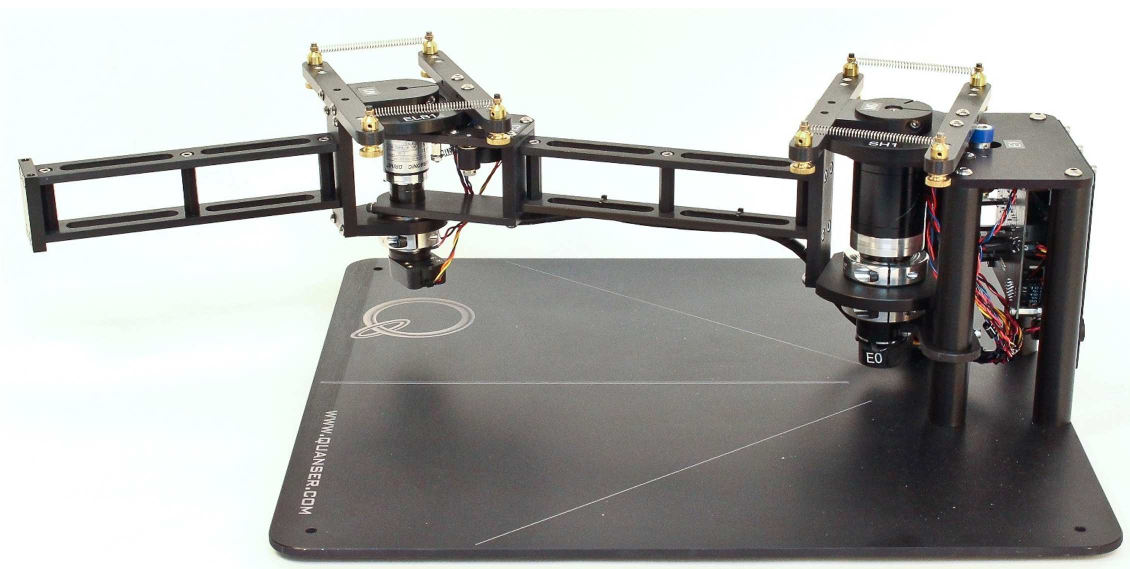
*Specialty Plant: 2DSFJ Robot*

# *2-DOF Serial Flexible Joint Robot*




# Reference Manual

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# 1. QUARC Application Examples

## 1.1. About QUARC

QUARC is Quanser's state-of-the-art rapid prototyping and production system for real-time control. QUARC integrates seamlessly with Simulink to allow Simulink models to be run in real-time on a variety of targets, such as Windows, and QNX. For details on installing QUARC, please see [1]. For information about using the QUARC software please see [2]. With QUARC interfacing to hardware becomes as easy as placing blocks in your Simulink diagram.

***CAUTION:***
For safety reasons when interfacing to the 2DSFJ drive #1, the *Saturation* block should be used to limit the current sent to the corresponding DC motor. Its parameters should have the following values: *Upper Limit*: **0.94** A, and *Lower Limit*: **-0.94** A. These settings are illustrated in Figure 1 and agree with the system specifications given in Error: Reference source not found.

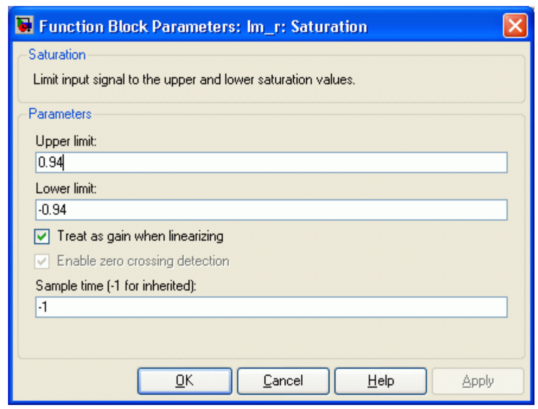

Figure 1 Saturation Settings For Drive #1

## 1.2. Two-DOF Serial Flexible Joint Robot – Decoupled System: Vibration Control

This Section outlines the design of a control system to reduce the vibration of the Two-Degree-Of-Freedom Serial Flexible Joint robot. A decoupled approach is used, which is to say that link coupling of the serial mechanism is neglected. Both drives are commanded independently of each other, each using a separate state-feedback control loop obtained using Linear Quadratic Regulation.

### 1.2.1. Operating Procedure

***Note:***

Before running the following example, ensure that the system is cabled and configured as detailed in Section 4.1. Also ensure that all four encoders of the 2DSFJ system are working properly before proceeding. The controller tuning described hereafter assumes that no load is attached to the 2DSFJ end-effector.

Open the Simulink model named *q_2DSFJ_robot_QuaRC.mdl*. Align the two flexible joints in their central position and start the *q_2DSFJ_robot_QuaRC.mdl* model. Please note that you need to run the setup script called *setup_2DSFJ_robot.m* prior to running the model as this file sets the required parameters for proper operation of the Simulink model and the hardware itself.

Each of the two flexible joints (i.e., stage 1 and stage 2) should now be tracking two ± 20-degree angular position trajectories in the form of square waves.. Typical system responses should look similar to the ones represented in the Scopes shown in Figures 2, 3, 4 and 5 where the square waves frequency is 0.1 Hz. In order to command each link tip of the device to a desired position, each of the two actuators (harmonic drives) has its own position control loop. Each uses a state-feedback control scheme tuned with the Linear-Quadratic Regulator (LQR) algorithm. The vibration of both links should be significantly minimized by the two state-feedback controllers. If the system does not track the prescribed trajectory, please review your wiring or contact Quanser technical support as detailed in the Obtaining Support section of this document.

Figures 2, 3, 4, and 5, depict the responses of the 2DSFJ system from the same run. For example Figure 2 corresponds to the Scope located at *q_2DSFJ_robot_QuaRC/2-DOF SFJ Robot + Q8: Actual Plant/Stage 1/Scopes/theta11 (deg)*, which plots the reference/desired (green trace), simulated (red trace), and actual position (blue trace) responses of the 2DSFJ $\theta_{11}$ angular output in degrees. It can be seen in Figures 2, 3, 4, and 5 that both square wave trajectories are out of phase. This is done so that link coupling can be best observed. It is uncompensated for, as the implemented controller design assumed a decoupled system.

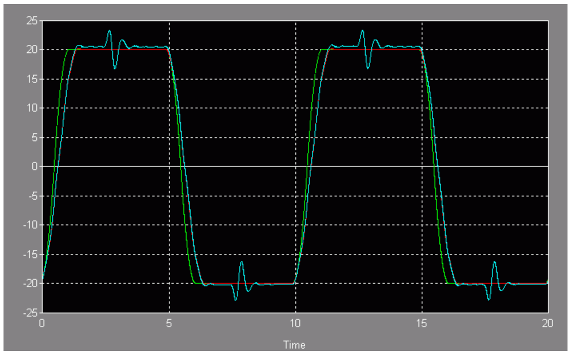


Figure 2 Drive #1 Load Shaft Angular Response: $\theta_{11}$

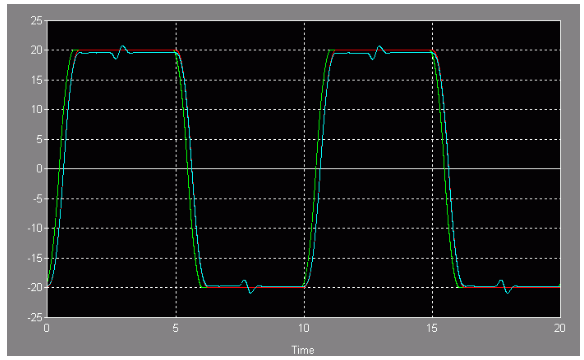


Figure 3 Flexible Joint #1 Angular Response: $\theta_{12}$

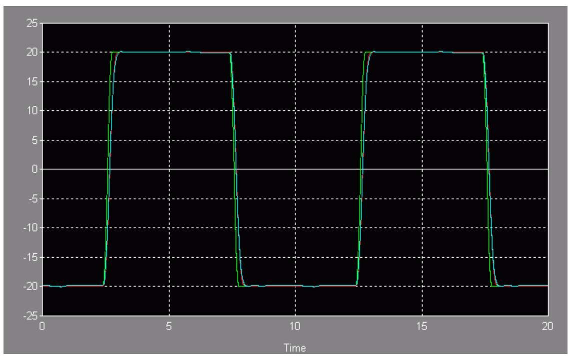


Figure 4 Drive #2 Load Shaft Angular Response: $\theta_{21}$

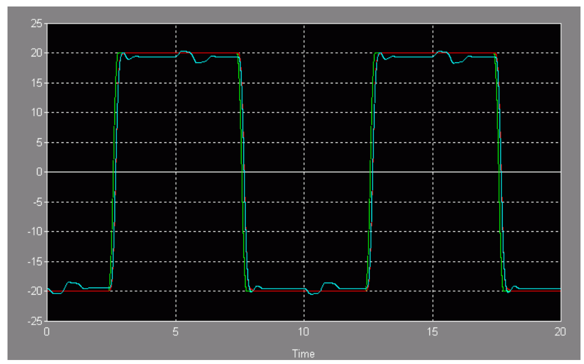


Figure 5 Flexible Joint #2 Angular Response: $\theta_{22}$

Moreover, the square wave setpoint generation for each drive is done using the *Continuous Sigmoid* block (which is located under *QuaRC Targets* | *Sources* | *Sigmoids* library in the Simulink Library Browser) in order to limit the setpoint maximum velocity and maximum acceleration. This is done so that the physical limitations of the system are respected. It results that both command currents never go into saturation. Also the maximum flexible joint deflection is limited, as shown by the two Scopes represented in Figures 6 and 7 (obtained from the same run as previously described).

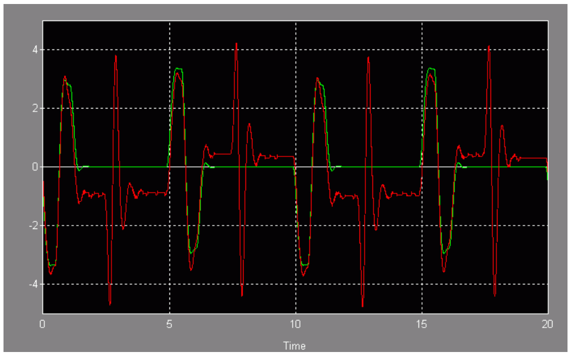

Figure 6 Flexible Joint #1 Deflection: $d\theta_1 = \theta_{12} - \theta_{11}$

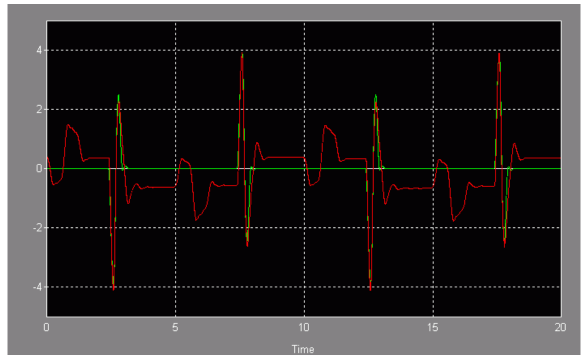

Figure 7 Flexible Joint #2 Deflection: $d\theta_2 = \theta_{22} - \theta_{21}$

In Figures 8, 9, 10, and 11, the controller switches between Full-State Feedback (FSF) and Partial-State Feedback (PSF) once every other square wave period. This is done to best demonstrate the improvement due to FSF, when compared to PSF, in terms of vibration minimization and speed of response of the first flexible joint output angle, as seen in Figures 8, 9. During this run, the second flexible joint controller tries to regulate a constant zero position. The coupling due to the first flexible joint can also be seen in Figures 10 and 11. In Full-State Feedback (FSF) mode for flexible joint #1, all four system position states are used by the control law. In Partial-State Feedback (PSF) mode, the flexible joint angle and angular velocity feedbacks are removed (i.e., multiplied by zero) and only drive #1 output shaft position is controlled. In PSF, the torsional load is actually ignored by the controller and it can then be considered as being in open-loop, oscillating at its natural frequency.

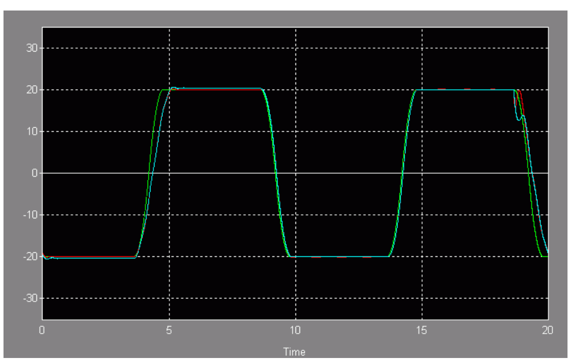

Figure 8 Drive #1 Response – FSF vs. PSF: $\theta_{11}$

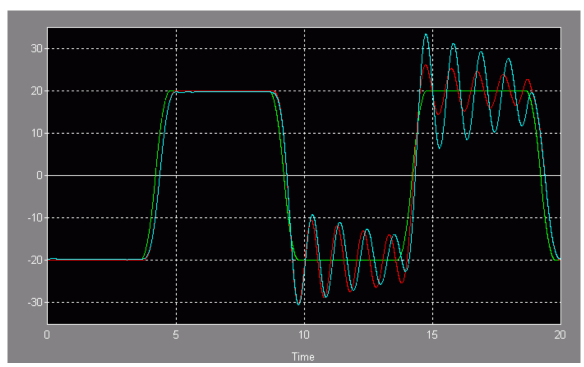

Figure 9 Flexible Joint #1 Response – FSF vs. PSF: $\theta_{12}$

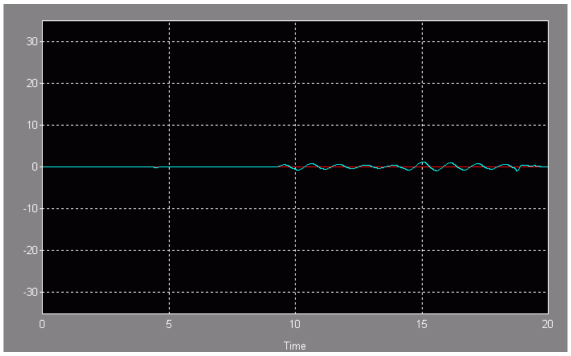


Figure 10 Drive #2 Response – FSF vs. PSF: $\theta_{21}$

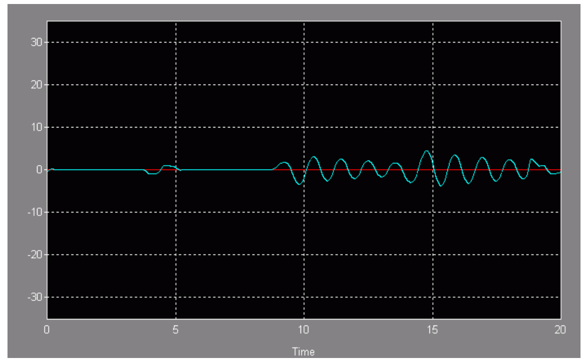


Figure 11 Flexible Joint #2 Response – FSF vs. PSF: $\theta_{22}$

The two state-feedback controller gain vectors and the model parameters are initialized in the MATLAB workspace by running the file *setup_2DSFJ_robot.m* in the MATLAB prompt. In order to control each flexible joint output to a desired position, the LQR algorithm is used. You can modify both *q_2DSFJ_robot_QuaRC.mdl* and/or *setup_2DSFJ_robot.m* files and re-generate the corresponding real-time code using QuaRC. To compile the real-time code corresponding to the controller diagram, use the *QuaRC | Build* option from the Simulink menu bar. After successful compilation click on *QuaRC | Start* in order to start running the real-time code on the actual plant.

Moreover, the Simulink-implemented controller model comes with two position watchdogs. They would stop the controller if the first flexible joint deflection exceeds ± 25° (as set by default by the *DTH1_MAX* variable in the setup script) or if the second flexible joint goes beyond ± 25° (as set by default by the *DTH2_MAX* variable in the setup script).

Running the *setup_2DSFJ_robot.m* script can also, if the corresponding flag variables (e.g., *SYS_ANALYSIS_1*, *PLOT_RESPONSE_1*) are enabled, simulate, analyse, and plot the Two-Degree-Of-Freedom Serial Flexible Joint system responses, using the MATLAB Control System Toolbox. For example the simulated magnitude Bode response plots for the second flexible joint, in both FSF and PSF modes, are represented in Figures 12 and 13. Comparing Figure 12 with Figure 13 shows the elimination of the resonance peak of the torsional system position response ($\theta_{22}$).

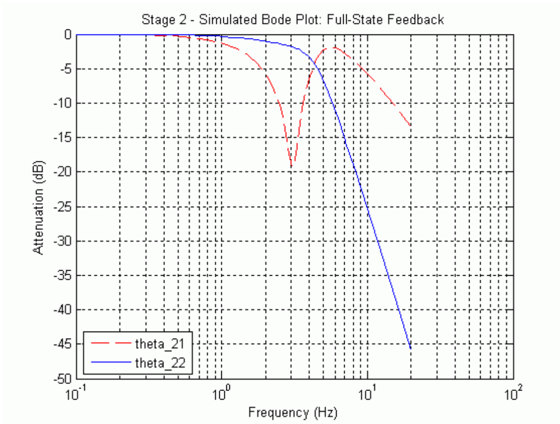

Figure 12 2nd Flexible Joint Magnitude Bode Plot: FSF

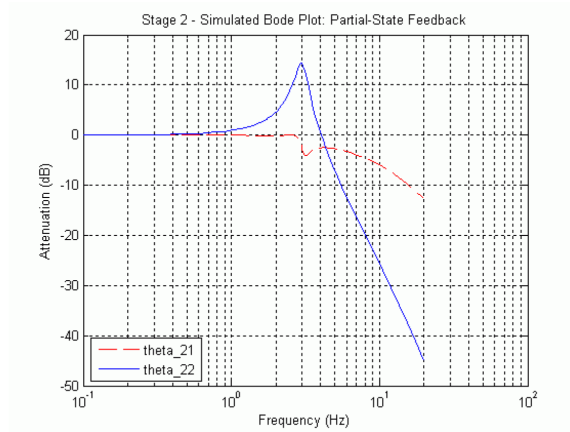

Figure 13 2nd Flexible Joint Magnitude Bode Plot: PSF

## 1.2.2. LQR Controller Design

A schematic of the Two-Degree-Of-Freedom Serial Flexible Joint (2DSFJ) system is represented in Figure 14. It depicts the two flexible joints connected in series and each actuated by its own drive system.

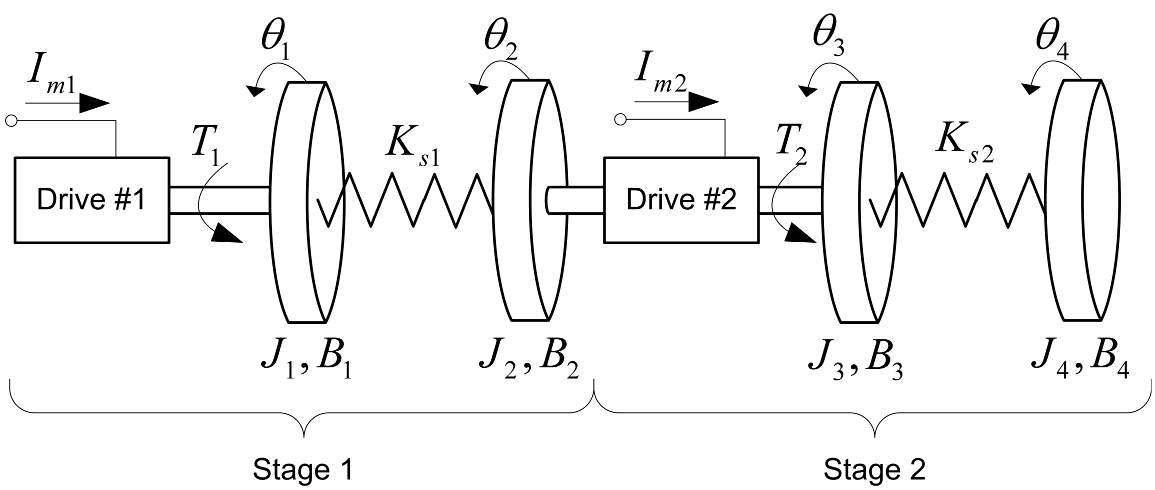

Figure 14 Schematic of the Two-Degree-Of-Freedom Serial Flexible Joint (2DSFJ) System

where $K_{s1}$ and $K_{s2}$ are the first and second flexible joint torsional stiffness constants, $I_{m1}$ and $I_{m2}$ the drive currents, $J_i$ (for $i$ = 1, 2, 3, 4) the intermediary load moments of inertia, and $B_i$ (for $i$ = 1, 2, 3, 4) the intermediary load viscous damping coefficients.

***Sign Convention:***

The positive direction of rotation, as illustrated in Figure 14 for all four load angles è$_i$ (for *i* = 1, 2, 3, 4) is chosen to be to CounterClockWise (CCW) when looking at the robot from top.

In the controller design procedure described hereafter, the 2DSFJ system is considered de-coupled and split into two separate and independent stages: Stage 1 and Stage 2, as depicted in Figure 14. Each stage has its own LQR state-feedback control loop.

Let us first consider the stage 1 system of the 2DSFJ plant. Its schematic is represented in Figure 15.

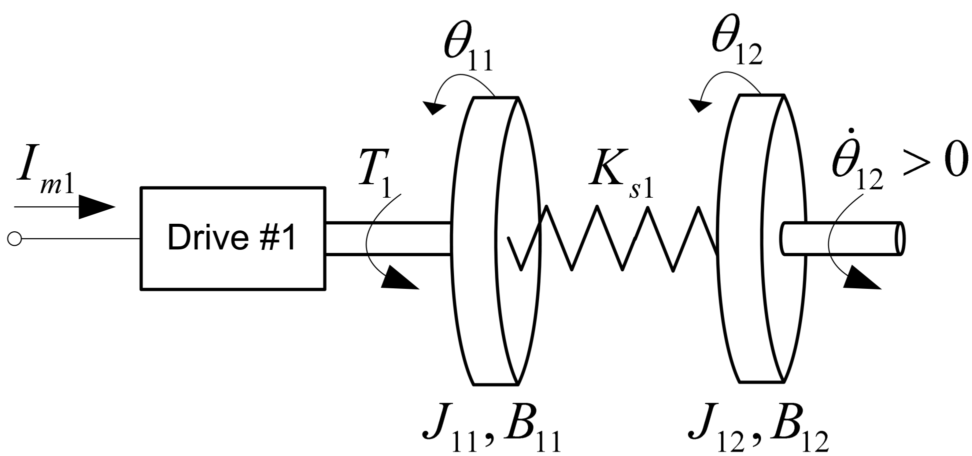


Figure 15 Schematic of the 2DSFJ Robot Stage 1 System

Table 1 provides a nomenclature of the symbols used in the 2DSFJ Stage 1 system mathematical modeling, as presented in this manual.

| ***Symbol*** | ***Description*** | ***Units*** |
|---|---|---|
| $I_{m1}$ | First (Shoulder) Motor Armature Current | A |
| $K_{t1}$ | First (Shoulder) Drive Torque Constant | N.m/A |
| $T_1$ | Torque Produced by Drive #1, at the Load Shaft | N.m |
| $\theta_{11}$ | First (Shoulder) Driving Shaft Absolute Angular Position | rad |
| $\frac{d}{dt}\theta_{11}(t)$ | First (Shoulder) Driving Shaft Absolute Angular Velocity | rad/s |
| $\theta_{12}$ | First Rigid Link Absolute Angular Position | rad |
| $\frac{d}{dt}\theta_{12}(t)$ | First Rigid Link Absolute Angular Velocity | rad/s |
| $J_{11}$ | First Flexible Joint Actuated Transition Equivalent Moment Of Inertia | $kg.m^2$ |
| $B_{11}$ | First Flexible Joint Actuated Transition Equivalent Viscous Damping Coefficient | N.m.s/rad |
| $J_{12}$ | First Flexible Joint Load Transition Equivalent Moment Of Inertia (Compounded With The Stage 2 System) | $kg.m^2$ |
| $B_{12}$ | First Flexible Joint Load Transition Equivalent Viscous Damping Coefficient (Compounded With The Stage 2 System) | N.m.s/rad |
| $K_{s1}$ | First Flexible Joint Torsional Stiffness Constant | N.m/rad |

Table 1 First Stage Of The 2-DOF SFJ Robot Model Nomenclature

Reference [4] details and derives the general dynamic equations of the 2DSFJ Stage 1 system. The Lagrange's method is used to obtain the dynamic model of the system. In the described modeling, the system's state vector, $X_1$, is chosen to include the generalized coordinates as well as their first-order time derivatives. It is defined by its transpose, as shown below:

$$X_1^T = \left[ \theta_{11}(t), \theta_{12}(t), \frac{d}{dt}\theta_{11}(t), \frac{d}{dt}\theta_{12}(t) \right]$$

The system input, $U_1$, is the current to the first motor, that is to say:

$$U_1 = I_{m1}$$

The state-space matrices $A_1$ and $B_1$ are defined to give a dynamic representation of the 2DSFJ Stage 1 system, such that:

$$\frac{\partial}{\partial t} X_1 = A_1 X_1 + B_1 U_1$$

From the system's two equations of motion, the $A_1$ matrix can be determined as follows:

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_{s1}}{J_{11}} & \frac{K_{s1}}{J_{11}} & -\frac{B_{11}}{J_{11}} & 0 \\ \frac{K_{s1}}{J_{12}} & -\frac{K_{s1}}{J_{12}} & 0 & -\frac{B_{12}}{J_{12}} \end{bmatrix}$$

Likewise, the transpose of the $B_1$ matrix characterizing the system can be seen below:

$$B_1^T = \begin{bmatrix} 0 & 0 & \frac{K_{t1}}{J_{11}} & 0 \end{bmatrix}$$

To control the stage 1 system position, a state-feedback controller is implemented according to the following feedback control law:

$$I_{m1} = -K_1 X_1$$

where $K_1$ is the gain vector for the stage 1 system.

The design file *setup_2DSFJ_robot.m* calculates the state-feedback gain $K_1$ using the LQR tuning algorithm. You may edit the file to change the system closed-loop behaviour. By default, the m-file returns the following state-feedback gains:

$$K_1 = [76.57, 81.55, 2.86, 23.02]$$

Likewise, let us now consider the stage 2 system of the 2DSFJ plant. Its schematic is represented in Figure 16.

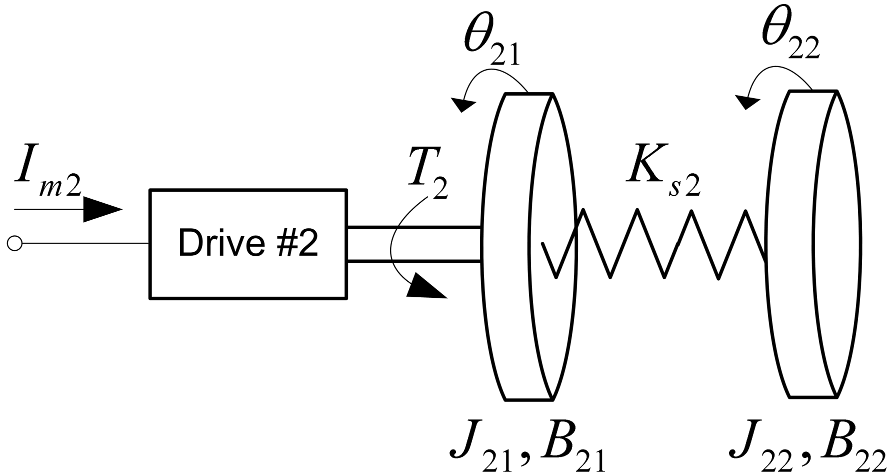


Figure 16 Schematic of the 2DSFJ Robot Stage 2 System

Table 2 provides a nomenclature of the symbols used in the 2DSFJ Stage 2 system mathematical modeling, as presented in this manual.

| Symbol | Description | Units |
|---|---|---|
| $I_{m2}$ | Second (Elbow) Motor Armature Current | A |
| $K_{t2}$ | Second (Elbow) Drive Torque Constant | N.m/A |
| $T_2$ | Torque Produced by Drive #2, at the Load Shaft | N.m |
| $\theta_{21}$ | Second (Elbow) Driving Shaft Angular Position Relative To Link #1 | rad |
| $\frac{d}{dt}\theta_{21}(t)$ | Second (Elbow) Driving Shaft Angular Velocity Relative To Link #1 | rad/s |
| $\theta_{22}$ | Second Rigid Link Angular Position Relative To Link #1 | rad |
| $\frac{d}{dt}\theta_{22}(t)$ | Second Rigid Link Angular Velocity Relative To Link #1 | rad/s |
| $J_{21}$ | Second Flexible Joint Actuated Transition Equivalent Moment Of Inertia | $kg.m^2$ |
| $B_{21}$ | Second Flexible Joint Actuated Transition Equivalent Viscous Damping Coefficient | N.m.s/rad |
| $J_{22}$ | Second Flexible Joint Load Transition Equivalent Moment Of Inertia | $kg.m^2$ |
| $B_{22}$ | Second Flexible Joint Load Transition Equivalent Viscous Damping Coefficient | N.m.s/rad |
| $K_{s2}$ | Second Flexible Joint Torsional Stiffness Constant | N.m/rad |

Table 2 Second Stage Of The 2-DOF SFJ Robot Model Nomenclature

Reference [5] details and derives the general dynamic equations of the 2DSFJ Stage 2 system. The Lagrange's method is used to obtain the dynamic model of the system. In the described modeling, the system's state vector, $X_2$, is chosen to include the generalized coordinates as well as their first-order time derivatives. It is defined by its transpose, as shown below:

$$X_2^T = \left[ \theta_{21}(t), \theta_{22}(t), \frac{d}{dt}\theta_{21}(t), \frac{d}{dt}\theta_{22}(t) \right]$$

The system input, $U_2$, is the current to the second motor, that is to say:

$$U_2 = I_{m2}$$

The state-space matrices $A_2$ and $B_2$ are defined to give a dynamic representation of the 2DSFJ Stage 2 system, such that:

$$\frac{\partial}{\partial t} X_2 = A_2 X_2 + B_2 U_2$$

From the system's two equations of motion, the $A_2$ matrix can be determined as follows:

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_{s2}}{J_{21}} & \frac{K_{s2}}{J_{21}} & -\frac{B_{21}}{J_{21}} & 0 \\ \frac{K_{s2}}{J_{22}} & -\frac{K_{s2}}{J_{22}} & 0 & -\frac{B_{22}}{J_{22}} \end{bmatrix}$$

Likewise, the transpose of the $B_2$ matrix characterizing the system can be seen below:

$$B_2^T = \begin{bmatrix} 0 & 0 & \frac{K_{t2}}{J_{21}} & 0 \end{bmatrix}$$

To control the stage 2 system position, a state-feedback controller is implemented according to the following feedback control law:

$$I_{m2} = -K_2 X_2$$

where $K_2$ is the gain vector for the stage 2 system.

The design file *setup_2DSFJ_robot.m* calculates the state-feedback gain $K_2$ using the LQR tuning algorithm. You may edit the file to change the system closed-loop behaviour. By default, the m-file returns the following state-feedback gains:

$$K_2 = [47.95, -7.13, 0.67, 2.90]$$

## 2. References

[1] QUARC Installation Guide

[2] QUARC HTML MATLAB Help Pages

[3] Dynamic Equations For The First Stage Of The Serial Flexible Joint (2DSFJ) Robot – Maple Worksheet or HTML File.

[4] Dynamic Equations For The Second Stage Of The Serial Flexible Joint (2DSFJ) Robot – Maple Worksheet or HTML File.

## 3. Obtaining Support

To obtain support from Quanser, go to http://www.quanser.com and click on the Tech Support link. Fill in the form with all the requested software and hardware information as well as a description of the problem encountered. Also, make sure your e-mail address and telephone number are included. Submit the form and a technical support person will contact you.