

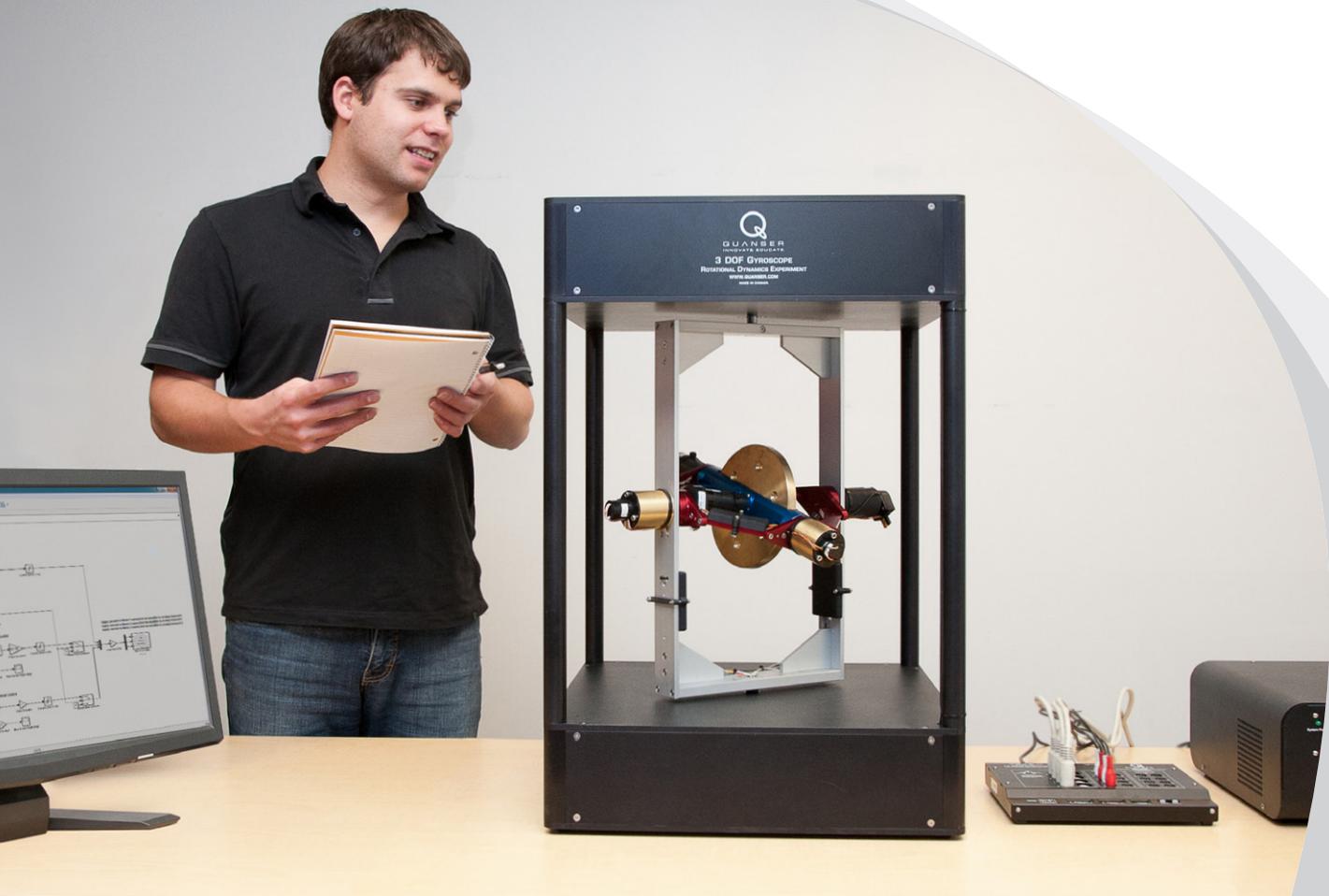


LABORATORY GUIDE

3 DOF Gyroscope Experiment for MATLAB®/Simulink® Users

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1 INTRODUCTION

Gyroscopes have become of great practical interest as they are used in control and guidance systems for air, sea, and space vehicles. The Quanser 3 DOF Gyroscope system can be actuated about all of its frames using the mounted motors while encoders measure the angle about each axis. In addition, the rotor itself is actuated and measured in the same manner.

The outer rectangular frame, outer red gimbal and the inner blue gimbal are designed such that they can be individually fixed in place upon desire. This allows the users to perform a variety of different experiments using the device. In this laboratory, the gyroscopic effect will be employed to control the angle of the red gimbal by applying the control command about the blue gimbal. The gray outer rectangular frame will be fixed (see the 3 DOF Gyroscope User Manual [5] for instructions on how to fix each frame). In order to do this, the rotor has to have acquired enough angular momentum (RPM) for the gyroscopic effect to take place. Therefore a controller is required to control the angular speed of the disk while another is required to control the red gimbal angle.

Topics Covered

- Obtain a state-space representation of the open-loop system.
- Design a state-feedback gain for the closed-loop system using the Linear-Quadratic Regulator (LQR) optimization.
- Simulate the system and ensure it is stabilized using the designed state-feedback control.
- Implement your state-feedback controller on the 3 DOF Gyroscope system and evaluate its actual performance.

Prerequisites

In order to successfully carry out this laboratory, the user should be familiar with the following:

1. Hardware and software requirements given in Section 4.
2. Modeling and state-space representation.
3. State-feedback design using Linear-Quadratic Regulator (LQR) optimization.
4. Basics of [Simulink®](#).

2 BACKGROUND

2.1 Modeling

2.1.1 Model Convention

The reference coordinate frame for the 3 DOF Gyroscope is shown in Figure 2.1.

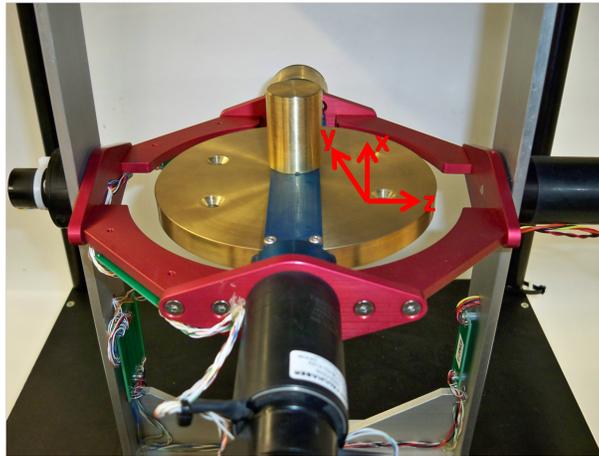


Figure 2.1: 3 DOF Gyroscope coordinate frame

2.1.2 Equations of Motion

The equations of motion representing the angular rate of the red gimbal, ψ , and the outer blue gimbal, θ , are ([1]):

$$\begin{aligned} J_y \ddot{\theta} + h \dot{\psi} &= \tau_y \\ J_z \ddot{\psi} - h \dot{\theta} &= 0 \end{aligned} \quad (2.1)$$

where

$$\begin{aligned} J_y &= 0.0039 \text{ kg}\cdot\text{m}^2 \\ J_z &= 0.0223 \text{ kg}\cdot\text{m}^2 \\ h &= 0.44 \text{ kg}\cdot\text{m}^2/\text{s}. \end{aligned}$$

The moment of inertia about the y-axis angle, θ , is J_y and the moment of inertia about the z-axis angle (red gimbal), ψ , is denoted as J_z . The constant h is calculated based on the moment of inertia of the gyroscope rotor about its own axis and its velocity. Because the outer gray rectangular frame is fixed, the only actuated axis is the y-axis. The control input in the single-input single-output (SISO) system is the torque applied in the y-axis, τ_y .

2.1.3 Linear State-Space Model

The linear state-space equations are

$$\dot{x} = Ax + Bu \quad (2.2)$$

and

$$y = Cx + Du \quad (2.3)$$

where x is the state, u is the control input, A , B , C , and D are state-space matrices.

For this system, the state and output are defined

$$x^T = [\dot{\theta} \quad \psi \quad \dot{\psi} \quad \int \psi] \quad (2.4)$$

and

$$y = \psi$$

Solving for the acceleration terms in the linear equations of motion given in Equation 2.1 we get

$$\begin{aligned} \ddot{\theta} &= \frac{-h}{J_y} \dot{\psi} + \frac{1}{J_y} \tau_y \\ \ddot{\psi} &= \frac{h}{J_z} \dot{\theta} \end{aligned}$$

Substituting the state in , we obtain the following state-space matrices:

$$A = \begin{bmatrix} 0 & 0 & -\frac{h}{J_y} & 0 \\ 0 & 0 & 1 & 0 \\ \frac{h}{J_z} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

and

$$B = \begin{bmatrix} \frac{1}{J_y} \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The system output (control variable) is the red gimbal angle which is the second entry in the system state vector i.e., ψ . Based on this, the C and D matrices in the output equation are

$$C = [0 \quad 1 \quad 0 \quad 0]$$

and

$$D = [0]$$

The velocities of the red and blue gimbal angles can be computed in the digital controller, e.g., by taking the derivative and filtering the result through a high-pass filter. The integral state can be computed by integrating the red gimbal angle measurement in the digital controller.

2.2 Control

In Section 2.1, we found a linear state-state space model that represents the 3 DOF Gyroscope system. This model is used to investigate the stability properties of the system in Section 2.2.1. In Section 2.2.2, the notion of controllability is introduced. The Linear Quadratic Regulator (LQR) algorithm is a common way to find the control gain and is discussed in Section 2.2.3. Lastly, Section 2.2.4 describes the state-feedback control used to control the red gimbal position.

2.2.1 Stability

The stability of a system can be determined from its poles ([6]):

- Stable systems have poles only in the left-hand plane.
- Unstable systems have at least one pole in the right-hand plane and/or poles of multiplicity greater than 1 on the imaginary axis.

- Marginally stable systems have one pole on the imaginary axis and the other poles in the left-hand plane.

The poles are the roots of the system's characteristic equation. From the state-space, the characteristic equation of the system can be found using

$$\det(sI - A) = 0 \quad (2.5)$$

where $\det()$ is the determinant function, s is the Laplace operator, and I the identity matrix. These are the *eigenvalues* of the state-space matrix A .

2.2.2 Controllability

If the control input, u , of a system can take each state variable, x_i where $i = 1 \dots n$, from an initial state to a final state then the system is controllable, otherwise it is uncontrollable ([6]).

Rank Test The system is controllable if the rank of its controllability matrix

$$T = [B \ AB \ A^2B \ \dots \ A^{n-1}B] \quad (2.6)$$

equals the number of states in the system,

$$\text{rank}(T) = n. \quad (2.7)$$

2.2.3 Linear Quadratic Regular (LQR)

If (A,B) are controllable, then the Linear Quadratic Regulator optimization method can be used to find a feedback control gain. Given the plant model in Equation 2.2, find a control input u that minimizes the cost function

$$J = \int_0^{\infty} x(t)'Qx(t) + u(t)'Ru(t) dt, \quad (2.8)$$

where Q and R are the weighting matrices. The weighting matrices affect how LQR minimizes the function and are, essentially, tuning variables.

Given the control law $u = -Kx$, the state-space in Equation 2.2 becomes

$$\begin{aligned} \dot{x} &= Ax + B(-Kx) \\ &= (A - BK)x \end{aligned}$$

2.2.4 Feedback Control

The feedback control loop in Figure 2.2 is designed to stabilize the red gimbal to a desired position, ψ_d .

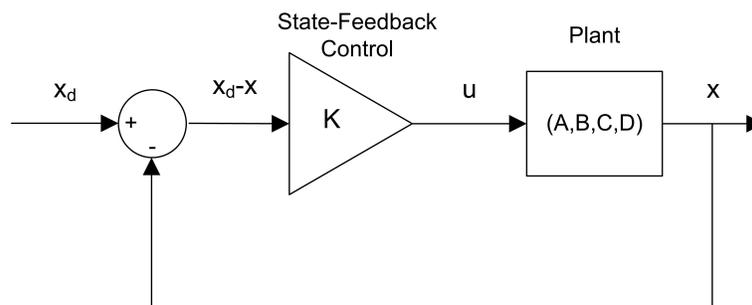


Figure 2.2: State-feedback control loop

The reference state is defined

$$x_d = [0 \ \psi_d \ 0 \ 0]$$

and the controller is

$$u = K(x_d - x). \quad (2.9)$$

Note that if $x_d = 0$ then $u = -Kx$, which is the control used in the LQR algorithm.

3 LAB EXPERIMENTS

3.1 Simulation

In this section we will use the Simulink diagram shown in Figure 3.1 to simulate the closed-loop control of the 3 DOF Gyroscope system. The system is simulated using the model summarized in Section 2.1. The Simulink model uses the state-feedback control described in Section 2.2.4. The feedback gain K is found using the Matlab LQR command (LQR is described briefly in Section 2.2.3).

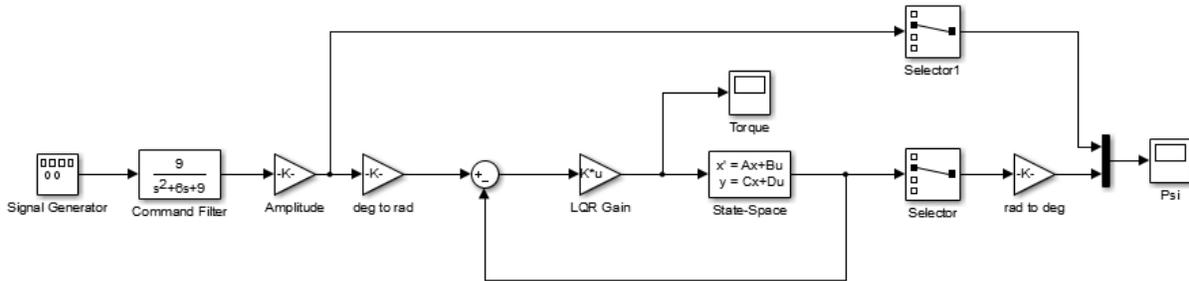


Figure 3.1: Simulink model used to simulate 3 DOF Gyroscope.

IMPORTANT: Before you can conduct these experiments, you need to make sure that the lab files are configured according to your setup. If they have not been configured already, then you need to go to Section 4 to configure the lab files first.

3.1.1 Procedure

Follow these steps to simulate the 3 DOF Gyroscope:

1. Make sure the LQR weighting matrices in `setup_3dgyro.m` are set to

$$Q = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 16 & 0 & 0 \\ 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0.0001 \end{bmatrix}$$

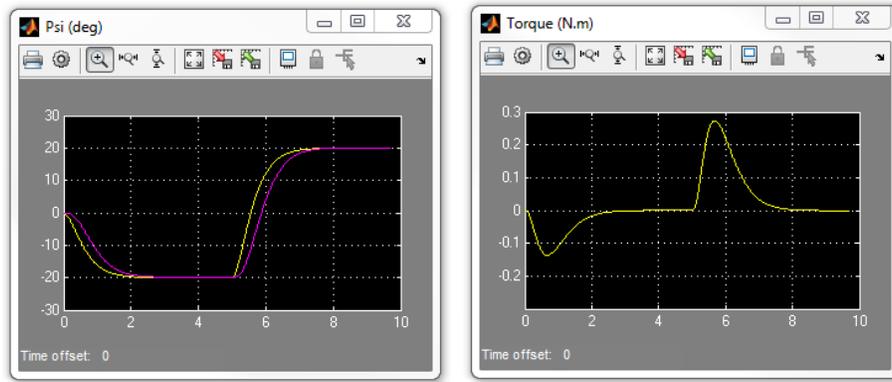
and $R = 5$.

2. Run the script to generate the gain

$$K = [0.65 \quad 1.79 \quad 0.12 \quad 0.0045].$$

Remark: When tuning the LQR, $Q(2,2)$ effects the red gimbal proportional gain while $Q(1,1)$ effects the red gimbal derivative gain (which reduces the overshoot). $Q(4,4)$ affects the red gimbal integral gain which is used to minimize the steady state error.

3. Open the red gimbal position scope, *Psi (deg)*, and the torque scope, *Torque (N.m)*.
4. Start the simulation. The scopes should be displaying responses similar to Figure 3.2.
5. The scopes should be displaying a response similar to Figure 3.2.



(a) Red Gimbal Position

(b) Torque

Figure 3.2: Simulated closed-loop response.

3.2 Implementation

The `q_3dgyro_lqr` Simulink diagram shown in Figure 3.3 is used to perform the red gimbal position control on the 3 DOF Gyroscope. The *HIL Interfacing* subsystem contains drivers that interface with the DC motor and sensors of the 3 DOF Gyroscope system.

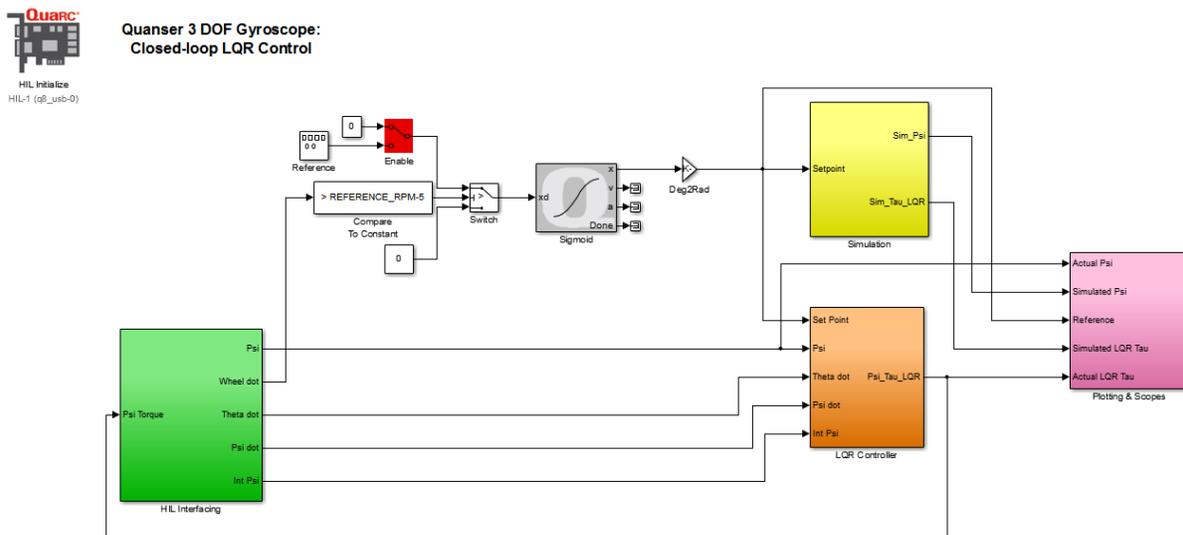


Figure 3.3: Simulink model used to run controller on the 3 DOF Gyroscope.

IMPORTANT: Before you can conduct these experiments, you need to make sure that the lab files are configured according to your setup. If they have not been configured already, then you need to go to Section 4 to configure the lab files first.

3.2.1 Procedure

Follow this procedure:

1. Run the `setup_3dgyro.m` script using the LQR weighting matrices that you used in the simulation in Section 3.1.

2. Open the red gimbal position scope, *Reference vs. Simulated vs. Actual Psi (deg)*, and the torque scope, *Simulated vs. Actual LQR Tau (N.m)*.
3. In the Simulink diagram, go to QUARC | Build.
4. Set the *Enable* switch to the constant 0 to disable the reference signal until the flywheel has reached it's required speed.
5. Place the 3 DOF Gyroscope flywheel, blue and red gimbals in their home starting position. Ensure that the flywheel's motor and the mass counter balances on the red gimbal (identified with red squares in Figure 3.4) are pointing toward the same direction (e.g., all facing up or all facing down).

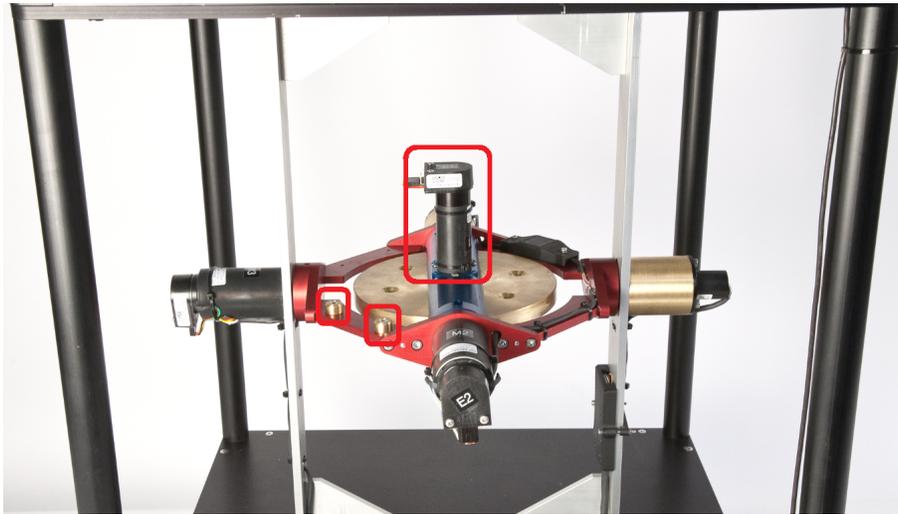


Figure 3.4: Starting position for the 3 DOF Gyroscope.

6. You will need to gently hold on to the blue and red gimbals and keep them in place for the first few seconds after the model has started running while the flywheel speeds up to the required RPM as show in Figure 3.5

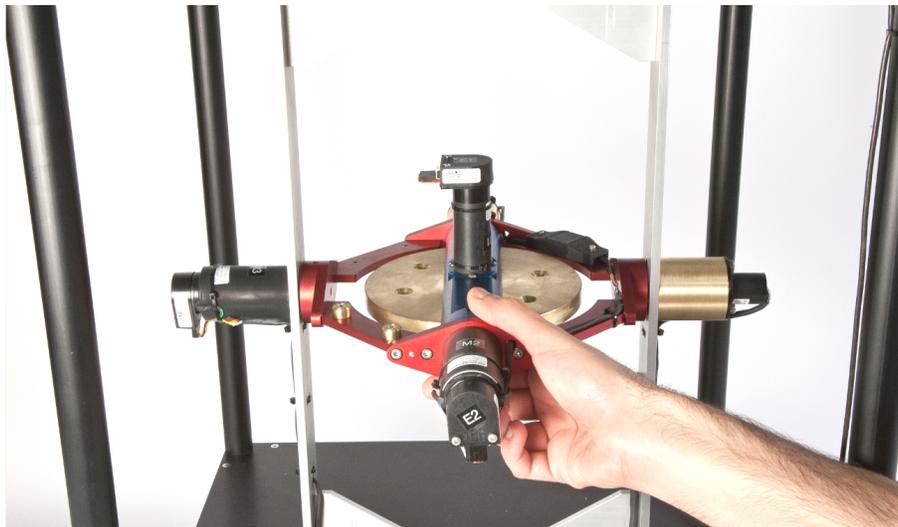
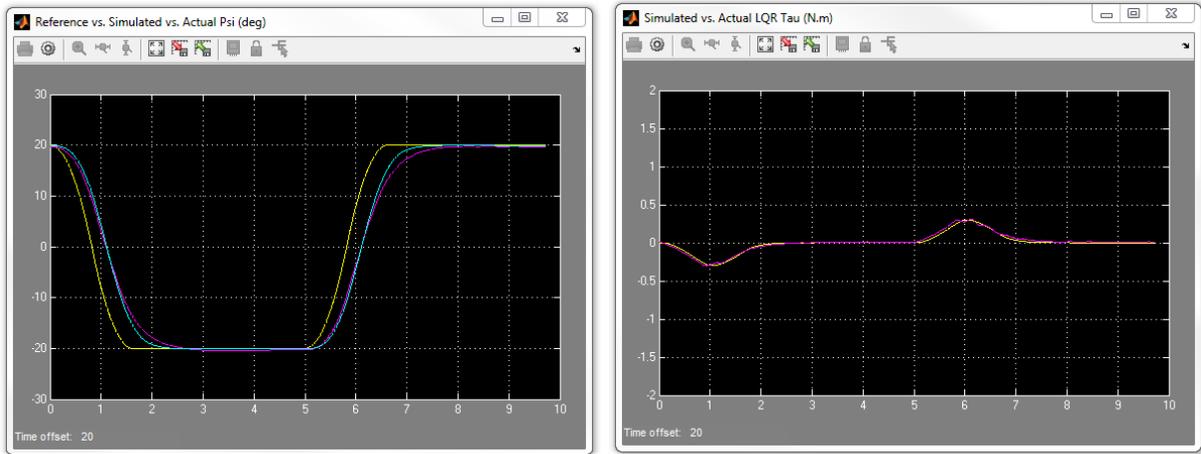


Figure 3.5: Holding the blue and red gimbals

7. Click on QUARC | Start to run the controller while holding the blue and red gimbals in their home position as instructed above.
8. After a few seconds, release the gimbals and change the *Enable* switch to start the 20 degree signal.

9. The red gimbal should now be going back and forth between the commanded positions at the frequency specified. Examine the position of the red gimbal in the *Reference vs. Simulated vs. Actual Psi (deg)* scope. You can also view the commanded motor torque in the *Simulated vs. Actual LQR Tau (N.m)* scope. The scopes should be displaying responses similar to Figure 3.6.



(a) Red Gimbal Position

(b) Torque

Figure 3.6: Typical response when running control on 3 DOF Gyroscope system

3.2.2 Analysis

An example of the closed-loop position response, is shown in Figure 3.7. You can generate this using the plot command after running the q_3dgyro_lqr QUARC controller.

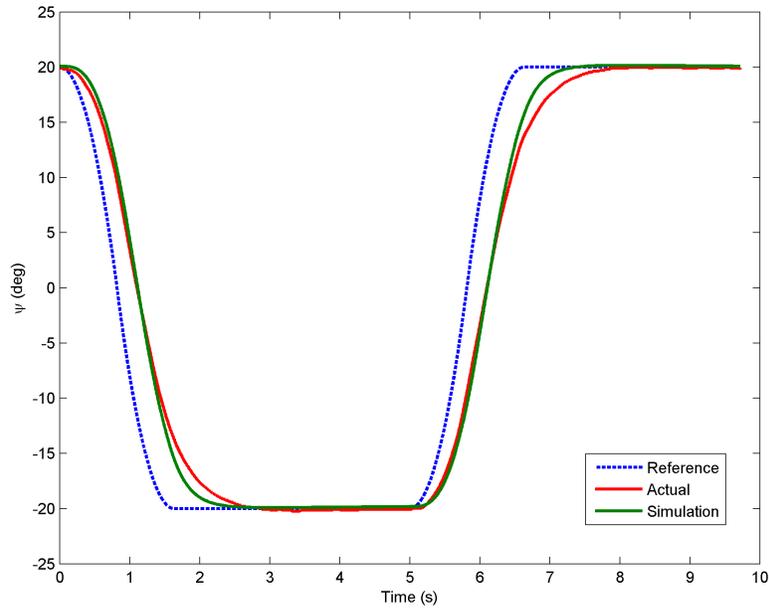


Figure 3.7: 3 DOF Gyroscope balance control response

The red gimbal is able to track the desired position quite well, though the response is slightly slower than the simulated response due to unmodeled non-linear phenomena present in the actual hardware.

4 SYSTEM REQUIREMENTS

Required Software

- Microsoft Visual Studio (MS VS)
- Matlab® with Simulink®, Real-Time Workshop, and the Control System Toolbox
- QUARC®

See the QUARC® software compatibility chart in [4] to see what versions of MS VS and Matlab are compatible with your version of QUARC and for what OS.

Required Hardware

- Data acquisition (DAQ) device **with four encoder inputs** and that is compatible with QUARC®. This includes Quanser DAQ boards such as Q8-USB, QPID, and QPIDe and some National Instruments DAQ devices. For a full listing of compliant DAQ cards, see Reference [2].
- Quanser 3 DOF Gyroscope
- Quanser AMPAQ-L4 power amplifier, or equivalent.

Before Starting Lab

Before you begin this laboratory make sure:

- QUARC® is installed on your PC, as described in [3].
- DAQ device has been successfully tested (e.g., using the test software in the Quick Start Guide or the *Analog Loopback Demo*).
- 3 DOF Gyroscope and amplifier are connected to your DAQ board as described its User Manual [5].

4.1 Overview of Files

File Name	Description
3 DOF Gyroscope User Manual.pdf	This manual describes the hardware of the 3 DOF Gyroscope system and explains how to setup and wire the system for the experiments.
3 DOF Gyroscope Laboratory Guide.pdf	This document demonstrates how to obtain the linear state-space model of the system, simulate the closed-loop system, and implement controllers on the 3 DOF Gyroscope plant using QUARC®.
setup_3dgyro.m	The main Matlab script that sets the 3 DOF Gyroscope motor and sensor parameters, the 3 DOF Gyroscope configuration-dependent model parameters, and the 3 DOF Gyroscope sensor parameters. Run this file only to setup the laboratory.
s_3dgyro_lqr	Simulink file that simulates the closed-loop control of a 3 DOF Gyroscope system using state-feedback control.
q_3dgyro_lqr	Simulink file that implements the state-feedback control on the 3 DOF Gyroscope system using QUARC®.

Table 4.1: Files supplied with the 3 DOF Gyroscope

4.2 Setup for Simulation

Before beginning the in-lab procedure outlined in Section 3.1, the `s_3dgyro_lqr` Simulink diagram and the `setup_3dgyro.m` script must be configured.

Follow these steps:

1. Load the Matlab software.
2. Browse through the *Current Directory* window in Matlab and find the folder that contains the file `setup_3dgyro.m`.
3. Open the `setup_3dgyro.m` script.
4. **Configure setup_3dgyro**: Make sure the script is setup to match this setup:
 - `CONTROL_TYPE = 'AUTO'`;
 - Ensure other parameters such as `VMAX_DAC` match your system configuration.
5. Run `setup_3dgyro.m` to setup the Matlab workspace.
6. Open the `s_3dgyro_lqr.mdl` Simulink diagram, shown in Figure 3.1.

4.3 Setup for Running on 3 DOF Gyroscope

Before performing the in-lab exercises in Section 3.2, the 3 DOF Gyroscope system and the `setup_3dgyro.m` script must be configured.

Follow these steps to get the system ready for this lab:

1. Lock the silver rectangular frame of the 3 DOF Gyroscope in place as detailed in the 3 DOF Gyroscope User Manual [5].
2. Configure and run `setup_3dgyro.m` as explained in Section 4.2.
3. Open the `q_3dgyro_lqr.mdl` Simulink diagram, shown in Figure 3.3.
4. **Configure DAQ**: Ensure the HIL Initialize block in the Simulink model is configured for the DAQ device that is installed in your system. See [2] for more information on configuring the HIL Initialize block.
5. You are now ready to run and tune the LQR controller as outlined in Section 3.2.1.

REFERENCES

- [1] Robert H. Canon. *Dynamics of Physical Systems*. McGraw Hill Book Company, New York.
- [2] Quanser Inc. *QUARC User Manual*.
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