles	OL				

Table 2.1: Results

## 3 CONTROL DESIGN

## 3.1 Specifications

The time-domain requirements are:

**Specification 1:** Servo angle 4% settling time:  $t_s <= 0.5 \text{ s.}$ 

**Specification 2:** Servo angle percentage overshoot:  $PO \le 5$  %.

**Specification 3:** Maximum link angle deflection:  $|\alpha| <= 12.5 \text{ deg.}$ 

**Specification 4:** Maximum control effort / voltage:  $|V_m| <= 10 \text{ V}$ .

Desired closed-loop poles:

• Damping ratio:  $\zeta = 0.6$ .

• Natural frequency:  $\omega_n = 20$  rad/s.

• Non-dominant poles:  $\{p_3 = -30, p_4 = -40\}.$ 

Design a feedback controller that places the poles of the closed-loop system to the desired locations indicated. The servo settling time, overshoot, link deflection, and control effort time-domain requirements are to be satisfied when the servo is tracking a  $\pm 20$  degree angle square wave. Note for the settling time specifications, the servo angle must reach 4% of its final, steady-state angle in the alloted time. For instance if the measured steady-state servo angle is 20 degrees, then it must settle within  $20 \pm 0.8$  degrees in 0.5 seconds.

## 3.2 Background

In Section 2.2, we found a linear state-space model that represents the Rotary Flexible Joint system. This model is used to investigate the stability properties of the Flexible Joint system in Section 3.2.1. In Section 3.2.2, the notion of controllability is introduced. The procedure to transform matrices to their companion form is described in Section 3.2.3. Once in their companion form, it is easier to design a gain according to the pole-placement principles, which is discussed in Section 3.2.4. Lastly, Section 3.2.6 describes the state-feedback control used to control the servo position while minimizing link deflection that is introduced by the flexible joint.

## 3.2.1 Stability

The stability of a system can be determined from its poles ([9]):

- Stable systems have poles only in the left-hand plane.
- Unstable systems have at least one pole in the right-hand plane and/or poles of multiplicity greater than 1 on the imaginary axis.
- Marginally stable systems have one pole on the imaginary axis and the other poles in the left-hand plane.

The poles are the roots of the system's characteristic equation. From the state-space, the characteristic equation of the system can be found using

$$\det(sI - A) = 0 \tag{3.1}$$

where det() is the determinant function, s is the Laplace operator, and I the identity matrix. These are the eigenvalues of the state-space matrix A.



### 3.2.2 Controllability

If the control input, u, of a system can take each state variable,  $x_i$  where  $i = 1 \dots n$ , from an initial state to a final state then the system is controllable, otherwise it is uncontrollable ([9]).

Rank Test The system is controllable if the rank of its controllability matrix

$$T = \begin{bmatrix} B & AB & A^2B \dots A^nB \end{bmatrix}$$
 (3.2)

equals the number of states in the system,

$$rank(T) = n. ag{3.3}$$

## 3.2.3 Companion Matrix

If (A,B) are controllable and B is  $n \times 1$ , then A is similar to a companion matrix ([1]). Let the characteristic equation of A be

$$s^n + a_n s^{n-1} + \ldots + a_1.$$

Then the companion matrices of A and B are

$$\tilde{A} = \begin{bmatrix} 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ -a_1 & -a_2 & \cdots & -a_{n-1} & -a_n \end{bmatrix}$$
(3.4)

and

$$\tilde{B} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{3.5}$$

Define

$$W = T\tilde{T}^{-1}$$

where T is the controllability matrix defined in Equation 3.2 and

$$\tilde{T} = [\tilde{B} \ \tilde{B}\tilde{A} \dots \tilde{B}\tilde{A}^n].$$

Then

$$W^{-1}AW = \tilde{A}$$

and

$$W^{-1}B = \tilde{B}.$$

### 3.2.4 Pole Placement

If (A,B) are controllable, then pole placement can be used to design the controller. Given the control law u = -Kx, the state-space in Equation 2.13 becomes

$$\dot{x} = Ax + B(-Kx)$$
$$= (A - BK)x$$

To illustate how to design gain K, consider the following system

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 3 & -1 & -5 \end{bmatrix} \tag{3.6}$$

and

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{3.7}$$

Note that A and B are already in the companion form. We want the closed-loop poles to be at [-1-2-3]. The *desired* characteristic equation is therefore

$$(s+1)(s+2)(s+3) = s^3 + 6s^2 + 11s + 6$$
(3.8)

For the gain  $K = [k_1 \ k_2 \ k_3]$ , apply control u = -Kx and get

$$A - KB = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 3 - k_1 & -1 - k_2 & -5 - k_3 \end{bmatrix}.$$

The characteristic equation of A - KB is

$$s^{3} + (k_{3} + 5)s^{2} + (k_{2} + 1)s + (k_{1} - 3)$$
(3.9)

Equating the coefficients between Equation 3.9 and the desired polynomial in Equation 3.8

$$k_1 - 3 = 6$$
  
 $k_2 + 1 = 11$   
 $k_3 + 5 = 6$ 

Solving for the gains, we find that a gain of  $K = [9 \ 10 \ 1]$  is required to move the poles to their desired location.

We can generalize the procedure to design a gain K for a controllable (A,B) system as follows:

**Step 1** Find the companion matrices  $\tilde{A}$  and  $\tilde{B}$ . Compute  $W = T\tilde{T}^{-1}$ .

**Step 2** Compute  $\tilde{K}$  to assign the poles of  $\tilde{A} - \tilde{B}\tilde{K}$  to the desired locations. Applying the control law u = -Kx to the general system given in Equation 3.4,

$$\tilde{A} = \begin{bmatrix} 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \\ -a_1 - k_1 & -a_2 - k_2 & \cdots & -a_{n-1} - k_{n-1} & -a_n - k_n \end{bmatrix}$$
(3.10)

**Step 3** Find  $K = \tilde{K}W^{-1}$  to get the feedback gain for the original system (A,B).

**Remark:** It is important to do the  $\tilde{K} \to K$  conversion. Remember that (A,B) represents the actual system while the companion matrices  $\tilde{A}$  and  $\tilde{B}$  do not.



### 3.2.5 Desired Poles

The rotary inverted pendulum system has four poles. As depicted in Figure 3.1, poles  $p_1$  and  $p_2$  are the complex conjugate *dominant* poles and are chosen to satisfy the natural frequency,  $\omega_n$ , and damping ratio,  $\zeta$ , specifications given in Section 3.1. Let the conjugate poles be

$$p_1 = -\sigma + j\omega_d \tag{3.11}$$

and

$$p_2 = -\sigma - j\omega_d \tag{3.12}$$

where  $\sigma = \zeta \omega_n$  and  $\omega_d = \omega_n \sqrt{1-\zeta^2}$  is the *damped* natural frequency. The remaining closed-loop poles,  $p_3$  and  $p_4$ , are placed along the real-axis to the left of the dominant poles, as shown in Figure 3.1.

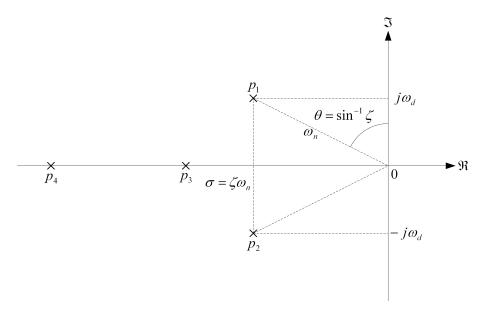


Figure 3.1: Desired closed-loop pole locations

#### 3.2.6 Feedback Control

The feedback control loop that in Figure 3.2 is designed to stabilize the servo to a desired position,  $\theta_d$ , while minimizing the deflection of the flexible link.

The reference state is defined

$$x_d = [\theta_d \ 0 \ 0 \ 0] \tag{3.13}$$

and the controller is

$$u = K(x_d - x). ag{3.14}$$

Note that if  $x_d = 0$  then u = -Kx, which is the control used in the control algorithm.

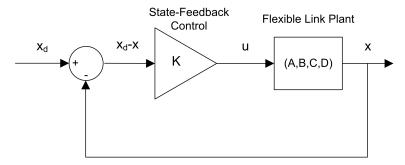


Figure 3.2: State-feedback control loop



## 3.3 Pre-Lab Questions

- 1. Based on your analysis in Section 2.3, is the system stable, marginally stable, or unstable? Did you expect the stability of the Rotary Flexibe Joint system to be as what was determined? If desired, use your experience with the actual device in the Modeling Laboratory in Section 2.3.
- 2. Using the open-loop poles, find the characteristic equation of the system.
- 3. Give the corresponding companion matrices  $\tilde{A}$  and  $\tilde{B}$ . Do not compute the transformation matrix W (this will be computed using software in the lab).
- 4. Find the location of the two dominant poles,  $p_1$  and  $p_2$ , based on the specifications given in Section 3.1. Place the other poles at  $p_3 = -30$  and  $p_4 = -40$ . Finally, give the *desired* characteristic equation.
- 5. When applying the control  $u=-\tilde{K}x$  to the companion form, it changes  $(\tilde{A},\tilde{B})$  to  $(\tilde{A}-\tilde{B}\tilde{K},\tilde{B})$ . Find the gain  $\tilde{K}$  that assigns the poles to their new desired location.

## 3.4 Lab Experiments

## 3.4.1 Control Design

The pole placement algorithm discussed in Section 3.2 is implemented in software to find the control gain that places the closed-loop poles at their desired location.

#### **Experimental Setup**

The ROTFLEX Control Design VI is shown running in Figure 3.3. It finds the necessary control gain in two ways: manually using companion matrices and using functions from the Control Design Module.

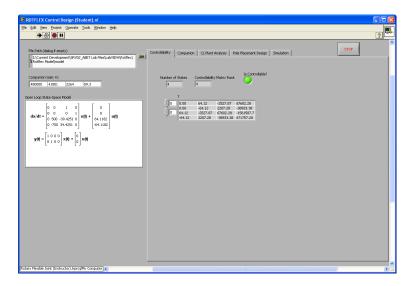


Figure 3.3: SRV02 RP PP Control Design (Student).vi when running

- 1. Under the Control Design and Simulation folder, open ROTFLEX Control Design (Student) VI.
- 2. Run the VI. It should look similarly as shown in Figure 3.3 (except using your model). The VI will prompt you to find a model file (unless you selected a model in *File Path* already). Find the model you saved from the previous modeling lab.
- 3. Go to the Controllability Test tab. Is the system controllable? Explain why.
- 4. Click on the *Companion* tab. The *Companion* matrices  $\tilde{A}$  and  $\tilde{B}$  for the model loaded are automatically found (denoted as Ac and Bc in LabVIEW). In order to find the gain K, we need to find the transformation matrix  $W = T\tilde{T}^{-1}$  (note:  $\tilde{T}$  is denoted as  $T_c$  in the VI). Stop the VI. In the VI block diagram, go to the *Companion* case. As shown in Figure 3.4, the block diagram is incomplete. Using the VIs from the *Diagram Disabled Structure*, calculate the controllability matrix T, the companion controllability matrix Tc, and the inverse of Tc. Show your completed block diagram and the resulting T, Tc, Inverse Tc, and Tc
- 5. Enter the companion gain,  $\tilde{K}$ , you found in the pre-lab in the *Companion Gain: Kc* input box on the VI front panel. Run the VI again to calculate the feedback gain K and record its value in Table 3.1.
- 6. Click on the *CL Plant Analysis*. This shows the closed-loop poles of the system, i.e., the eigenvalues of A-BK. Record the closed-loop poles of the system when using the gain K calculated above. Have the poles been placed to their desired locations? If not, then go back and re-investigate your control design until you find a gain that positions the poles to the required location.
- 7. Select the Pole Placement Design. No gain is generated because, as shown in Figure 3.5, the block diagram is not complete. Back in the Companion tab, the necessary matrices to find gain K were found manually through matrix operations. All that work can instead be done using the Pole Placement VI. Add the Pole Placement VI.



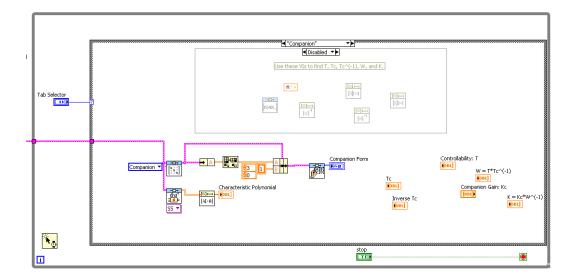


Figure 3.4: Incomplete block diagram of Companion tab

to the block diagram so the gain is automatically generated from the state-space model and the desired poles. The *Pole Placement* VI is found in the *Control Design* palette. Run the VI again and verify that the gain is the same as generated before.

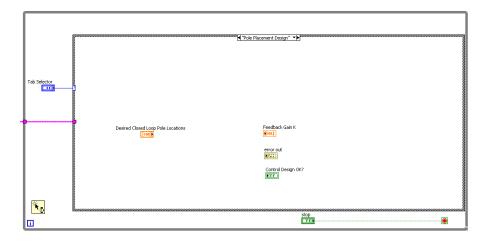


Figure 3.5: Incomplete block diagram of Pole Placement tab

#### 3.4.2 Control Simulation

Using the linear state-space model of the system and the designed control gain, the closed-loop response can be simulated. This way, we can test the controller and see if it satisfies the given specifications before running it on the hardware platform.

#### **Experimental Setup**

The *ROTFLEX Control Simulation* VI shown in Figure 3.6 is used to simulate the closed-loop response of the Flexible Joint using the control developed in Section 3.3.

The Signal Generator generates a square wave with an amplitude and frequency specified by the *Amplitude (deg)* and *Frequency (Hz)* controls. These are used to change the desired servo position command. By default, the servo command is a 0.33 Hz square wave of  $\pm$  20 degrees. This is passed through a *Rate Limiter* block to smooth the

signal (see the VI block diagram). The state-feedback gain K is set in the *Feedback Gain* control box on the front panel. The VI loads the state-space model specified by the *File Dialog* box.

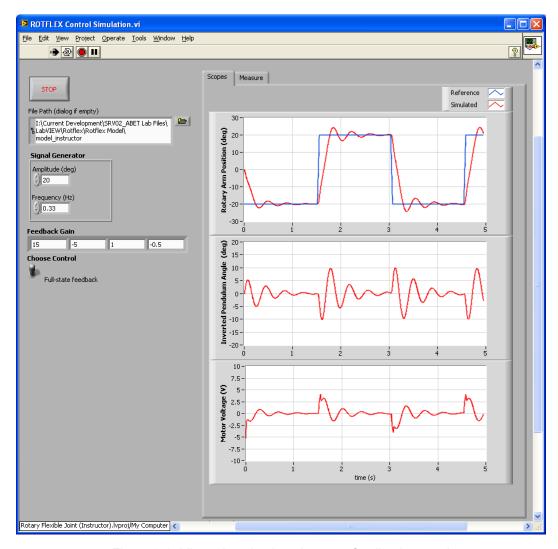


Figure 3.6: VI used to simulate the state-feedback control

**IMPORTANT:** Before you can conduct this experiment, you need to make sure that the lab files are configured according to your system setup. If they have not been configured already, go to Section 4.2 to configure the files first. Make sure the model you found in Section 2.3.3 is in *File Dialog* and the control gain K you found is entered in the *Feedback Gain* control.

- 1. In Rotary Flexible Joint (Student).lvproj, open the ROTFLEX Control Simulation VI.
- 2. Click the File Path control and find the model you saved from Section 2.3.2.
- 3. Enter the gain *K* you found in Section 3.4.1 in *Feedback Gain* control of the VI.
- 4. Set *Amplitude (deg)* to 20 and *Frequency (Hz)* to 0.33 to generate a ±20 degree 0.33 Hz square wave desired servo angle.
- 5. Make sure the *Choose Control* switch is set to the UPWARD *Full-State Feedback* position.
- 6. Run the VI. It should look similarly as shown in Figure 3.6 (except using your model and gain). The VI will prompt you to find a model file (unless you selected a model in *File Path* already).
- 7. Stop the VI, by clicking on the Stop button, once you have a suitable response.



- 8. Plot the simulated response of the servo, link, and motor input voltage obtained using your obtained gain *K* in a Matlab figure and attach it to your report.
- Measure the settling time and percentage overshoot of the simulated servo response, the maximum link deflection, and the voltage used. Does the response satisfy the specifications given in Section 3.1? Use the cursors in the graphs found in the *Measure* tab to take your measurements.

## 3.4.3 Control Implementation

In this experiment, the servo position is controlled while minimizing the link deflection using the control found in Section 3.4.1. Measurements will then be taken to ensure that the specifications are satisfied.

#### **Experimental Setup**

The ROTFLEX Control VI for NI-DAQmx shown in Figure 3.8 is used to run the state-feedback control on the Quanser Flexible Joint system. The VI interfaces with the DC motor and sensors of the system and implements the feedback developed in Section 3.4.1.

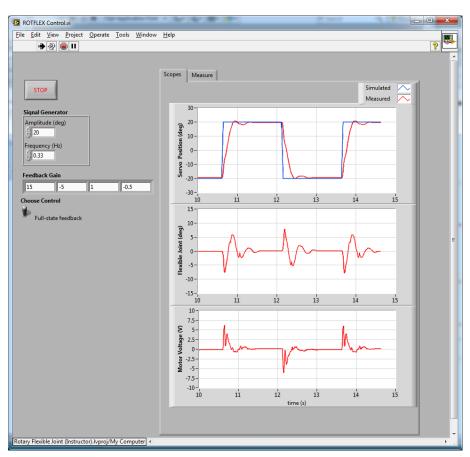


Figure 3.7: VI used to implement state-feedback control

**IMPORTANT:** Before you can conduct this experiment, you need to make sure that the lab files are configured according to your system setup. If they have not been configured already, then go to Section 4.2 to configure the lab files first.

- 1. In Rotary Flexible Joint (Student).lvproj, open the ROTFLEX Control VI. Make sure the VI is configured for your data-acquisition system.
- 2. Enter the gain K you found in Section 3.4.1 in Feedback Gain control of the VI.

- 3. Make sure the Manual Switch is set to the UPWARD Full-State Feedback position.
- 4. Set Amplitude (deg) to 20 and Frequency (Hz) to 0.33 to generate a  $\pm$ 20 degree 0.33 Hz square wave desired servo angle.
- 5. Run the VI. The flexible link should be tracking the default  $\pm$  20 degree signal. It should look similarly as shown in Figure 3.8 (except using your model and gain).
- 6. Stop the controller once you have obtained a representative response.
- 7. Plot the responses from the theta (deg), alpha (deg), and Vm (V) scopes.
- 8. Measure the settling time and percent overshoot of the measured servo response and the maximum link deflection. Does the response satisfy the specifications given in Section 3.1? As with the simulation, use the cursors in the graphs in the *Measure* tab to take your measurements.

### 3.4.4 Partial-State Feedback Control Implementation

In this section, the partial-state feedback response of the system is assessed and compared with the full-state feedback control in Section 3.4.3.

- 1. As in Section 3.4.3, open the ROTFLEX Control VI (for your DAQ configuration).
- 2. Ensure the control gain you settled on in Section 3.4.3 is entered in the Feedback Gain control.
- 3. Make sure the Choose Control switch is set to the Partial-State Feedback (downward) position.
- 4. Run the VI. The link should be tracking the default  $\pm 20$  degree square wave.
- 5. Stop the controller once you have obtained a representative response.
- 6. As in Section 3.4.3, attach plots representing the SRV02 angle and flexible link angle response, as well as the input voltage.
- 7. Measure the settling time and percentage overshoot of the measured servo response and the maximum link deflection. Does the response satisfy the specifications given in Section 3.1?
- 8. Examine the difference between the partial-state feedback (PSF) response and the full-state feedback (FSF) response. Explain why PSF control behaves this way by looking at the VI block diagram.



## 3.5 Results

Fill out Table 3.1 with your answers from your control lab results - both simulation and implementation.

Description	Symbol	Value	Units		
Pre Lab Questions					
Desired poles	DP				
Companion Gain	$ ilde{K}$				
Simulation: Control Design					
Transformation Matrix	W				
Control Gain	K				
Closed-loop poles	CLP				
Simulation: Closed-Loop System					
Maximum deflection	$ \alpha _{max}$		deg		
Maximum voltage	$ V_m _{max}$		V		
Implementation					
Control Gain	K				
Maximum deflection	$ \alpha _{max}$		deg		
Maximum voltage	$ V_m _{max}$		V		

Table 3.1: Results

## 4 SYSTEM REQUIREMENTS

#### **Required Hardware**

- Data-aquisition (DAQ) device that is compatible with LabVIEW™ and Quanser Rapid Control Prototyping Toolkit®, e.g., Q1\_cRIO, Q2-USB, Q8-USB, QPID/QPIDe, certain NI PCI/PCIe DAQ Devices.
- Quanser SRV02-ET rotary servo. See Reference [2].
- Quanser VoltPAQ power amplifier, or equivalent (e.g. Reference [4] for VoltPAQ User Manual).

#### **Required Software**

- NI LabVIEW™
- · NI LabVIEW Control Design and Simulation Module
- · NI LabVIEW MathScript RT Module
- For NI CompactRIO users:
  - NI LabVIEW Real-Time Module
  - NI LabVIEW FPGA Module
  - NI-RIO Drivers
- Quanser Rapid Control Prototyping Toolkit<sup>®</sup>



## 4.1 Overview of Files

File Name	Description
Flexible Joint User Manual.pdf	This manual describes the hardware of the Rotary Flexi-
	ble Joint system and explains how to setup and wire the
	system for the experiments.
Flexible Joint Lab Manual (Student).pdf	This laboratory guide contains pre-lab questions and lab
	experiments demonstrating how to design and implement
	a position controller on the Quanser SRV02 Flexible Joint plant using LabVIEW™.
Rotary Flexible Joint (Student).lvproj	LabVIEW project containing all the VIs the student needs
	to complete the labs.
ROTFLEX Modeling (Student).vi	Generates the state-space model of ROTFLEX system.
	Note: VI is incomplete.
ROTFLEX Sine Sweep (Student).vi	This VI feeds a Sine Sweep signal to the servo and mea-
	sures the corresponding Flexible Joint angle. The mea-
	sured response can then be used to find the natural fre-
	quency of the link. Note: FFT MathScript is not complete.
ROTFLEX Modeling Validation.vi	VI used to compare the Flexible Joint state-space model
	with the measured response from the actual system.
ROTFLEX Control Design (Student).vi	Finds the feedback control gain $K$ based on desired pole
	location. Note: VI is incomplete.
ROTFLEX Control Simulation.vi	Simulates the Flexible Joint system when using a full or
	partial state-feedback control.
ROTFLEX Control.vi	Implements the closed-loop state-feedback controller on
	the Quanser Rotary Flexible Joint system using Lab-
	VIEW™ . Full or partial-state feedback control can be se-
	lected.

Table 4.1: Files supplied with the SRV02 Flexible Joint Control Laboratory.

## 4.2 Software Setup

Follow these steps to get the system ready for this lab:

- 1. Load the LabVIEW™ software.
- 2. Open the LabVIEW project called Rotary Flexible Joint (Student).lvproj shown in Figure 4.1.
- 3. **Control Design and Simulation Labs**: To run a control design or simulation based VI, go to the *Control Design and Simulation* folder and open the VI that corresponds to the lab.
- Finding Stiffness Lab: To find the natural frequency of the Flexible Joint as in Section 2.3.1, open the ROT-FLEX Sine Sweep VI.
- 5. **Model Validation Lab:** When going through the lab explained in Section 2.3.3 to validate your model, open the *ROTFLEX Model Validation* VI.
- 6. **Control Implementation Lab:** When going through the control implementation lab in Section 3.4.3 or Section 3.4.4, open the *ROTFLEX Control* VI.
- 7. **Configure DAQ:** Before running the VI, make sure you set the correct *Board type* (e.g., 'q1\_cRIO', 'q2\_usb', 'q8\_usb', 'qpid', or 'qpid\_e') in the HIL Initialize block.
- 8. **Quanser CompactRIO Users:** Before running the VI, make sure you can connect to your CompactRIO through the Measurement & Automation software. See the SRV02 cRIO User Manual ([5]).

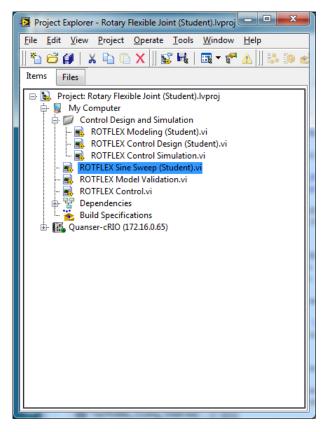


Figure 4.1: Flexible Joint LabVIEW Project (Student version).

9. **Channel Configuration:** For any of the DAQ-based VIs, the encoder input and ouput channels are set, by default, to match the wiring in the Rotary Flexible Joint User Manual ([7]). If the wiring is different on your system, make sure the VI uses the correct channels. For instance, if your joint encoder is connected to Encoder Input Channel #2 on your DAQ, then set the channels in the *HIL Read Encoder* VI to read from [0,2] (instead of [0,1]).



## 5 LAB REPORT

This laboratory contains two groups of experiments, namely,

- 1. Modeling the Quanser Rotary Flexible Joint system, and
- State-feedback control.

For each experiment, follow the outline corresponding to that experiment to build the *content* of your report. Also, in Section 5.3 you can find some basic tips for the *format* of your report.

## 5.1 Template for Content (Modeling)

#### I. PROCEDURE

- 1. Finding Stiffness
  - Briefly describe the main goal of the experiment.
  - Briefly describe the experiment procedure in Step 3 in Section 2.3.1.
- 2. State-Space Model
  - · Briefly describe the main goal of the experiment.
  - Briefly describe creating the model in Step 6 in Section 2.3.2.
- 3. Model Validation
  - Briefly describe the main goal of the experiment.
  - Briefly describe the model validation experiment in Step 6 in Section 2.3.3.

#### II. RESULTS

Do not interpret or analyze the data in this section. Just provide the results.

- 1. Frequency-response plot from step 3 in Section 2.3.1.
- 2. Model validation plot from step 6 in Section 2.3.3.
- 3. Provide applicable data collected in this laboratory (from Table 2.1).

#### III. ANALYSIS

Provide details of your calculations (methods used) for analysis for each of the following:

- 1. Measured link stiffness in step 6 in Section 2.3.1.
- 2. Model discrepancies given in step 8 in Section 2.3.3.

#### **IV. CONCLUSIONS**

Interpret your results to arrive at logical conclusions for the following:

1. How does the model compare with the actual system in step 7 of Section 2.3.3, State-space model validation.

## 5.2 Template for Content (Control)

#### I. PROCEDURE

- 1. Control Design
  - · Briefly describe the main goal of the control design.
  - Briefly describe the control design procedure in Step 4 in Section 3.4.1.

#### 2. Simulation

- Briefly describe the main goal of the simulation.
- Briefly describe the simulation procedure in Step 8 in Section 3.4.2.
- 3. Full-State Feedback Implementation
  - · Briefly describe the main goal of this experiment.
  - Briefly describe the experimental procedure in Step 7 in Section 3.4.3.
- 4. Partial-State Feedback Implementation
  - · Briefly describe the main goal of this experiment.
  - Briefly describe the experimental procedure in Step 6 in Section 3.4.4.

#### II. RESULTS

Do not interpret or analyze the data in this section. Just provide the results.

- 1. Matrices from Step 4 in Section 3.4.1, Find transformation matrix.
- 2. Response plot from step 8 in Section 3.4.2, Full-state feedback controller simulation.
- 3. Response plot from step 7 in Section 3.4.3, for *Full-state feedback controller implementation*.
- 4. Response plot from step 6 in Section 3.4.4, for Partial-state feedback controller implementation.
- 5. Provide applicable data collected in this laboratory (from Table 3.1).

#### III. ANALYSIS

Provide details of your calculations (methods used) for analysis for each of the following:

- 1. Controllability of system in Step 3 in Section 3.4.1.
- 2. Closed-loop poles in Step 6 in Section 3.4.1.
- 3. LabVIEW block diagram used to generate the control gain in Step 7 in Section 3.4.1.
- 4. Settling time, percent overshoot, link deflection, and input voltage Step 9 in Section 3.4.2, *Full-state feedback controller simulation*.
- 5. Settling time, percent overshoot, link deflection, and input voltage in Step 8 in Section 3.4.3, *Full-state feedback controller implementation*.
- 6. Settling time, percent overshoot, link deflection, and input voltage in Step 7 in Section 3.4.3, *Partial-state feedback controller implementation*.
- 7. Comparison between partial-state and full-state feedback in step 8 in Section 3.4.4.

#### IV. CONCLUSIONS

Interpret your results to arrive at logical conclusions for the following:



- 1. Whether the controller meets the specifications in Step 9 in Section 3.4.2, *Full-state feedback controller simulation*.
- 2. Whether the controller meets the specifications in Step 8 in Section 3.4.3, for *Full-state feedback controller implementation*.
- 3. Whether the controller meets the specifications in Step 7 in Section 3.4.4, for *Partial-state feedback controller implementation*.

## 5.3 Tips for Report Format

#### **PROFESSIONAL APPEARANCE**

- Has cover page with all necessary details (title, course, student name(s), etc.)
- Each of the required sections is completed (Procedure, Results, Analysis and Conclusions).
- · Typed.
- · All grammar/spelling correct.
- · Report layout is neat.
- · Does not exceed specified maximum page limit, if any.
- · Pages are numbered.
- Equations are consecutively numbered.
- Figures are numbered, axes have labels, each figure has a descriptive caption.
- Tables are numbered, they include labels, each table has a descriptive caption.
- Data are presented in a useful format (graphs, numerical, table, charts, diagrams).
- No hand drawn sketches/diagrams.
- · References are cited using correct format.



## REFERENCES

- [1] Bruce Francis. Ece1619 linear systems course notes (university of toronto), 2001.
- [2] Quanser Inc. SRV02 User Manual, 2009.
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