

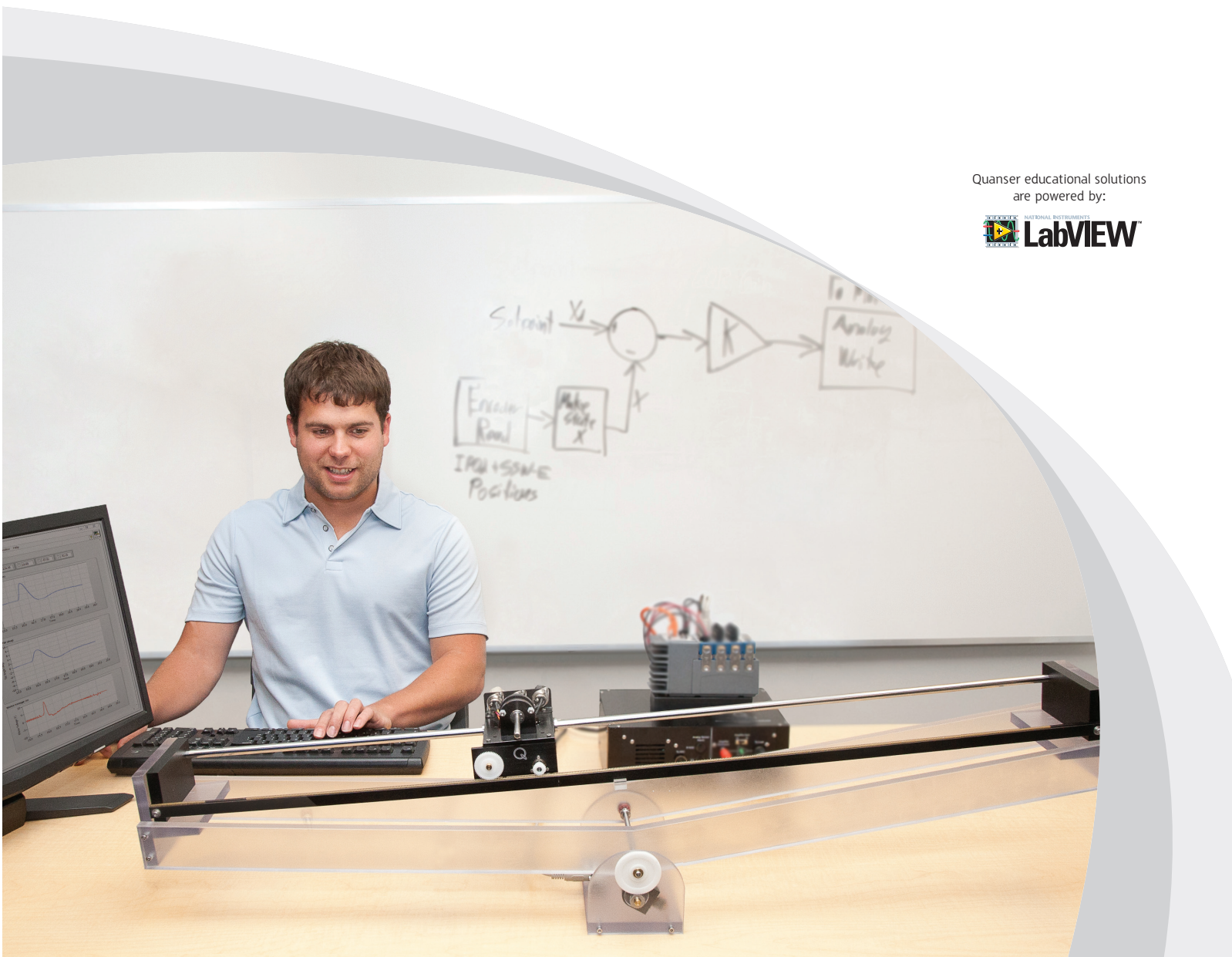


LABORATORY GUIDE

Seesaw Experiment for LabVIEW™ Users

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1 INTRODUCTION

This laboratory manual describes how to design a state-feedback control system that can balance a seesaw using the IP02 linear servo cart.

The plant has two main components: the Quanser IP02 linear motion plant and the Quanser SEESAW-Emodule. The seesaw is composed of a long polycarbonate platform hinged onto a central pivot. An encoder is integrated into the pivot to measure the linear seesaw tilt angle.

Topics Covered

- Obtain a state-space representation of the open-loop system.
- Design and tune an LQR-based state-feedback controller satisfying the closed-loop system's desired design specifications.
- Simulate the system and ensure it is stabilized using the designed state-feedback control.
- Implement the state-feedback controller on the SEESAW system and evaluate its actual performance.

Prerequisites

In order to successfully carry out this laboratory, the user should be familiar with the following:

1. See the system requirements in Section 4 for the required hardware and software.
2. Modeling and state-space representation.
3. State-feedback design using Linear-Quadratic Regular (LQR) optimization.
4. Basics of **LabVIEW™**.
5. LabVIEW Integration lab detailed in Appendix A in the IP02 Lab Workbook [2].

2 BACKGROUND

2.1 Modeling

2.1.1 Model Convention

The linear seesaw model is shown in Figure 2.1. The SEESAW-E supports the IP02 Linear Servo Base Unit. The positive sense of rotation is defined to be counter-clockwise (CCW), when facing the linear cart and seesaw pinions. The positive direction of linear displacement of the IP02 cart is to the right when facing the cart. Finally, the zero angle tilt, $\theta = 0$, of the seesaw corresponds to the seesaw perfectly horizontal.

The IP02 cart location is at linear position x_c , while the absolute position of the cart centre of mass is located at coordinates, (X_c, Y_c) . The IP02 cart assembly has mass, m_c , and is actuated by an applied force, F_c . The total mass of the linear seesaw and the IP02 track is m_{sw} , with moment of inertia, J_{sw} . The centre of mass of the linear seesaw and IP02 track is located at absolute coordinates, (X_{sw}, Y_{sw}) . The distance from the linear seesaw pivot and the centre of mass of the linear seesaw and IP02 track is, D_c . The IP02 track is located distance, D_t , above the linear seesaw pivot.

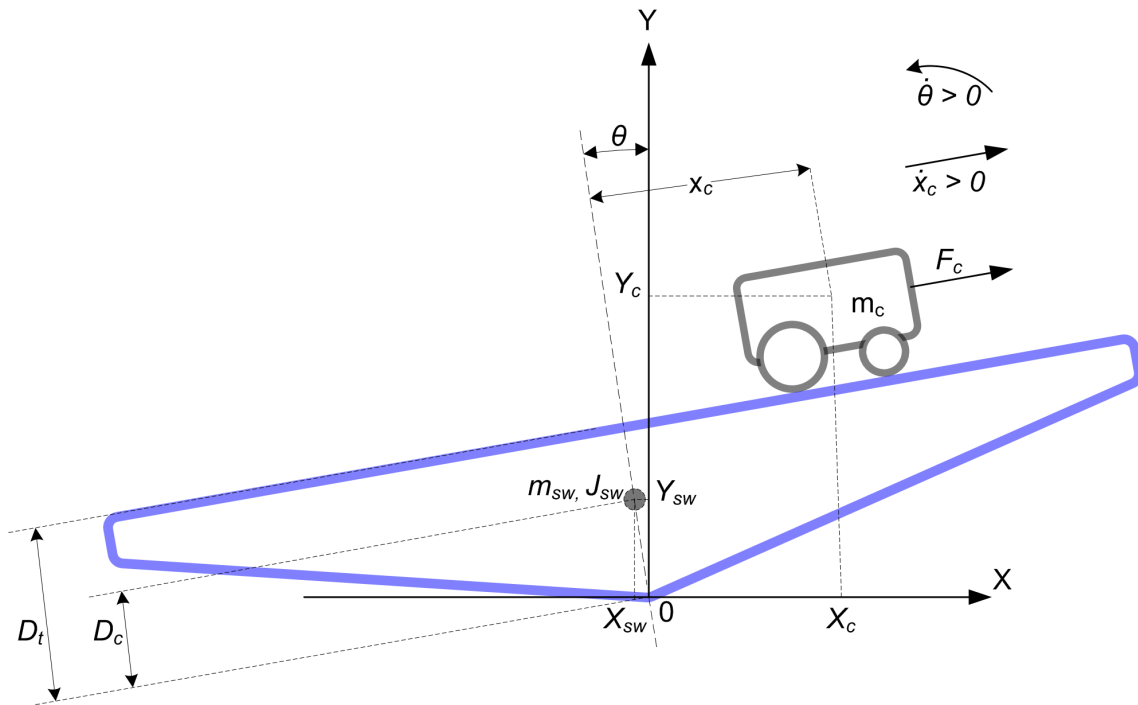


Figure 2.1: Linear Seesaw conventions

2.1.2 Nonlinear Equations of Motion

Instead of using classical mechanics, the Lagrange method is used to find the equations of motion of the system. This systematic method is often used for more complicated systems such as robot manipulators with multiple joints.

More specifically, the equations that describe the motions of the IP02 cart and the seesaw with respect to the servo motor voltage, i.e. the dynamics, will be obtained using the Euler-Lagrange equation:

$$\frac{\partial^2 L}{\partial t \partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = Q_i$$

The variables q_i are called *generalized coordinates*. For this system let

$$q(t)^\top = [x_c(t) \quad \theta(t)]$$

where, as shown in Figure 2.1, $\theta(t)$ is the seesaw tilt angle and $x_c(t)$ is the linear cart position. The corresponding velocities are

$$\dot{q}(t)^\top = \left[\frac{\partial x_c(t)}{\partial t} \quad \frac{\partial \theta(t)}{\partial t} \right]$$

Note: The dot convention for the time derivative will be used throughout this document, e.g., $\dot{\theta} = \frac{d\theta}{dt}$. The time variable t will also be dropped from θ and x_c , i.e., $\theta = \theta(t)$, $x_c = x_c(t)$.

With the generalized coordinates defined, the Euler-Lagrange equations for the rotary pendulum system are

$$\begin{aligned} \frac{\partial^2 L}{\partial t \partial \dot{x}_c} - \frac{\partial L}{\partial x_c} &= Q_1 \\ \frac{\partial^2 L}{\partial t \partial \dot{\theta}} - \frac{\partial L}{\partial \theta} &= Q_2 \end{aligned}$$

The Lagrangian of the system is described

$$L = T - V$$

where T is the total kinetic energy of the system and V is the total potential energy of the system. Thus the Lagrangian is the difference between a system's kinetic and potential energies.

The generalized forces Q_i are used to describe the non-conservative forces (e.g., friction) applied to a system with respect to the generalized coordinates. In this case, the generalized force acting on the linear cart is

$$Q_1 = F_c - B_{eq}\dot{x}_c, \quad (2.1)$$

and the force acting on the seesaw is

$$Q_2 = -B_{sw}\dot{\theta}. \quad (2.2)$$

Our control variable is the input servo motor voltage, V_m . Opposing the applied force resultant from the motor voltage is the equivalent damping, corresponding to the term B_{eq} . Since the seesaw is not actuated, the only force acting on the link is the damping, B_{sw} .

The Euler-Lagrange equations is a systematic method of finding the equations of motion, i.e., EOMs, of a system. Once the kinetic and potential energy are obtained and the Lagrangian is found, then the task is to compute various derivatives to get the EOMs. After going through this process, the nonlinear equations of motion for the system can be obtained. **See the supplied Maple worksheet (or its equivalent HTML representation) for the complete derivation.**

Based on the system schematic shown in Figure 2.1 and the generalized forces Equation 2.1 and Equation 2.2, the first Lagrange equation can be expressed as:

$$m_c \ddot{x}_c - m_c D_t \ddot{\theta} - m_c x_c \dot{\theta}^2 + g m_c \sin(\theta) = F_c - B_{eq} \dot{x}_c,$$

and the second Lagrange equation can be more explicitly expressed as shown below:

$$m_c \ddot{\theta} x_c^2 + 2m_c \dot{x}_c \dot{\theta} + g m_c \cos(\theta) x_c - m_c D_t \ddot{x}_c + (J_{sw} + m_c D_t^2) \ddot{\theta} + g(-m_c D_t \sin(\theta) - m_{sw} D_c \sin(\theta)) = -B_{sw} \dot{\theta}$$

Solving the two Lagrange equations for the second-order time derivative of the Lagrangian coordinates results in the following non-linear equations:

$$\ddot{x}_c = \frac{1}{(m_c x_c^2 + J_{sw})m_c} \left(m_c^2 x_c^3 \dot{\theta}^2 + (F_c m_c - g m_c^2 \sin(\theta) - B_{eq} \dot{x}_c m_c) x_c^2 + (-2m_c^2 D_t \dot{x}_c \dot{\theta} + (m_c^2 D_t^2 + m_c J_{sw}) \dot{\theta}^2 - m_c^2 D_t g \cos(\theta)) x_c + (-m_c D_t^2 B_{eq} - B_{eq} J_{sw}) \dot{x}_c - m_c D_t g m_{sw} D_c \sin(\theta) \right)$$

and

$$\ddot{\theta} = \frac{1}{m_c x_c^2 + J_{sw}} \left((D_t m_c \dot{\theta}^2 - 2m_c \dot{x}_c \dot{\theta} - g m_c \cos(\theta)) x_c + D_t F_c - D_t B_{eq} \dot{x}_c + g m_{sw} D_c \sin(\theta) - B_{sw} \dot{\theta} \right)$$

The force applied to the linear cart, F_c , is generated by the servo motor as described by the equation

$$F_c = \left(\frac{\eta_g K_g K_t}{R_m r_{mp}} \right) \left(-\frac{K_g K_m \dot{x}_c}{r_{mp}} + \eta_m V_m \right) \quad (2.3)$$

See [1] for a description of the corresponding IP02 parameters (e.g. such as the back-emf constant, K_m).

2.1.3 Linearizing

Here is an example of how to linearize a two-variable nonlinear function called $f(z)$. Variable z is defined

$$z^\top = [z_1 \ z_2]$$

and $f(z)$ is to be linearized about the operating point

$$z_0^\top = [a \ b]$$

The linearized function is

$$f_{lin} = f(z_0) + \left(\frac{\partial f(z)}{\partial z_1} \right) \bigg|_{z=z_0} (z_1 - a) + \left(\frac{\partial f(z)}{\partial z_2} \right) \bigg|_{z=z_0} (z_2 - b)$$

2.1.4 Linear State-Space Model

The linear state-space equations are

$$\dot{x} = Ax + Bu \quad (2.4)$$

and

$$y = Cx + Du \quad (2.5)$$

where x is the state, u is the control input, A , B , C , and D are state-space matrices. For the linear seesaw system, the state and output are defined

$$x^\top = [x_c \ \theta \ \dot{x}_c \ \dot{\theta}]$$

and

$$y^\top = [x_1 \ x_2].$$

After linearizing the nonlinear equations of motion about the zero tilt (balanced) position, and substituting the state given in Equation 2.1.4, we obtain the following state-space matrices:

$$A = \frac{1}{J_{sw}} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -m_c D_t g & \frac{-g m_c J_{sw} + m_c D_t g m_{sw} D_c}{m_c} & \frac{-m_c D_t^2 B_{eq} - B_{eq} J_{sw}}{m_c} & -D_t B_{sw} \\ -g m_c & g m_{sw} D_c & -D_t B_{eq} & -B_{sw} \end{bmatrix}$$

and

$$B = \frac{1}{J_{sw}} \begin{bmatrix} 0 \\ 0 \\ \frac{J_{sw} + m_c D_t^2}{m_c} \\ D_t \end{bmatrix}.$$

In the output equation, only the position of the linear servo cart and seesaw tilt angle are being measured. Based on this, the C and D matrices in the output equation are

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (2.6)$$

and

$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (2.7)$$

Note: The velocities of the servo and seesaw angle can be computed in the digital controller, e.g., by taking the derivative and filtering the result through a high-pass filter.

2.2 Control

In Section 2.1, we found a linear state-state space model that represents the SEESAW-E system. This model is used to investigate the stability properties of the system in Section 2.2.1. In Section 2.2.2, the notion of controllability is introduced. Using the Linear Quadratic Regular algorithm, or LQR, is a common way to find the control gain and is discussed in Section 2.2.3. Lastly, Section 2.2.4 describes the state-feedback control used to control the servo position while minimizing link deflection.

2.2.1 Stability

The stability of a system can be determined from its poles ([4]):

- Stable systems have poles only in the left-hand plane.
- Unstable systems have at least one pole in the right-hand plane and/or poles of multiplicity greater than 1 on the imaginary axis.
- Marginally stable systems have one pole on the imaginary axis and the other poles in the left-hand plane.

The poles are the roots of the system's characteristic equation. From the state-space, the characteristic equation of the system can be found using

$$\det(sI - A) = 0 \quad (2.8)$$

where $\det()$ is the determinant function, s is the Laplace operator, and I the identity matrix. These are the *eigenvalues* of the state-space matrix A .

2.2.2 Controllability

If the control input, u , of a system can take each state variable, x_i where $i = 1 \dots n$, from an initial state to a final state then the system is controllable, otherwise it is uncontrollable ([4]).

Rank Test The system is controllable if the rank of its controllability matrix

$$T = [B \ AB \ A^2B \ \dots \ A^nB] \quad (2.9)$$

equals the number of states in the system,

$$\text{rank}(T) = n. \quad (2.10)$$

2.2.3 Linear Quadratic Regular (LQR)

If (A,B) are controllable, then the Linear Quadratic Regular optimization method can be used to find a feedback control gain. Given the plant model in Equation 2.4, find a control input u that minimizes the cost function

$$J = \int_0^{\infty} x(t)' Q x(t) + u(t)' R u(t) dt, \quad (2.11)$$

where Q and R are the weighting matrices. The weighting matrices affect how LQR minimizes the function and are, essentially, tuning variables.

Given the control law $u = -Kx$, the state-space in Equation 2.4 becomes

$$\begin{aligned} \dot{x} &= Ax + B(-Kx) \\ &= (A - BK)x \end{aligned}$$

2.2.4 Feedback Control

The feedback control loop that in Figure 2.2 is designed to balance the seesaw by regulating the seesaw tilt angle, θ .

The reference state is defined

$$x_d = [0 \quad 0 \quad 0 \quad 0]$$

and the controller is therefore

$$u = -Kx, \quad (2.12)$$

which is the control used in the LQR algorithm.

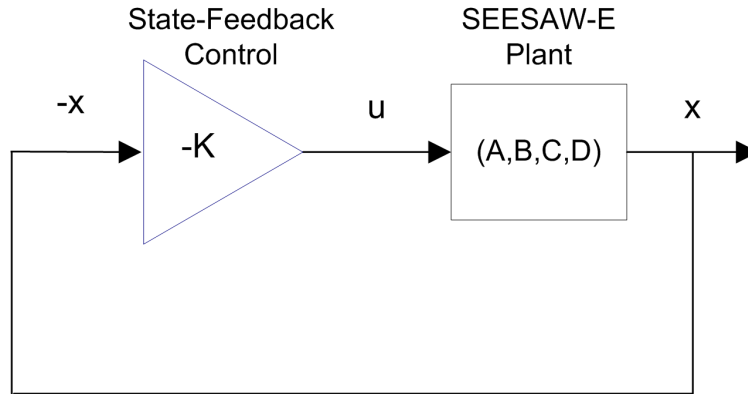


Figure 2.2: State-feedback control loop

To eliminate servo tracking error, we can augment the system to include an integrator such that

$$\dot{\eta} = \begin{bmatrix} A & 0 \\ 1 & 0 \end{bmatrix} \eta + \begin{bmatrix} B \\ 0 \end{bmatrix} u$$

where A and B are the state-space matrices defined in Section 2.1.4 and the states are

$$\eta^T = [x_c \quad \theta \quad \dot{x}_c \quad \dot{\theta} \quad \int \theta dt]$$

This introduces the integration terms $\eta_7(t) = \int \theta dt$ to the feedback controller

$$u = K(-\eta).$$

3 LAB EXPERIMENTS

3.1 Simulation

In this section we will use the VI shown in Figure 3.1 to simulate the closed-loop control of the SEESAW-E system. The system is simulated using the linear model summarized in Section 2.1. The VI uses the state-feedback control described in Section 2.2.4. The feedback gain K is found using the LQR command from the *Control Design and Simulation Toolkit* (LQR is described briefly in Section 2.2.3). The goal is to make sure the gain used successfully stabilizes the system (i.e., keeps it balanced), and does not saturate the dc motor.

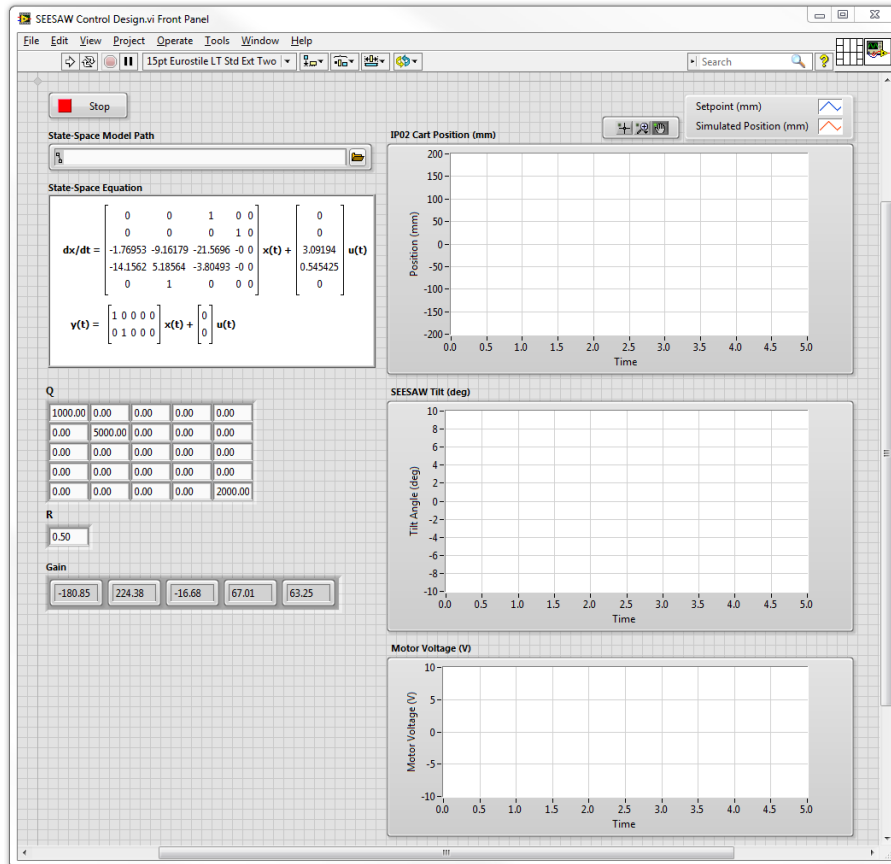


Figure 3.1: VI used to simulate SEESAW-E.

IMPORTANT: Before you can conduct these experiments, you need to make sure that the lab files are configured according to your setup. If they have not been configured already, then you need to go to Section 4 to configure the lab files first.

3.1.1 Procedure

Follow these steps to simulate the system:

1. Open and run *SEESAW Control Design.vi* as described in Section 4. Make sure you choose your model file using the *Model Path* control. The model file is generated using the *SEESAW Modeling VI* by entering the state space model and exporting the resultant model file.

- By default, the Q matrix is set to identity matrix. Set the LQR weighting matrices to

$$Q = \begin{bmatrix} 1000 & 0 & 0 & 0 & 0 \\ 0 & 5000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2000 \end{bmatrix}$$

and

$$R = 0.5.$$

- This automatically generates the gain

$$K = \begin{bmatrix} 180.797 & -224.3485 & 16.6684 & -66.9969 & -63.2456 \end{bmatrix}.$$

LQR Tuning: When tuning the LQR, we start with the identity matrix. To put more emphasis on the seesaw tilt angle θ , we set $Q(2, 2) = 5000$. The last diagonal element, $Q(5, 5)$ is set to 2000 to generate an integral gain for the seesaw tilt regulation.

- Run the VI. The scopes should be displaying responses similar to Figure 3.2.

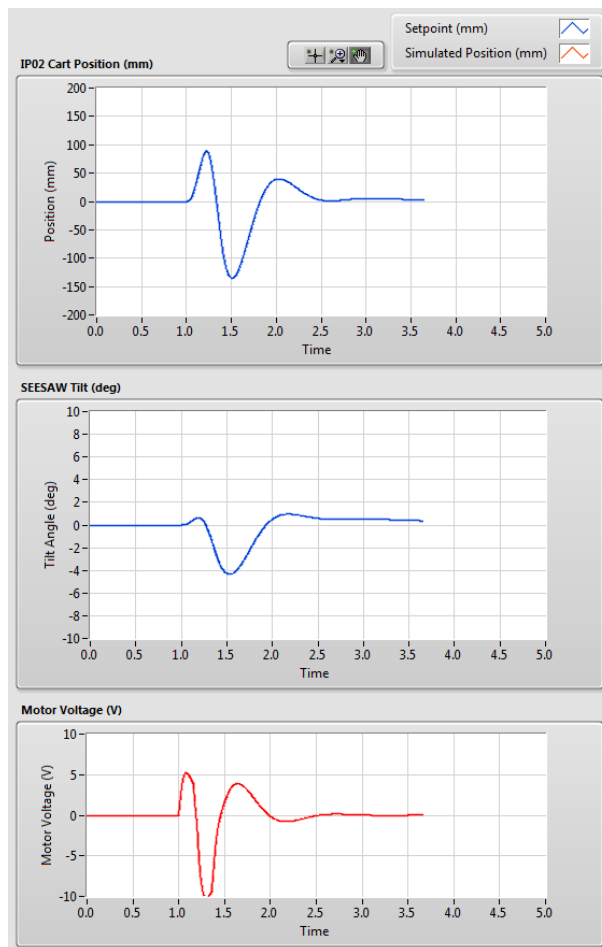


Figure 3.2: Simulated closed-loop response.

- This is an iterative design process. You can update your Q and R matrices, and acquire a new control gain K on-the-fly.
- Click on the *STOP* button to stop running the VI.

3.1.2 Analysis

As shown by the response in Figure 3.2, the seesaw maintains its balanced horizontal position despite the applied 5 degree disturbance. Further analysis can be performed using the *Graph Palette*.

3.2 Implementation

The *SEESAW Balance Control VI* shown in Figure 3.3 is used to perform the balance control on the SEESAW. The VI contains **Quanser Rapid Control Prototyping Toolkit®** blocks that interface with the dc motor and sensors of the SEESAW system.

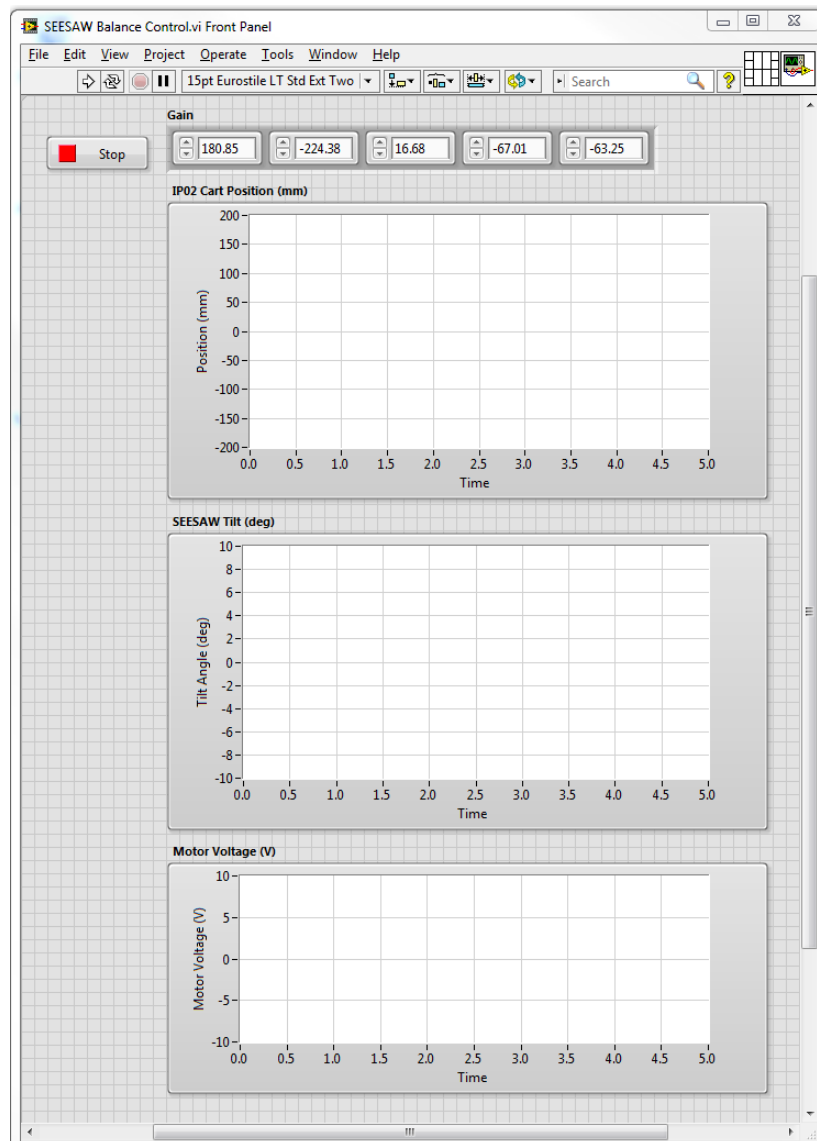


Figure 3.3: VI used to run controller on the SEESAW.

IMPORTANT: Before you can conduct these experiments, you need to make sure that the lab files are configured according to your setup. If they have not been configured already, then you need to go to Section 4 to configure the lab files first.

3.2.1 Procedure

Follow this procedure:

1. Make sure gain K is set to the gain you found and simulated in Section 3.1.
2. **Make sure that the seesaw is horizontal, and the cart is close to the centre of the track.**
3. Run the VI.
4. Release the seesaw. The scopes should be displaying responses similar to Figure 3.4.

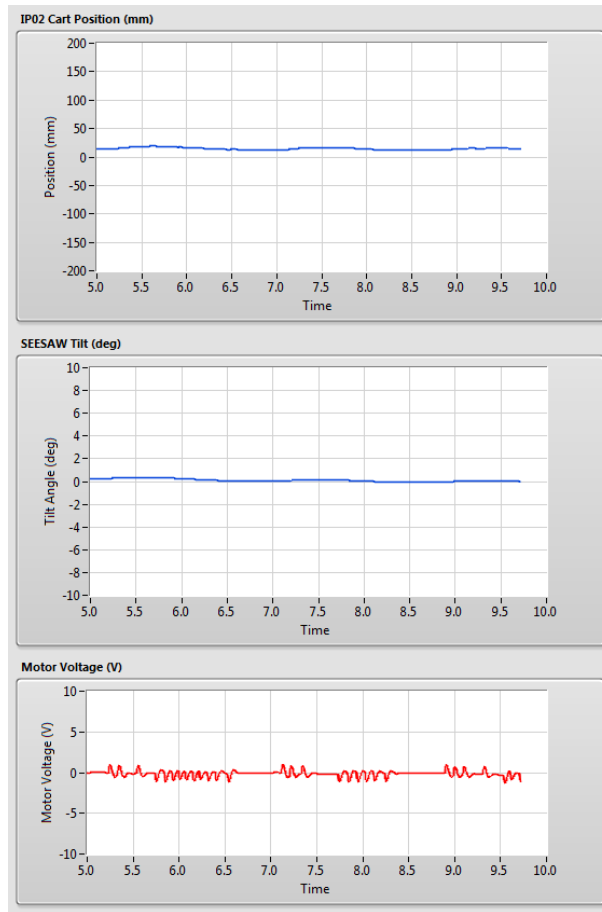


Figure 3.4: Typical response when balancing the SEESAW system

5. In order to verify that the LQR tuned response satisfies your design specifications, an external disturbance needs to be introduced. Once the system has stabilized, gently tap the seesaw to introduce a disturbance of about ± 5 degrees from the horizontal equilibrium position. The system response should resemble the scopes shown in Figure 3.5.

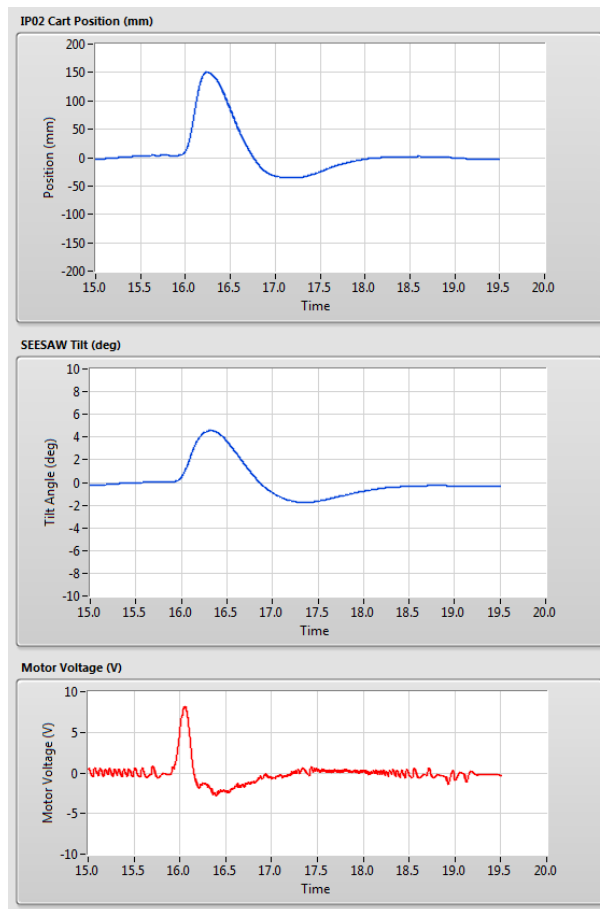


Figure 3.5: Typical response when disturbing the SEESAW system

6. To stop the experiment, click on the *Stop* button but **make sure you catch the seesaw before it swings down**. This will prevent the assembly from hitting the table surface.

3.2.2 Analysis

The balance control response is shown in Figure 3.6.

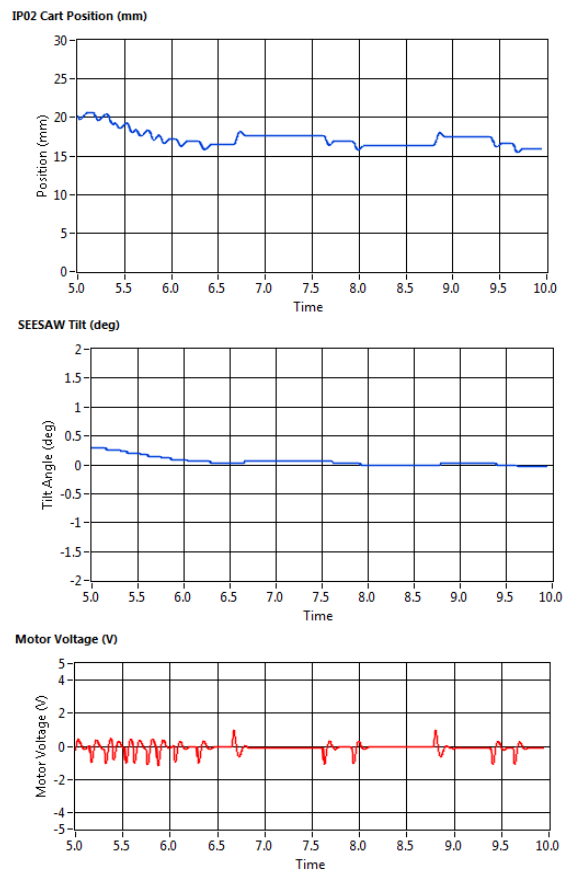


Figure 3.6: SEESAW balance control response

Due to the friction in the system, the servo oscillates back-and-forth slightly to balance the seesaw. Because of the integrator, the seesaw eventually stabilizes about the 0 degree setpoint.

4 SYSTEM REQUIREMENTS

Required Software

Make sure **LabVIEW™** is installed with the following required add-ons:

1. **LabVIEW™**
2. NI-DAQmx
3. NI **LabVIEW™** Control Design and Simulation Module
4. NI **LabVIEW™** MathScript RT Module
5. **Quanser Rapid Control Prototyping Toolkit®**

Note: Make sure the Quanser Rapid Control Prototyping (RCP) Toolkit is installed after LabVIEW. See the RCP Toolkit Quick Start Guide for more information.

Required Hardware

- Data acquisition (DAQ) device **with 2x encoder inputs** and that is compatible with **Quanser Rapid Control Prototyping Toolkit®**.
- Quanser IP02 linear servo.
- Quanser SEESAW-E (positioned underneath the IP02).
- Quanser VoltPAQ-X1 power amplifier, or equivalent.

Before Starting Lab

Before you begin this laboratory make sure:

- **LabVIEW™** is installed on your PC.
- DAQ device has been successfully tested (e.g., using the test software in the Quick Start Guide or the *Analog Loopback Demo*).
- SEESAW-E and amplifier are connected to your DAQ board as described its User Manual [3].

4.1 Overview of Files

File Name	Description
SEESAW-E User Manual.pdf	This manual describes the hardware of the SEESAW system and explains how to setup and wire the system for the experiments.
SEESAW-E Laboratory Manual.pdf	This document demonstrates how to obtain the linear state-space model of the system, simulate the closed-loop system, and implement controllers on the SEESAW plant using LabVIEW™.
SEESAW Project.lvproj	LabVIEW project that contains all the VIs required for the lab.
SEESAW Modeling.vi	VI used to generate the linear state-space model of the SEESAW system.
SEESAW Control Design.vi	VI used to design the LQR state-feedback gain and simulate the SEESAW system.
SEESAW Balance Control.vi	VI that implements the state-feedback control on the SEESAW system.
SEESAW.mws	Maple worksheet used to develop the model for the SEESAW experiment. Waterloo Maple 9, or a later release, is required to open, modify, and execute this file.
SEESAW.html	HTML presentation of the Maple Worksheet. It allows users to view the content of the Maple file without having Maple 9 installed. No modifications to the equations can be performed when in this format.

Table 4.1: Files supplied with the SEESAW

4.2 Setup for Simulation

Before beginning the in-lab procedure outlined in Section 3.1, the modeling and control design VIs must be configured.

Follow these steps:

1. Load LabVIEW™.
2. Open the *SEESAW Project.lvproj* LabVIEW project, shown in Figure 4.1.
3. Open the *SEESAW Modeling.vi* shown in Figure 4.2.
4. The seesaw and IP02 parameters are already set, by default. Run the VI to generate the linear state-space model.
5. In *Model Name*, enter the name of the model you and click on OK. This will save the state-space model under the folder *Model Files*. You can close this VI now.
6. Open the *SEESAW Control Design* VI, shown in Figure 3.1.
7. Using the *File Path* control, select the model file.
8. Run the VI. The state-space model should load. You are now ready to design your LQR control and simulate the closed-loop response.

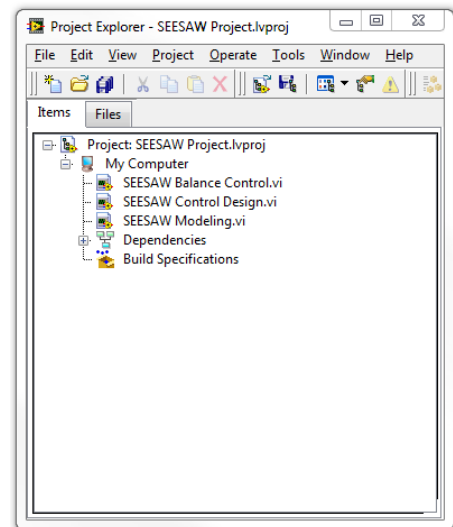


Figure 4.1: LabVIEW SEESAW-E Project

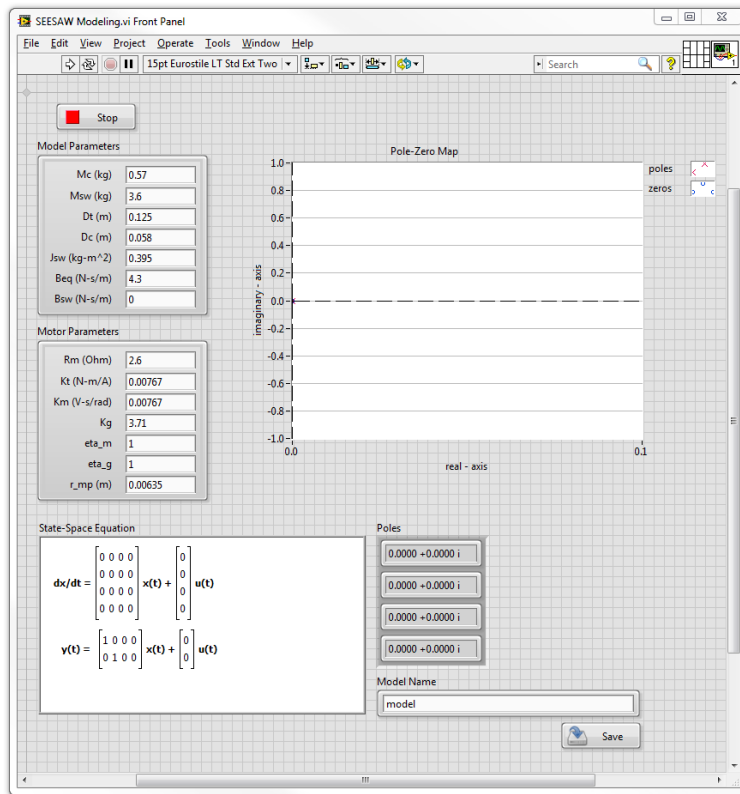


Figure 4.2: SEESAW-E Modeling VI

4.3 Setup for Running on SEESAW

Before performing the in-lab exercises in Section 3.2, the SEESAW system and the *SEESAW Balance Control.vi* must be configured properly.

Follow these steps to get the system ready for this lab:

1. Setup the IP02 with the SEESAW module as detailed in the SEESAW User Manual [3].
2. **Make sure that the seesaw is horizontal, and the cart is close to the centre of the track.** For more information, go to the SEESAW User Manual [3].
3. Open the *SEESAW Balance Control.vi*, shown in Figure 3.3.
4. Set gain K control in the VI to the value found in Section 4.2 (or another gain you want to test on the system).
5. **Configure DAQ:** Ensure the HIL Initialize block is configured for the DAQ device that is installed in your system. To do this, go to the block diagram (CTRL-E) and double click on the **HIL Initialize** Express VI shown in Figure 4.3.

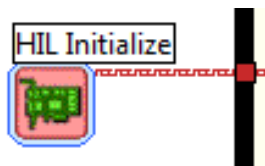


Figure 4.3: HIL Initialize Express VI

6. Under the Main tab, select the data acquisition device that is installed on your system in the *Board type* section. For example, in Figure 4.4 the Q2-USB is chosen.

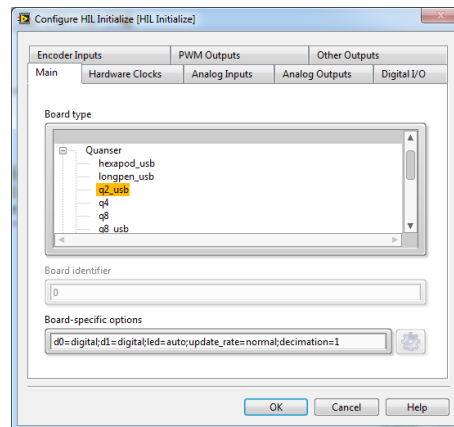


Figure 4.4: Select DAQ board that will be used to control system

REFERENCES

- [1] Quanser Inc. *IP02 User Manual*, 2009.
- [2] Quanser Inc. *IP02 Lab Workbook (LabVIEW)*, 2012.
- [3] Quanser Inc. *SEESAW-E User Manual*, 2012.
- [4] Norman S. Nise. *Control Systems Engineering*. John Wiley & Sons, Inc., 2008.

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