



# LABORATORY GUIDE

## 2 DOF Hover Experiment for LabVIEW™ Users

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# 1 INTRODUCTION

This laboratory guide summarizes a simple, linear dynamic model of the Quanser 3 DOF Hover and designs a state-feedback control system. The goal is control the position of pitch, roll, and yaw axes of the 3 DOF Hover with a smooth control signal that does not saturate the actuators.

## Topics Covered

- Obtain a state-space representation of the open-loop system.
- Design and tune an LQR-based state-feedback controller satisfying the closed-loop system's desired design specifications.
- Simulate the system and ensure it is stabilized using the designed state-feedback control.
- Implement the state-feedback controller on the 3 DOF Hover system and evaluate its actual performance.

## 1.1 Prerequisites

In order to successfully carry out this laboratory, the user should be familiar with the following:

- See the System Requirements given in Section 4 for the required hardware and software.
- Modeling and state-space representation.
- State-feedback design using Linear-Quadratic Regular (LQR) optimization.
- Basics of [LabVIEW™](#) .
- How to use basic operation in [Quanser Rapid Control Prototyping Toolkit®](#) software. Please see the RCP demos and help page for more information.

# 2 BACKGROUND

## 2.1 Modeling

### 2.1.1 Free-Body Diagram

The free-body diagram of the Quanser 3 DOF Hover is illustrated in Figure 2.1 and it accompanies the Maple worksheet named *3 DOF Hover Equations.mws* or its HTML equivalent *3 DOF Hover Equations.html*. The equations can be edited and re-calculated by executing the worksheet using the Maple 9 software.

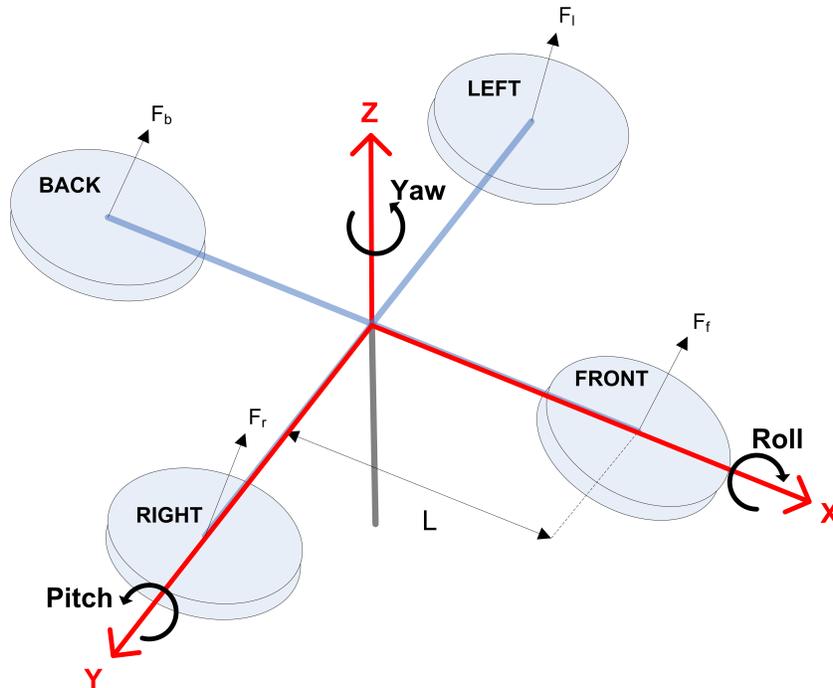


Figure 2.1: Simple free-body diagram of 3 DOF Hover

The 3 DOF Hover modeling conventions used are:

1. The 3 DOF Hover is horizontal (i.e., parallel with the ground) when the pitch and roll angles are zero,  $\theta_p = 0$  and  $\theta_r = 0$ .
2. Yaw angle increases positively,  $\dot{\theta}_y(t) > 0$  when the body rotates in the **counter-clockwise (CCW)** direction.
3. Pitch angle increases positively,  $\dot{\theta}_p > 0$  when rotated CCW.
4. Roll angle increases positively,  $\dot{\theta}_r > 0$ , when rotate CCW.

When a positive voltage is applied to any motor a positive thrust force is generated and this causes the corresponding propeller assembly to rise. The thrust force generated by the front, back, right, and left motors are denoted by  $F_f$ ,  $F_b$ ,  $F_r$ , and  $F_l$ , respectively. The thrust forces generated by the front and back motors primarily control the motions about the pitch axis while the right and left motors primarily move the hover about its roll axis. Notice that the pitch angle increases when the thrust force from the front motor is larger than back motor  $F_f > F_b$ . The roll angle increases when the thrust force from the right motor is larger than the left motor,  $F_r > F_l$ .

## 2.1.2 Pitch and Roll Axis Model

The dynamics for each axis can be described by the general equation

$$J\ddot{\theta} = \Delta FL$$

where  $\theta$  is the angle of the pivot,  $L$  is the distance between the propeller motor and the pivot on the axis,  $J$  is the moment of inertia about the axis, and  $\Delta F$  is the differential thrust-force. With the force diagram in Figure 2.2, we can model the pitch axis using the equation

$$J_p\ddot{\theta}_p = K_f(V_f - V_b) \quad (2.1)$$

where  $K_f$  is the thrust-force constant,  $V_f$  is the front motor voltage,  $V_b$  is the back motor voltage,  $\theta_p$  is the pitch angle, and  $J_p$  is the moment of inertia about the pitch axis. Notice that this follows the conventions in Figure 2.1, where the pitch angle increases when the front motor voltage is larger than the back motor.

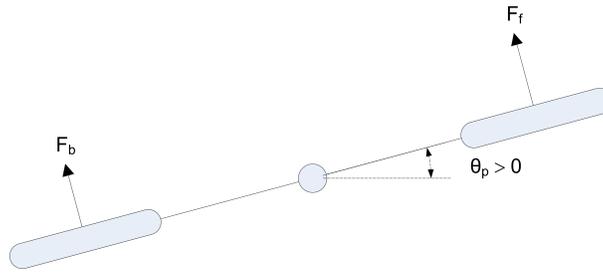


Figure 2.2: Free-body diagram of pitch axis

Similarly, for the roll axis we have

$$J_r\ddot{\theta}_r = K_f(V_r - V_l) \quad (2.2)$$

where  $K_f$  is the thrust-force constant,  $V_r$  is the right motor voltage,  $V_l$  is the left motor voltage,  $\theta_r$  is the roll angle, and  $J_r$  is the moment of inertia about the roll axis. The roll angle increases when the right motor voltage is larger than the left motor.

## 2.1.3 Yaw Axis Model

The motion about the yaw axis, illustrated in Figure 2.3, is caused by the difference in torques exerted by the two clockwise and two counter-clockwise rotating propellers

$$J_y\ddot{\theta}_y = \Delta\tau = \tau_l + \tau_r - \tau_f - \tau_b$$

where  $\tau_l$  and  $\tau_r$  are the torques generated by the left and right *clockwise* propellers and  $\tau_f$  and  $\tau_b$  are the torques exerted by the front and back counter-clockwise rotors. Note that the counter-clockwise torques are negative. The torque generated by all the propellers is assumed to be  $\tau = K_t V_m$ , where  $K_t$  is the thrust-torque constant and  $V_m$  is the motor voltage. Thus in terms of applied voltage, the yaw axis equation of motion is

$$J_y\ddot{\theta}_y = K_t(V_r + V_l) - K_t(V_f + V_b). \quad (2.3)$$

## 2.1.4 State-Space Model

The state-space representation is given by

$$\dot{x} = Ax + Bu$$

and

$$y = Cx + Du.$$

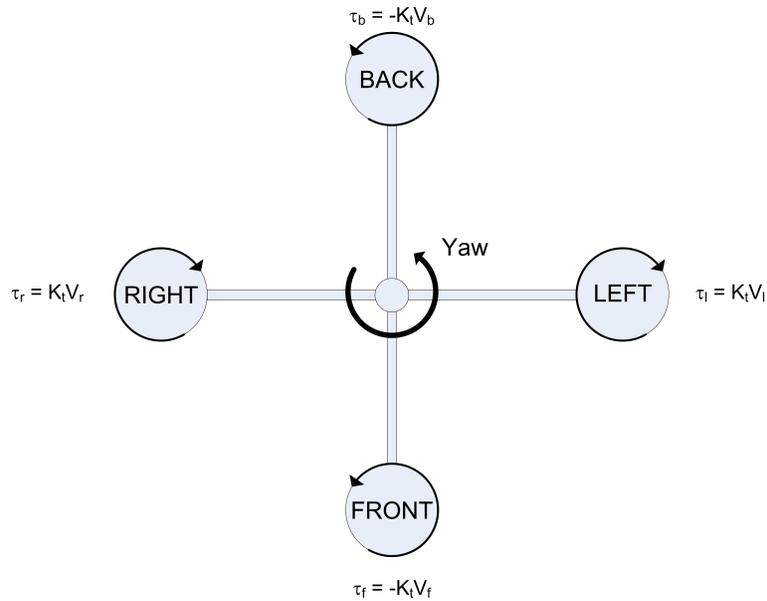


Figure 2.3: Free-body diagram of yaw axis

For the 3 DOF Hover, we define the state vector

$$x^T = [ \theta_y \quad \theta_p \quad \theta_r \quad \dot{\theta}_y \quad \dot{\theta}_p \quad \dot{\theta}_r ], \quad (2.4)$$

the output vector

$$y^T = [ \theta_y \quad \theta_p \quad \theta_r ]$$

and the control vector

$$u^T = [ V_f \quad V_b \quad V_r \quad V_l ].$$

Using the equations of motion given in equations 2.1, 2.2, and 2.3, the corresponding 3 DOF Hover state-space matrices (as derived in the Maple worksheet) are

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{K_t}{J_y} & -\frac{K_t}{J_y} & \frac{K_t}{J_y} & \frac{K_t}{J_y} \\ \frac{LK_f}{J_p} & -\frac{LK_f}{J_p} & 0 & 0 \\ 0 & 0 & \frac{LK_f}{J_r} & -\frac{LK_f}{J_r} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The model parameters used in the  $(A, B)$  matrices are defined in the Quanser 3 DOF Hover User Manual.

## 2.2 Control Design

### 2.2.1 State-Feedback

In this section a state-feedback controller is designed to regulate the pitch, roll, and yaw angles of the Quanser 3 DOF Hover to desired positions. The control gains are computed using the Linear-Quadratic Regulator algorithm in Section 2.2.2.

The state-feedback controller entering the motors is defined

$$u = [V_f \quad V_b \quad V_r \quad V_l]^\top = \begin{cases} K(x_d - x) + u_{\text{bias}} & \text{if } u \geq 0 \\ 0 & \text{if } u < 0 \end{cases} \quad (2.5)$$

where  $x$  is defined in Equation 2.4,  $K \in \mathbb{R}^{4 \times 6}$  is the control gain,

$$x_d = [\theta_{d,y} \quad \theta_{d,p} \quad \theta_{d,r} \quad 0 \quad 0 \quad 0]$$

is the setpoint vector (i.e., desired or reference angles) and

$$u_{\text{bias}}^\top = [V_{\text{bias}} \quad V_{\text{bias}} \quad V_{\text{bias}} \quad V_{\text{bias}}]$$

is the bias voltage, i.e., a fixed constant voltage applied to each motor. Adding a bias voltage to each propeller helps prevent the voltage from going below zero and being cutoff. By keeping the rotors in motion, this can also help make the system more responsive.



**Caution:** Due to the low resistance of the motor, switching between positive and negative voltage can cause permanent damage to the power amplifier. The controller given in Equation 2.5 only applies positive voltage to the motors.

Allowing only positive thrust also makes it resemble more closely to how actual VTOL and helicopter devices operate, i.e., their propellers cannot reverse direction.

### 2.2.2 Linear Quadratic Regulator

The control gains are computed using the Linear-Quadratic Regulator scheme. Using the feedback law

$$u = -Kx$$

the weighting matrices

$$Q = \begin{bmatrix} 500 & 0 & 0 & 0 & 0 & 0 \\ 0 & 350 & 0 & 0 & 0 & 0 \\ 0 & 0 & 350 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 0 \\ 0 & 0 & 0 & 0 & 0 & 20 \end{bmatrix}$$

and

$$R = \begin{bmatrix} 0.01 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 \\ 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0.01 \end{bmatrix}$$

and the state-space matrices  $(A, B)$  found previously, the control gain

$$K = \begin{bmatrix} -111.8 & 132.3 & 0 & -41.41 & 36.23 & 0 \\ -111.8 & -132.3 & 0 & -41.41 & -36.23 & 0 \\ 111.8 & 0 & 132.3 & 41.41 & 0 & 36.23 \\ 111.8 & 0 & -132.3 & 41.41 & 0 & -36.23 \end{bmatrix}$$

is calculated by minimizing the cost function

$$J = \int_0^{\infty} x^T Q x + u^T R u dt.$$

# 3 IN-LAB PROCEDURE

## 3.1 Controller Simulation

In this section we will use the LabVIEW VI shown in Figure 3.1 to simulate the closed-loop control of the Quanser 3 DOF Hover system. The system is simulated using the linear model summarized in Section 2.1. The VI model uses the state-feedback control described in Section 2.2. The feedback gain  $K$  is found using the LQR command (LQR is described briefly in Section 2.2.2). The goal is to make sure the generated gain can stabilize the system while not saturating the dc motors.

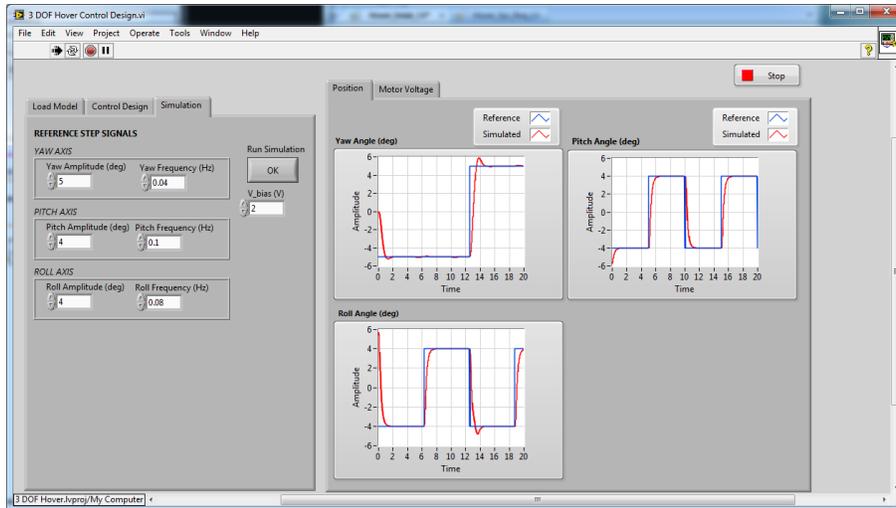


Figure 3.1: VI used to simulate 3 DOF Hover system

**IMPORTANT:** This VI loads the state-space model generated and saved from the *3 DOF Hover Modeling.vi*. Before you can conduct the closed-loop simulation, you need to make sure this VI is ran as explained in Section 4.

### 3.1.1 Objectives

- Investigate the closed-loop position control performance of the LQR using a linear model of the 3 DOF Hover system.
- Ensure the controller does not saturate the actuator.

### 3.1.2 Procedure

Follow these steps to simulate the closed-loop response of the 3 DOF Hover:

1. For details on how to set up the simulation, see Section 4.
2. Load the **LabVIEW™** software.
3. Open the VI called *3 DOF Hover Control Design.vi*, shown in Figure 3.1.
4. Run the VI.
5. Load the model generated by the *3 DOF Hover Modeling.vi*. When loaded, the *Load Model* tab should look similarly as shown in Figure 3.2.

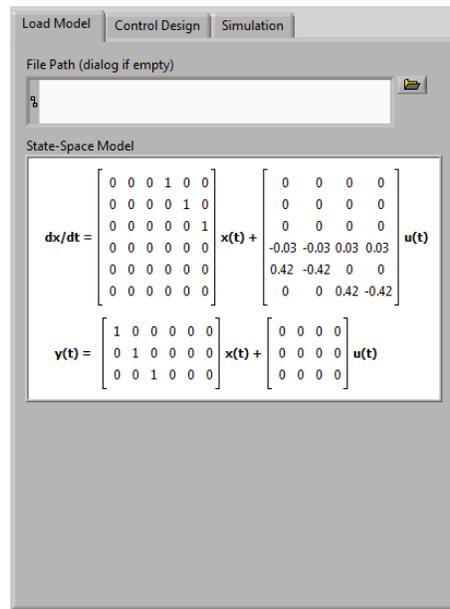


Figure 3.2: State-space model for 3 DOF Hover system

- Click on the *Control Design* tab to generate your control gain based on LQR. Sample gain generated is shown in Figure 3.3. You can change the  $Q$  and  $R$  weighting matrices to suit

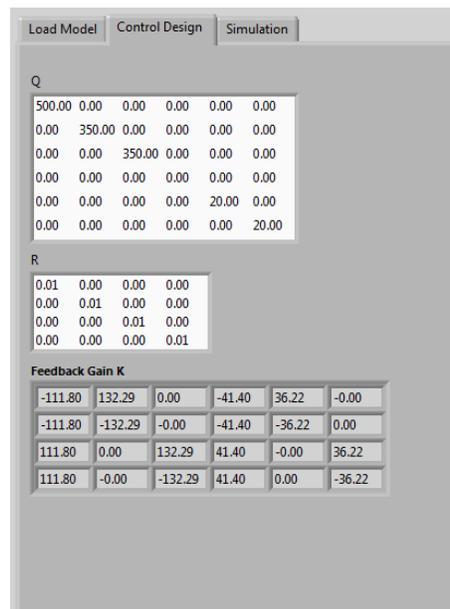


Figure 3.3: Control gain generated using LQR for 3 DOF Hover system

- Click on *Simulation* tab.
- To command a desired yaw step of  $\pm 5$  degrees at 0.04 Hz, pitch step of  $\pm 2$  degrees at 0.1 Hz frequency, and a roll angle step of  $\pm 2$  degrees at 0.08 Hz, set the front panel control boxes to the following:
  - Yaw Amplitude* to 5 deg and *Roll Frequency* to 0.04 Hz.
  - Pitch Amplitude* to 4 deg and *Pitch Frequency* to 0.1 Hz.
  - Roll Amplitude* to 4 deg and *Roll Frequency* to 0.08 Hz.

- Run the simulation by clicking on the *OK* button. The scopes should be displaying responses similar to Figure 3.4. The yaw, pitch, and roll angles should track the commanded square position signal scope. The control signal is shown on the *Motor Voltage* tab and it shows how the Saturation block only allows positive control voltages and limits them to a maximum of 20V (which can be changed). At low command angles, the voltages are changing about the bias voltage 2 V, i.e.,  $V_{bias} = 2V$ .

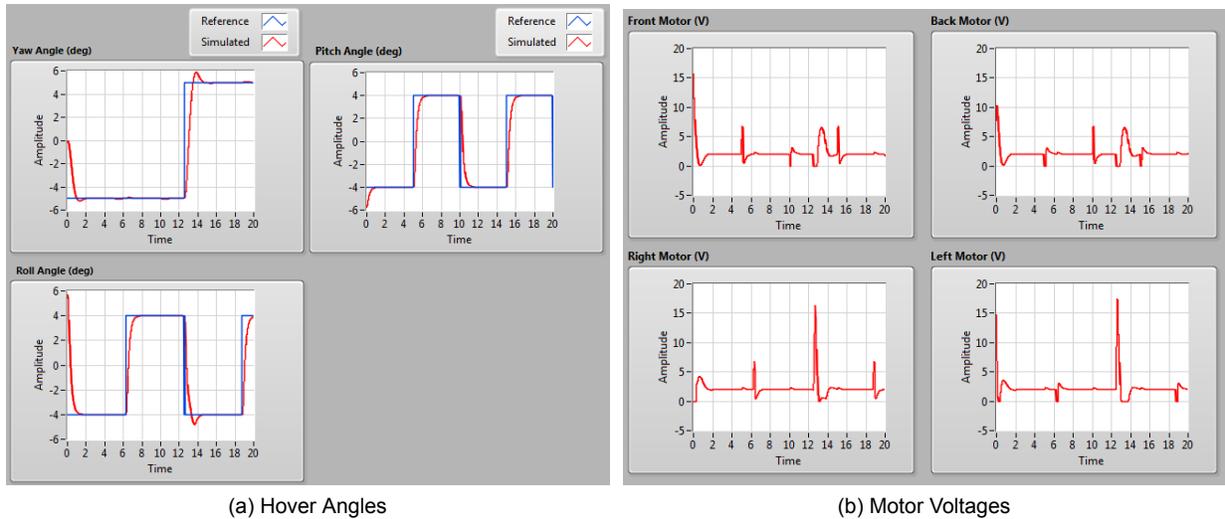


Figure 3.4: Sample simulated closed-loop response of 3 DOF Hover

## 3.2 Implementation

The VI shown in Figure 3.5 is used to perform the balance control on the 3 DOF Hover. The VI interface with the DC motor and encoders of the 3 DOF Hover system.

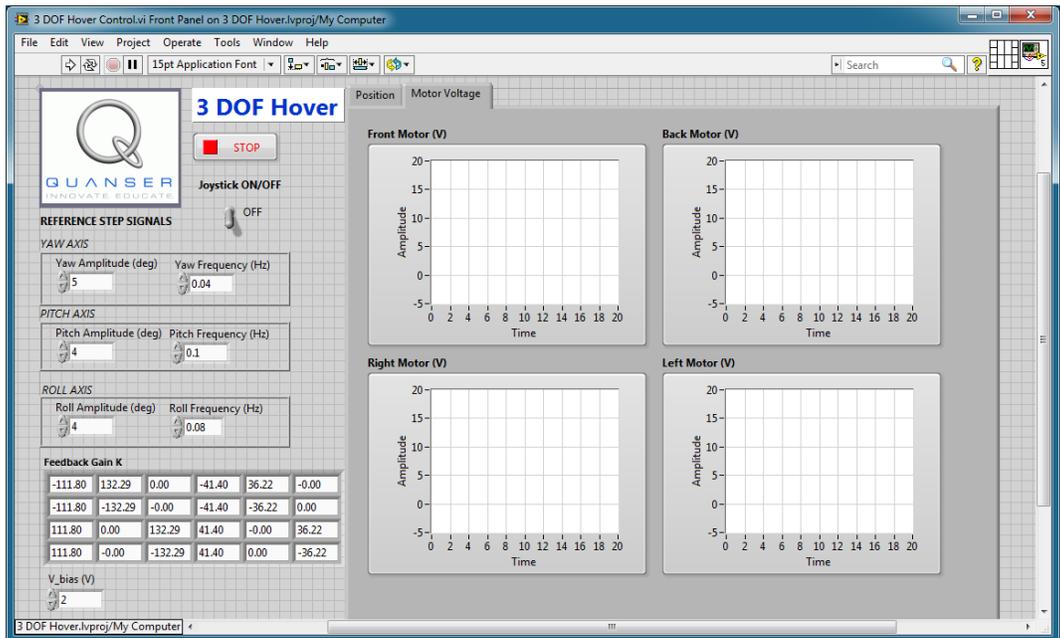


Figure 3.5: VI used to run controller on the 3 DOF Hover.

**IMPORTANT:** Before you can conduct these experiments, you need to make sure that the lab files are configured according to your setup. If they have not been configured already, then you need to go to Section 4 to configure the

lab files first.

### 3.2.1 Objectives

Implement the previously designed LQR in order to control the position of the 3 DOF Hover.

### 3.2.2 Procedure

Follow this procedure:

1. Make sure the gain that you used in the simulation in Section 3.1 is entered in the *Feedback Gain K* input box.
2. Set the *Joystick ON/OFF* switch to the OFF position to generate commands from the VI signal generators.
3. Run the VI. See the sample response shown in Figure 3.6 using a yaw step of  $\pm 5$  degrees at 0.04 Hz, a pitch of  $\pm 4$  degrees at 0.1 Hz, and a roll of  $\pm 4$  degrees at 0.08 Hz.

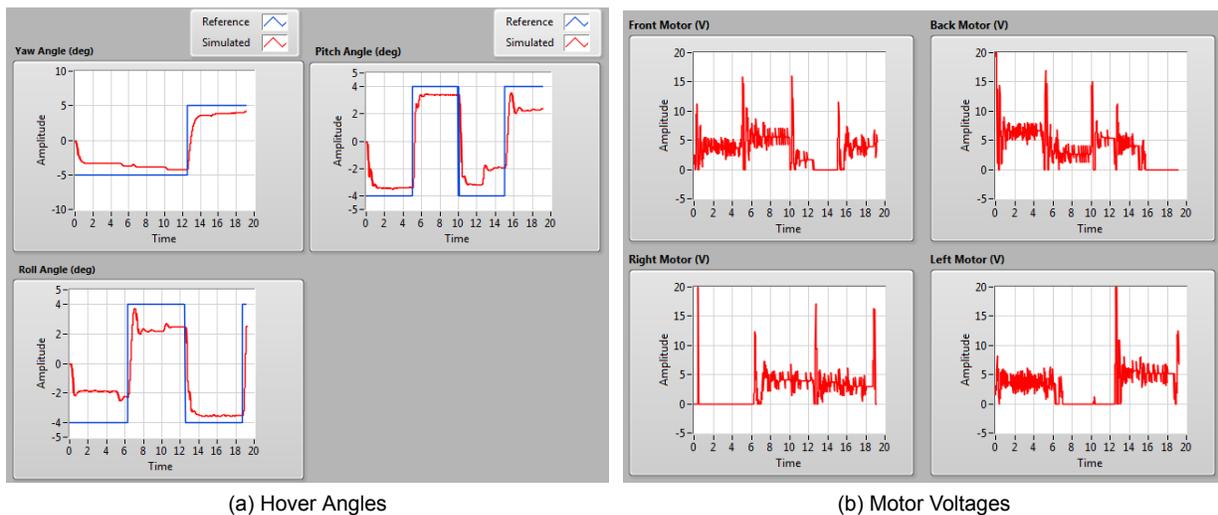


Figure 3.6: Sample closed-loop response of 3 DOF Hover

4. If your specifications have not been met, go back to the *3 DOF Hover Control Design.vi* to tune the LQR weighting matrices and generate a new control gain  $K$ .
5. The desired angle can be also be generated using a USB joystick described in 3 DOF Hover User Manual ([1]). To use the joystick, set the *Joystick ON/OFF* switch to the ON position.



**Caution:** Do not switch between Program and Joystick command when the controller is running. Change switch setting when the controller is OFF.

6. To stop the experiment, click on the Stop button.
7. Power off the amplifier(s).

### 3.2.3 Analysis

An example of the closed-loop balance response is shown in Figure 3.6.

The tracking performance of the pitch and roll are affected by the yaw control. When the yaw is stabilized at -5 degrees, the pitch step has a better peak time and steady-state error than when the yaw is stabilized at 5 degrees. This may be because when the yaw is at -5 degrees, the front and back motor have a higher average voltage which creates more overall thrust force and hence better control performance. Similarly, the roll angle tracks its desired angle better when the yaw is at 5 degrees, i.e., when the left and right motors have higher average voltage.

This is the drawback of the decoupled linear controller. It is based on a simple model and does not compensate for the coupling and other unmodeled effects in the system. Increasing  $\tilde{V}_{bias}$  does improve performance, but can draw too much current from the motors. The steady-state error can easily be improved using an integrator (an anti-windup strategy would be recommended) or a more advanced algorithm that compensates for disturbances and uncertainties (e.g., adaptive or robust type control).

# 4 SYSTEM REQUIREMENTS

## Required Software

Make sure **LabVIEW™** is installed with the following required add-ons:

1. **LabVIEW™**
2. NI-DAQmx
3. NI **LabVIEW™** Control Design and Simulation Module
4. NI **LabVIEW™** MathScript RT Module
5. **Quanser Rapid Control Prototyping Toolkit®**

**Note:** Make sure the Quanser Rapid Control Prototyping (RCP) Toolkit is installed after LabVIEW. See the RCP Toolkit Quick Start Guide for more information.

## Required Hardware

- Data acquisition (DAQ) device **with 3x encoder inputs and 4x analog output channels** that is compatible with **Quanser Rapid Control Prototyping Toolkit®**. This includes Quanser DAQ boards such as Q8-USB, QPID, and QPIDe and some National Instruments DAQ devices.
- Quanser 3 DOF Hover
- Quanser VoltPAQ-X4 power amplifier, or an equivalent amplifier solution for four motors.

## Before Starting Lab

Before you begin this laboratory make sure:

- **LabVIEW™** is installed on your PC.
- DAQ device has been successfully tested (e.g., using the test software in the Quick Start Guide or the *Analog Loopback Demo*).
- 3 DOF Hover and amplifier are connected to your DAQ board as described its User Manual [1].

## 4.1 Overview of Files

File Name	Description
3 DOF Hover User Manual.pdf	This manual describes the hardware of the 3 DOF Hover system and explains how to setup and wire the system for the experiments.
3 DOF Hover Laboratory Guide.pdf	This document demonstrates how to obtain the linear state-space model of the system, simulate the closed-loop system, and implement controllers on the 3 DOF Hover plant using LabVIEW™ .
3 DOF Hover Model.vi	VI used to generate the linear state-space model of the 3 DOF Hover system.
3 DOF Hover Hover Control Design.vi	VI that designs the LQR-based control and simulates the closed-loop control of the system using state-feedback control.
3 DOF Hover Control.vi	VI that implements the state-feedback control on the Quanser 3 DOF Hover system using LabVIEW™ .
3 DOF Hover Control (cRIO).vi	Same as 3 DOF Hover Control.vi except to be implemented on the NI CompactRIO.
Joystick_Read.vi	Only needed for NI CompactRIO users. This VI runs on the host PC to read the joystick and sends the corresponding positions to the 3 DOF Hover Control (cRIO).vi.
3 DOF Hover Equations.mws	Maple worksheet used to develop the model for the 3 DOF Hover experiment. Waterloo Maple 9, or a later release, is required to open, modify, and execute this file.
3 DOF Hover Equations.html	HTML presentation of the Maple Worksheet. It allows users to view the content of the Maple file without having Maple installed. No modifications to the equations can be performed when in this format.

Table 4.1: Files supplied with the 3 DOF Hover

## 4.2 Setup for Simulation

Before beginning the in-lab procedure outlined in Section 3.1, the *3 DOF Hover Modeling* and *3 DOF Hover Control Design* VI must be configured.

Follow these steps:

1. Load LabVIEW™ .
2. Open the *3 DOF Hover.lvproj* LabVIEW project, shown in Figure 4.1.
3. Open the *3 DOF Hover Modeling.vi* shown in Figure 4.2.
4. The 3 DOF Hover parameters are already set, by default. Run the VI to generate the linear state-space model.
5. In *Model Name*, enter the name of the model you and click on OK. This will save the state-space model under the folder *Hover Model*.
6. Close the 3 DOF Hover VI.
7. Open the *3 DOF Hover Control Design* VI, shown in Figure 3.1.

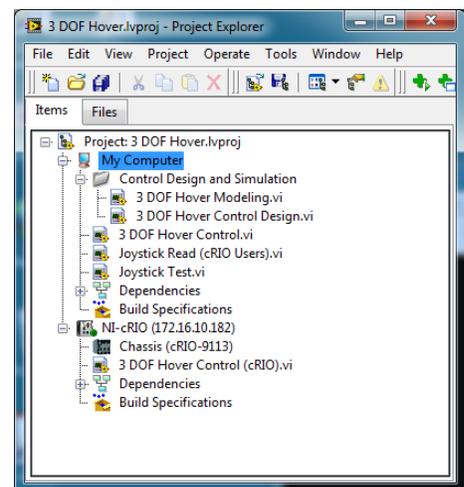


Figure 4.1: 3 DOF Hover LabVIEW Project

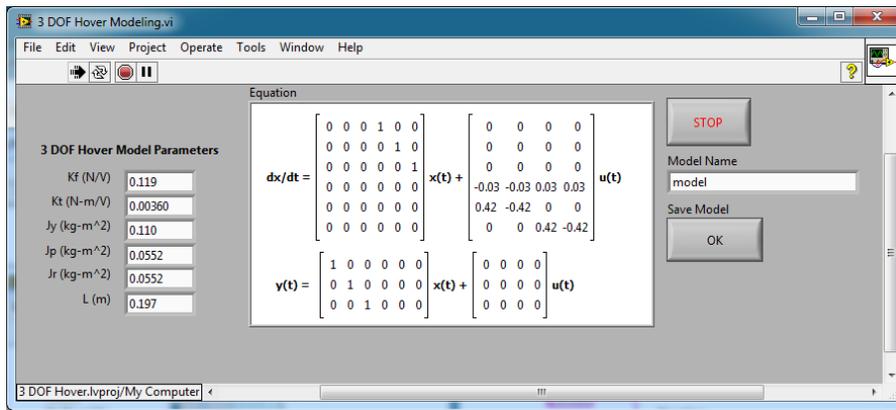


Figure 4.2: 3 DOF Hover Modeling VI

8. Using the *File Path* control, select the model file.
9. Run the VI. The state-space model should load. You are now ready to design your LQR control and simulate the closed-loop response.

### 4.3 Setup for Running on 3 DOF Hover

Before performing the in-lab exercises in Section 3.2, the 3 DOF Hoversystem and the *3 DOF Hover Control.vi* must be configured properly.

Follow these steps to get the system ready for this lab:

1. Setup the 3 DOF Hover system as detailed in the 3 DOF Hover User Manual ([1]).
2. **Make sure the 3 DOF Hover is balanced** before starting any controller. For more information, go to the 3 DOF Hover User Manual ([1]).
3. Open the *3 DOF Hover Control.vi*, shown in Figure 3.5.
4. Set gain *K* control in the VI to the value found in Section 4.2 (or another gain you want to test on the system).
5. **Configure DAQ:** Ensure the HIL Initialize block is configured for the DAQ device that is installed in your system. To do this, go to the block diagram (CTRL-E) and double click on the **HIL Initialize** Express VI shown in Figure 4.3.

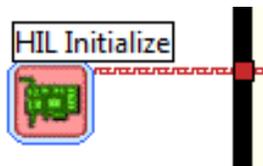


Figure 4.3: HIL Initialize Express VI

6. Under the Main tab, select the data acquisition device that is installed on your system in the *Board type* section. For example, in Figure 4.4 the Q8-USB is chosen.
7. Turn ON the power amplifier(s).
8. The desired angle can be also be generated using a USB joystick described in 3 DOF Hover User Manual ([1]). To use the joystick, set the Manual Switch shown in Figure 3.5 to the ON position. The rate at which the desired angle increases or decreases given a joystick position can be changed in the joystick sub-VI via the

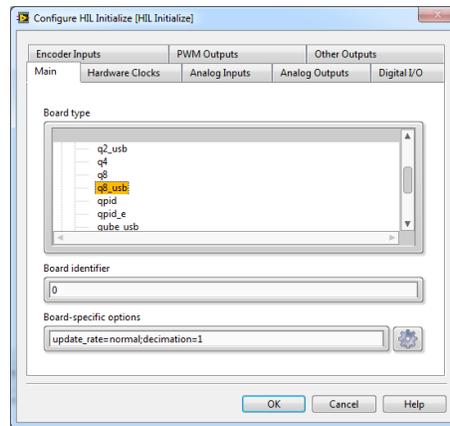


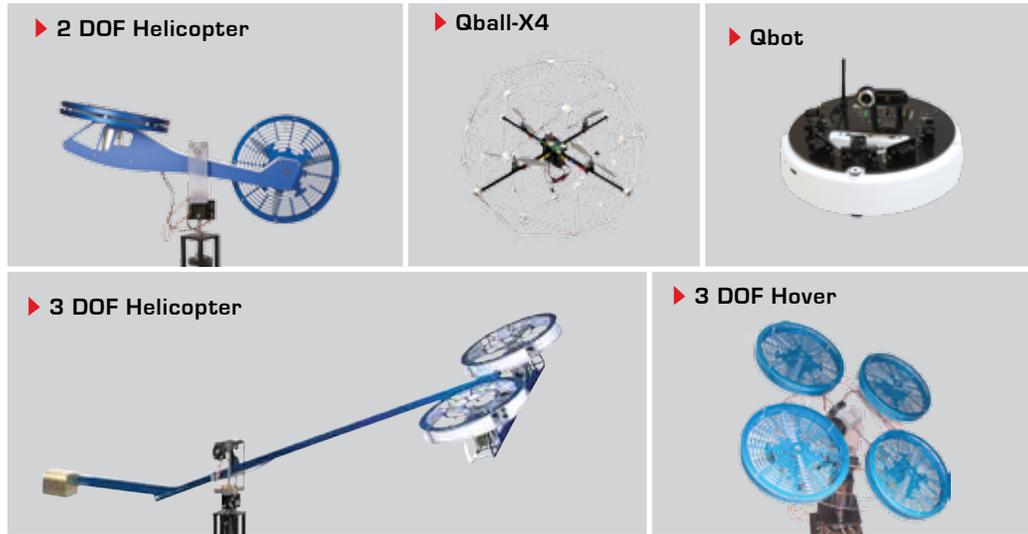
Figure 4.4: Select DAQ board that will be used to control system

block diagram. Note that there is also a Rate Limiter sub-VI used for all reference signals (i.e., from the signal generator in the VI and the joystick).

# REFERENCES

[1] Quanser Inc. *3 DOF Hover User Manual*, 2013.

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