



Linear Motion Servo Plant: AMD-1

Linear Experiment #9: Vibration Control

Active Mass Damper - One Floor (AMD-1)



Student Handout

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1. Objectives

As illustrated in Figure 1, below, the purpose of the AMD-1 experiment is to design a control system that dampens the vibration of a bench-scale building-like tall structure using an Active Mass Damper (AMD) mounted at the top. The Active Mass Damper - One Floor (AMD-1) experiment can be used in earthquake mitigation studies and to investigate Control-Structure Interaction (CSI). It is conceptually similar to active mass dampers used to suppress vibrations in tall structures (e.g. high-rise buildings) and to protect not only against earthquakes but also, for example, strong winds (e.g. hurricanes).

The Active Mass Damper – One Floor (AMD-1) plant is illustrated in Figure 1, below, and is fully described in Reference [1]. This laboratory takes advantage that the dynamics of the active mass (i.e. cart) are tightly coupled to these of the building-like structure to which it is attached. Therefore, the active mass can either be used to excite or to dampen the flexible structure vibration. The purpose of the AMD-1 laboratory is to design a switching-mode control system that first excites the vibration mode of the one-story structure and then dampens the structure oscillation.

During the course of this experiment, you will become familiar with the design of Proportional-plus-Velocity (PV) position controller to drive the linear cart (i.e. active mass) such that it excites the flexible structure natural vibrations. Then a vibration reduction control strategy will be designed in order to dampen the structure oscillations. Such a AMD control strategy will be implemented using a full-state feedback law based on a full-order observer. Pole placement will be used to tune the control scheme.



Figure 1 AMD-1 Experiment

At the end of the session, you should know the following:

- How to mathematically model the IP01- or IP02-based linear servo plant from first principles in order to obtain the open-loop transfer function, in the Laplace domain.
- How to design and tune a Proportional-Velocity (PV) position controller to meet the required design specifications.
- How to mathematically model the Active Mass Damper – One Floor (AMD-1) plant using the Lagrange's method and to obtain its state-space representation.
- How to design and tune a full-order state-observer based on the structure's acceleration feedback signal.
- How to design and tune a state-feedback controller satisfying the closed-loop system's desired design specifications.
- How to implement in real-time the total AMD-1 mode-switching (between vibration excitation and active mass damping) control scheme and evaluate its actual performance.

2. Prerequisites

To successfully carry out this laboratory, the prerequisites are:

- i) To be familiar with your Active Mass Damper – One Floor (AMD-1) main components (e.g. actuator, sensors), your data acquisition card (e.g. Q8, MultiQ), and your power amplifier (e.g. VoltPAQ), as described in References [1], [3], [4], and [5].
- ii) To have successfully completed the pre-laboratory described in Reference [2]. Students are therefore expected to be familiar in using QUARC to control and monitor the plant in real-time and in designing their controller through Simulink, as detailed in Reference [6].
- iii) To have successfully completed the laboratory described in Reference [7].
- iv) To be familiar with the complete wiring of your Active Mass Damper – One Floor (AMD-1) plant, as per dictated in Reference [1].
- v) To be familiar with the design theory of full-order state observers, as described for example in Reference [8].

3. References

- [1] *Active Mass Damper – One Floor (AMD-1) User Manual.*
- [2] *IP01 and IP02 – Linear Experiment #0: Integration with QUARC – Student Handout.*
- [3] *IP01 and IP02 User Manual.*
- [4] *Data Acquisition Card User Manual.*
- [5] *Power Amplifier User Manual.*
- [6] *QUARC User Manual* (type `doc quarc` in Matlab to access).
- [7] *IP01 and IP02 - Linear Experiment #1: PV Position Control – Student Handout.*

[8] P. R. Bélanger. *Control Engineering: A Modern Approach*. 1995, Saunders College Publishing.

4. Experimental Setup

4.1. Main Components

To setup this experiment, the following hardware and software are required:

- **Power Module:** Quanser VoltPAQ-X1, or equivalent.
- **Data Acquisition Board:** Quanser Q8-USB, QPID, or equivalent.
- **Active Mass Damper Plant:** Quanser Active Mass Damper – One Floor (AMD-1), as represented in Figure 1, above.
- **Real-Time Control Software:** QUARC, as detailed in Reference [6], or equivalent.

For a complete and detailed description of the main components comprising this setup, please refer to the manuals corresponding to your configuration.

4.2. Wiring

To wire up the system, please follow the default wiring procedure for your Active Mass Damper – One Floor (AMD-1) system, as fully described in Reference [1]. When you are confident with your connections, you can power up the amplifier.

5. Controllers Design Specifications

The Active Mass Damper – One Floor (AMD-1) experiment essentially consists of a switching-mode controller using a pre-defined logic sequence to alternate between a mode of repeatable structural excitation and the Active Mass Damping (AMD) mode to stiffen the structure. Each mode is achieved through its own closed-loop control scheme.

However, the power amplifier (e.g. VoltPAQ) should not go into saturation in any case. Also both controllers control effort, which is proportional to the motor input voltage V_m , should stay within the system's physical limitations.

5.1. Excitation Mode: PV Controller Design Specifications

The (self-)excitation mode uses the active mass to generate a repeatable disturbance to the AMD-1's building-like structure. In this case, the linear cart (i.e. active mass) control loop consists of a Proportional-Velocity (PV) position controller.

In the present laboratory (i.e. the pre-lab and in-lab sessions), you will design and implement a control strategy based on the Proportional-Velocity (PV) control scheme, in order for your linear cart system to satisfy the following performance closed-loop requirements:

1. The cart position Percent Overshoot, PO , should be less than 10%, i.e.:

$$PO \leq 10 \text{ [\%]} \quad [1]$$

2. The time to first peak should be less than 150 ms, i.e.:

$$t_p \leq 0.15 \text{ [s]} \quad [2]$$

5.2. Active Mass Damping (AMD) Mode

In the Active Mass Damping (AMD) mode, the linear cart is controlled by using a state-feedback law based on a full-order observer. This time around, the purpose of the control strategy is to dampen the AMD-1 top floor oscillations (created in the excitation mode) by stiffening the structure.

The general AMD design requirements are expressed below in terms of settling time and decayed amplitude of the floor deflection (x_f) for an initial deflection of around 30 mm:

$$t_s \leq 1.0 \text{ [s]} \quad \text{for} \quad |x_f| \leq 1.5 \text{ [mm]}$$

The particular pole placements presented in the following by Equations [3] and [4] were chosen for the AMD closed-loop system to meet the specifications stated above.

5.2.1. State-Feedback Design: Pole Locations

Determine the AMD-1 full-state feedback gain vector, K , such that the closed-loop poles (i.e. eigenvalues) due to the state-feedback law are placed at the following locations:

$$-6 + 15j, -6 - 15j, -8, -16 \quad [3]$$

5.2.2. Full-Order State Observer Design: Pole Locations

However in order to be based on a full-state feedback (i.e. all structural displacements and velocities) law, the AMD-1's active structural control strategy should be based on a state observer. This is explained by the fact that the AMD-1's floor deflection and velocity are not measured directly, only its acceleration is. This particular design for the AMD-1 plant is due to full-scale real-life applications where floor deflections and velocities are difficult to measure directly. However, they can be estimated from acceleration measurements, since accelerometers provide an affordable and reliable way of sensing a building dynamic behaviour.

Determine the AMD-1 full-order observer gain matrix, G , such that the closed-loop poles (i.e. eigenvalues) due to the observer error dynamics are placed at the following locations:

$$-20, -25, -30, -35 \quad [4]$$

Comparing Equations [3] and [4], it can be noted that the dynamics of the observer error (i.e. rate at which the estimation error goes to zero) are at least three times faster (i.e. farther to the left in the s-plane) than the plant itself. This should ensure that the estimator does not interfere with the plant's dynamics.

6. Pre-Lab Assignments

6.1. Assignment #1: Proportional-Velocity (PV) Controller Design

In this design procedure for the PV controller, the one-floor flexible structure on top of which the linear cart is mounted is ignored. The system is considered to consist of the IP01- or IP02-based linear servo plant alone. A schematic of the AMD-1 linear cart system's input and output is represented in Figure 7, below.

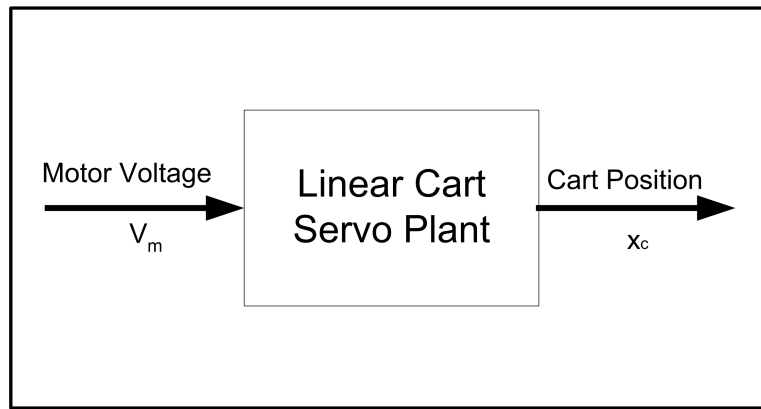


Figure 2 The AMD-1 Linear Cart Input and Output

1. Based on your previous work from Reference [7], write down the open-loop transfer function of your IP01- or IP02-based system, $G_c(s)$, as defined below:

$$G_c(s) = \frac{x_c(s)}{V_m(s)} \quad [5]$$

2. The Proportional-Velocity (PV) position controller implemented in this lab for your linear servo plant introduces two corrective terms: one is proportional (by K_p) to the cart position error while the other is proportional (by K_v) to the cart velocity. Equation [6], below, expresses the resulting PV control law:

$$V_m(t) = K_p (x_{c_d}(t) - x_c(t)) - K_v \left(\frac{d}{dt} x_c(t) \right) \quad [6]$$

3. Quickly re-iterate your previous work from Pre-Lab Assignment #3 (i.e. the *PV Controller Design* Section) of Reference [7] in order to determine the two PV controller's gains,

K_p and K_v , as functions of the second-order system's characteristic parameters ω_{nc} and ζ_c .

4. Using the two Hint formulae provided below, express ω_{nc} and ζ_c as functions of the two PV design specifications previously defined, PO and t_p .

Hint formula #1:

$$PO = 100 e^{\left(-\frac{\pi \zeta_c}{\sqrt{1 - \zeta_c^2}} \right)} \quad [7]$$

Hint formula #2:

$$t_p = \frac{\pi}{\omega_{nc} \sqrt{1 - \zeta_c^2}} \quad [8]$$

5. Determine the values of ω_{nc} and ζ_c corresponding to the desired PV design specifications, as defined previously. Determine, then, from your results the numerical values of K_p and K_v satisfying the desired time requirements of your closed-loop PV-plus-cart system.

6.2. AMD-1 System Representation and Notations

A schematic of the Active Mass Damper – One Floor (AMD-1) plant is represented in Figure 3, below. The AMD-1 scaled building is a Single-Degree-Of-Freedom (SDOF) structure (i.e. single story). The AMD-1 system's nomenclature is provided in Appendix A. As illustrated in Figure 3, the positive direction of horizontal displacement is towards the right when facing the system.

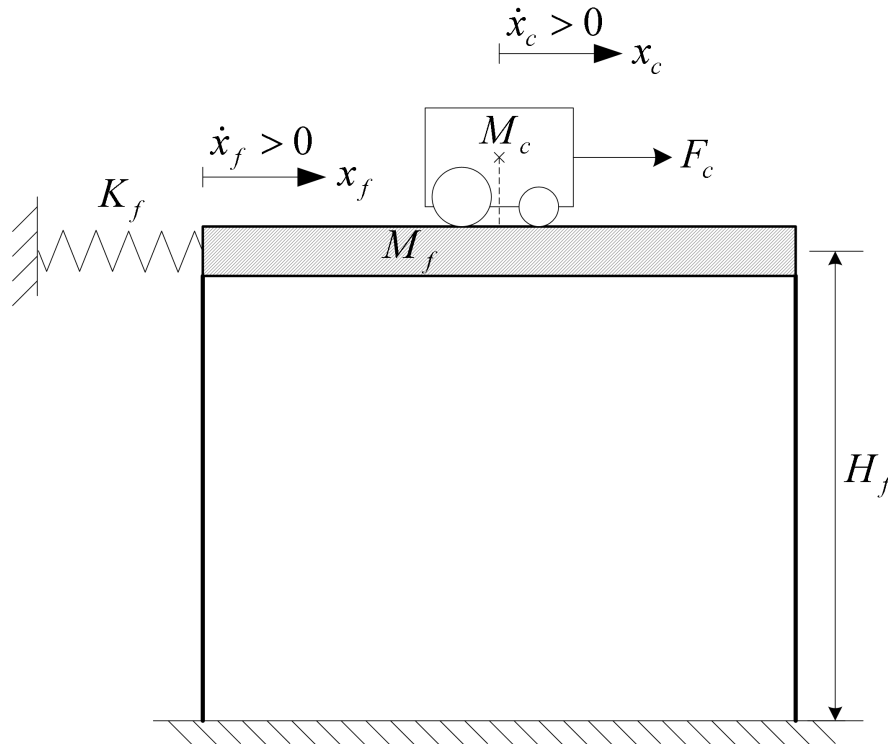


Figure 3 Schematic of the AMD-1 Plant

For small floor deflection angles, the AMD-1 "roof" is modelled as a standard linear spring-mass system, as represented in Figure 3 above. Using the system's specifications given in Reference [1], the floor assembly mass, M_f , can be calculated from:

$$M_f = M_r + M_{tf} \quad [9]$$

The AMD-1's top floor linear stiffness constant K_f , for small angular structure oscillations, is given in Reference [1]. K_f models the lateral stiffness of the structure. However, it is consistent with the following relationship, obtained from the standard Ordinary Differential Equation (ODE) describing the free-oscillatory motion of a mass:

$$K_f = 4 (M_f + M_c) \pi^2 f_n^2 \quad [10]$$

In the presented modelling approach, the structure viscous damping coefficient, B_f , is neglected.

6.3. Assignment #2: Determination of the AMD-1 System's Linear Equations Of Motion (EOM)

The determination of the AMD-1's structure-plus-cart equations of motion is derived in Appendix B. If Appendix B has not been supplied with this handout, derive the system's equations of motion following the system's schematic and notations previously defined and illustrated in Figure 3. Also, put the resulting EOM under the following format:

$$\frac{\partial^2}{\partial t^2} x_c = \left(\frac{\partial^2}{\partial t^2} x_c \right) \left(x_c, x_f, \frac{\partial}{\partial t} x_c, \frac{\partial}{\partial t} x_f, F_c \right) \quad [11]$$

and

$$\frac{\partial^2}{\partial t^2} x_f = \left(\frac{\partial^2}{\partial t^2} x_f \right) \left(x_c, x_f, \frac{\partial}{\partial t} x_c, \frac{\partial}{\partial t} x_f, F_c \right) \quad [12]$$

Hint #1:

By neglecting the Coulomb (a.k.a. static) friction of the cart system, the two EOM should be linear. They represent a pure spring-mass-damper system.

Hint #2:

You can use the method of your choice to model the system's dynamics. However, the modelling developed in Appendix B uses the energy-based Lagrangian approach. In this case, since the system has two Degrees-Of-Freedom (DOF), there should be two Lagrangian coordinates (a.k.a. generalized coordinates). The chosen two coordinates are namely: x_c and x_f . Also, the input to the system is defined to be F_c , the linear force applied by the motorized cart.

6.4. Assignment #3: AMD-1 State-Space Representation

In order to design and implement a state-feedback controller for our system, a state-space representation of that system needs to be derived. Moreover, it is reminded that state-space matrices, by definition, represent a set of linear differential equations that describe the system's dynamics. Since the two EOM of the AMD-1, as found in Assignment #2, should already be linear, they can be written under the state-space matrices representation.

Answer the following questions:

1. Determine from the system's two equations of motion, the state-space representation of our AMD-1 plant. That is to say, determine the state-space matrices A , B , C , and D verifying the following relationships:

$$\frac{\partial}{\partial t} X = A X + B U \quad \text{and} \quad Y = C X + D U \quad [13]$$

where X is the system's state vector. In practice, X is often chosen to include the generalized coordinates as well as their first-order time derivatives. In our case, X is defined such that its transpose is as follows:

$$X^T = \left[x_c(t), x_f(t), \frac{d}{dt} x_c(t), \frac{d}{dt} x_f(t) \right] \quad [14]$$

Furthermore, it is reminded that the system's measured output vector is:

$$Y^T = \left[x_c(t), \frac{d^2}{dt^2} x_f(t) \right] \quad [15]$$

Also in Equation [13], the input U is set in a first time to be F_c , the driving force of the linear motorized cart. Thus we have:

$$U = F_c \quad [16]$$

As a remark, it can be seen from Equations [15] and [16] that the AMD-1 system consists of two outputs for one input.

2. From the system's state-space representation previously found, transform the state-space matrices for the case where the system's input U is equal to the linear cart's DC motor voltage V_m , instead of the linear force F_c . The system's input U can now be expressed by:

$$U = V_m \quad [17]$$

Hint:

In order to convert the previously found force equation state-space representation to voltage input, it is reminded that the driving force, F_c , generated by the DC motor and acting on the cart through the motor pinion has already been determined in previous laboratories. As shown for example in Equation [B.9] of Reference [7], F_c can be

expressed by:

$$F_c = - \frac{\eta_g K_g^2 \eta_m K_t K_m \left(\frac{d}{dt} x_c(t) \right)}{R_m r_{mp}^2} + \frac{\eta_g K_g \eta_m K_t V_m}{R_m r_{mp}} \quad [18]$$

3. Evaluate the matrices A , B , C , and D found in question 2, that is to say in case the system's input U is equal to the cart's DC motor voltage, as expressed in Equation [16].

Hint:

Evaluate state-space matrices by using the model parameter values given in Reference [1]. Ask your laboratory instructor what system configuration you are going to use in your in-lab session. In case no additional information is provided, assume that the two additional masses are mounted on top of the linear cart.

4. Calculate the open-loop poles from the system's state-space representation, as previously evaluated in question 3. Is it stable? What is the type of the system? What can you infer regarding the system's dynamic behaviour? Do you see the need for a closed-loop controller? Explain.

Hint:

The characteristic equation of the open-loop system can be expressed as shown below:

$$\det(s I - A) = 0 \quad [19]$$

where $\det()$ is the determinant function, s is the Laplace operator, and I the identity matrix. Therefore, the system's open-loop poles can be seen as the eigenvalues of the state-space matrix A .

6.5. Assignment #4: Full-Order Observer

Since all the states contained in X , as defined in Equation [14], cannot be directly measured (e.g. x_f), a state observer needs to be built to estimate them. The system's variables directly measured are expressed in the output vector defined in Equation [15]. This laboratory focuses on the design of a full-order observer.

The full-order observer structure is defined as follows:

$$\frac{\partial}{\partial t} X_o = A X_o + B U + G (Y - Y_o) \quad [20]$$

and:

$$Y_o = C X_o + D U \quad [21]$$

It can be seen from Equations [20] and [21] that the state observer has for inputs the system's input(s) and output(s) and calculates as outputs the states estimates. The observer is basically a replica of the actual plant with a corrective term $G(Y-Y_o)$ multiplied by the observer gain matrix.

The obtained estimated state vector can then be used for state-feedback law, as expressed below:

$$(U = V_m) = -K X_o \quad [22]$$

Let us define the estimation error vector as follows:

$$X_e = X - X_o \quad [23]$$

Using Equations [13], [20], [21], and [23], the estimation error dynamically behaves accordingly to the following relationship:

$$\frac{\partial}{\partial t} X_e = (A - G C) X_e \quad [24]$$

Therefore from Equation [24], the estimation error will asymptotically go to zero if and only if $(A-GC)$ is stable, that is to say iff G is determined such as $(A-GC)$ has all its eigenvalues in the left-hand plane. However, a theorem shows that if (A,C) is observable, then $(A-GC)$ can always be made stable by a proper choice of G . Therefore, it is possible to estimate the state(s) of a system if and only if that system is observable. Please refer to your in-class notes as required.

Determine whether (A,C) is observable.

Hint:

By definition, a system is observable iff its observability matrix has full rank (i.e. number of states). The observability matrix is defined as follows:

$$W_o = [C, C A, C A^2, C A^3]^T \quad [25]$$

7. In-Lab Procedure

7.1. Experimental Setup And Wiring

Even if you do not configure the experimental setup entirely yourself, you should be at least completely familiar with it and understand it. If in doubt, refer to References [1], [4], [5], and/or [6].

The first task upon entering the lab is to ensure that the complete system is wired as fully described in Reference [1]. You should be familiar with the complete wiring and connections of your Active Mass Damper – One Floor (AMD-1) system. If you are still unsure of the wiring, please ask for assistance from the Teaching Assistant assigned to the lab. When you are confident with your connections, you can power up the amplifier. You are now ready to begin the lab.

7.2. Real-Time Implementation Of The AMD-1 Switching-Mode Controller

7.2.1. Objectives

- To implement in real-time with QUARC the previously designed PV position controller in order to command your actual AMD-1 linear servo plant.
- To design through pole placement a full-order observer for the actual AMD-1 system.
- To design through pole placement a state-feedback law for the actual AMD-1 system by using the obtained estimated state vector.
- To implement in real-time with QUARC the previously determined observer-based state-feedback to dampen the vibration of the AMD-1 structure by appropriately driving the active mass on top of it.

7.2.2. Experimental Procedure

Please follow the steps described below:

Step1. If you have not done so yet, you can start-up Matlab now. Depending on your system configuration, open the Simulink model file of name type *q_amd1_e.mdl*. You should obtain a diagram similar to the one shown in Figure 4, below.

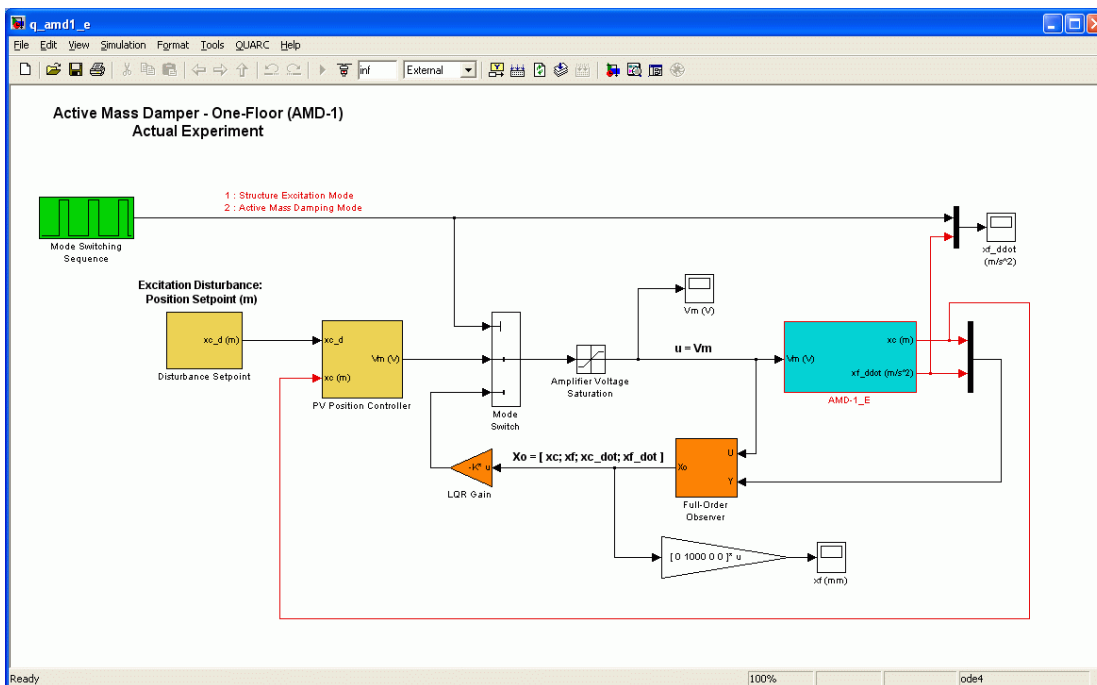


Figure 4 Real-Time Implementation of the AMD-1 Controller

In order to use your actual AMD-1 system, the controller diagram directly interfaces with your system hardware, as shown in Figure 5, below.

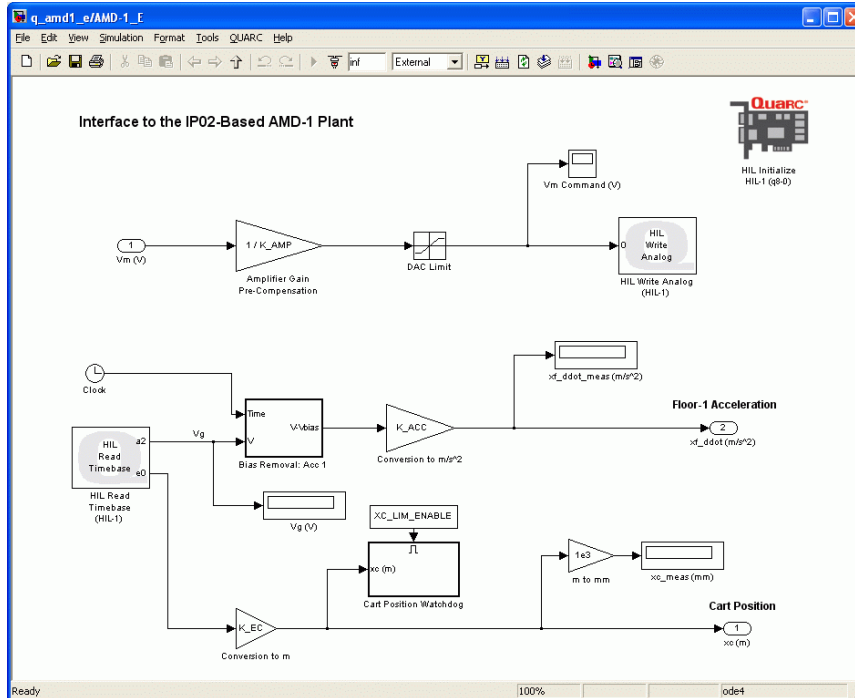


Figure 5 Interface Subsystem to the Actual AMD-1 Plant

Step2. **Configure DAQ:** Double-click on the HIL Initialize block shown in Figure 5, above, that is found in the *AMD-1* subsystem and ensure it is configured for the DAQ device that is installed in your system (e.g Q8-USB). See Reference [6] for more information on configuring the HIL Initialize block.

Step3. To familiarize yourself with the diagram, it is suggested that you open the model subsystems to get a better idea of their composing blocks as well as take note of the I/O connections.

The real-time model, shown in Figure 4 above, implements a switching-mode controller. A pre-defined *Mode-Switching Sequence* alternatively controls the linear cart using either a Proportional-Velocity (PV) position controller or an observer-based state-feedback law. In short, the PV controller is mostly used in the excitation mode to self-excite the vibration in the building-like structure. On the other hand, the state-feedback law is designed to dampen the structure oscillation by driving the active mass (i.e. cart). Both control loops are the actual implementations of the pre-laboratory assignments previously carried out.

Opening the *Disturbance Setpoint* sub-system should show a diagram similar to Figure 6. This diagram generates the cart position setpoint to follow in order to best excite the AMD-1's flexible structure. The setpoint basically consists of one full period of a sine wave of frequency 2.5 Hz (i.e. for a duration of 0.4 seconds) and amplitude 6 cm. The sinusoidal excitation's frequency has been chosen accordingly to the struc-

ture's natural frequency, as given in Reference [1]. This excitation repeats itself every 10 seconds.

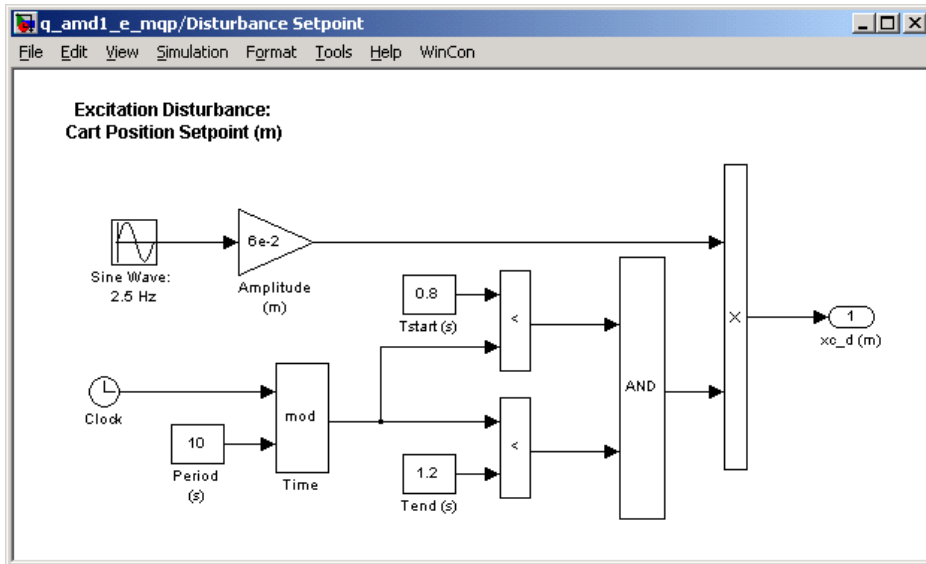


Figure 6 Excitation Mode: Cart Position Setpoint Generation

Figure 4, above, also includes a subsystem named *PV Position Controller*, which implements your linear cart PV controller's two feedback loops. This PV controller is based on the actual measurement of the cart position (coming from the encoder). The obtained position signal is then differentiated and low-pass filtered to attenuate any high-frequency noise.

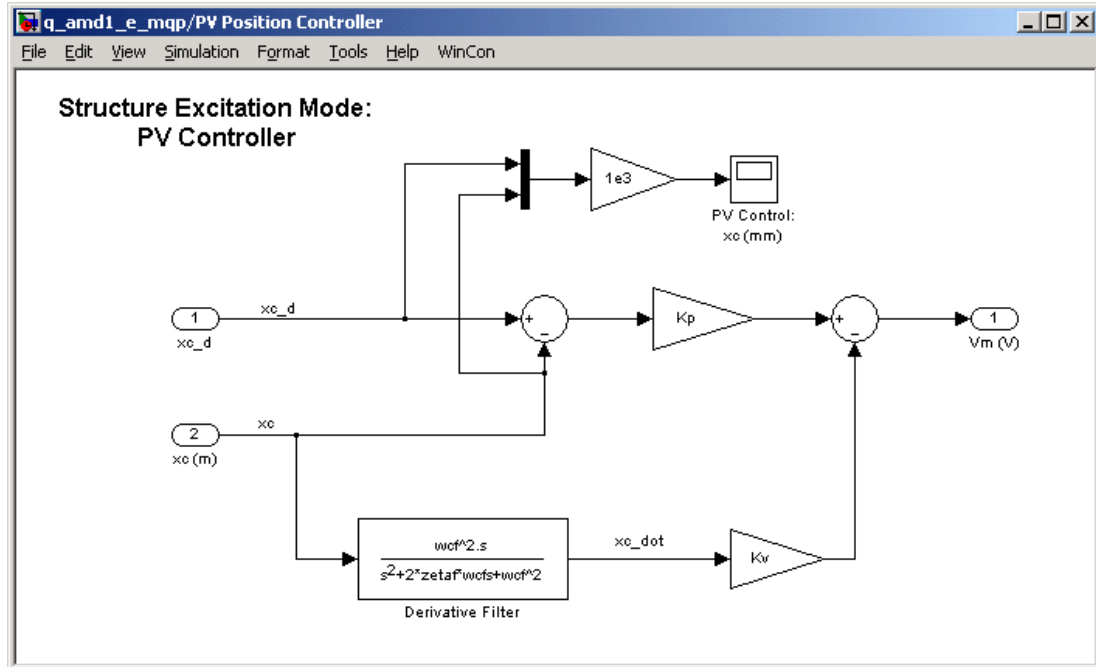


Figure 7 Diagram used for the Real-Time Implementation of the AMD-1 Cart PV Position Controller



CAUTION:

The velocity signal used in the control inner-loop of the actual linear cart is obtained by first differentiating the position signal (i.e. encoder counts), and then by low-pass filtering the obtained signal in order to eliminate its high frequency content. As a matter of fact, high-frequency noise, which is moreover amplified during differentiation, causes long-term damage to the motor. To protect your DC motor, the recommended cut-off frequency is 50 Hz.

When the *Mode-Switching Sequence* block outputs "2" instead of "1", that turns on the Active Mass Damping (AMD) capabilities of the linear cart located on top of the structure. The AMD behaviour is achieved by implementing a full-order-observer-based state-feedback control loop, as illustrated in Figure 4 above by the orange blocks. The full-order state observer defined by Equations [20] and [21] is implemented in the subsystem called *Full-Order Observer* and depicted in Figure 8, below.

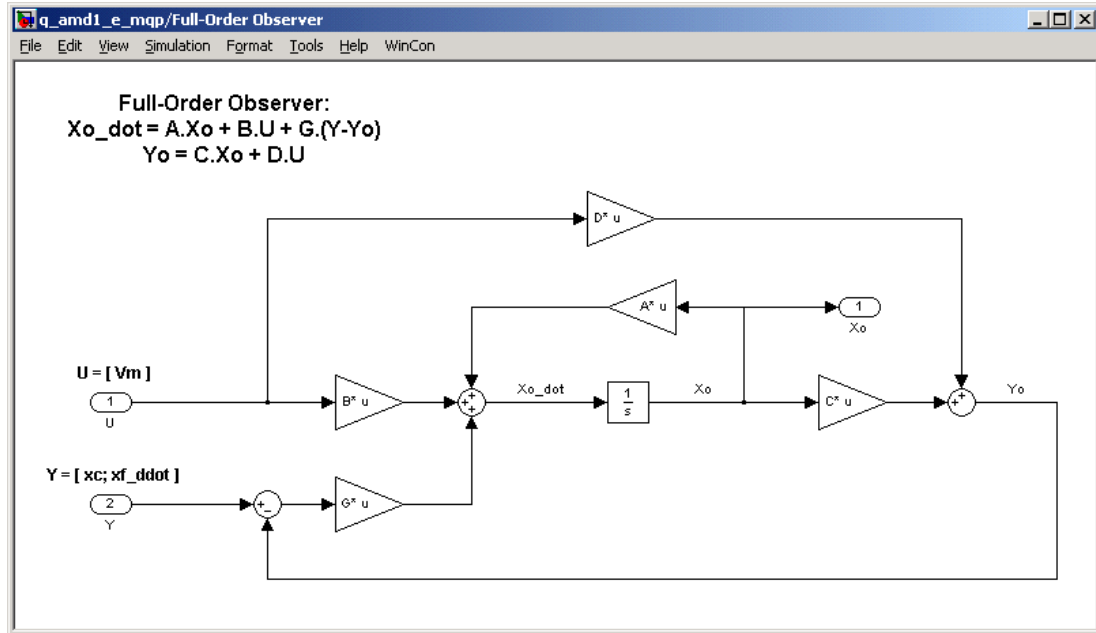


Figure 8 Actual AMD-1 Full-Order State Observer

Step4. Before being able to run the actual control loops, the different controller gains must be initialized in the Matlab workspace, since they are to be used by the Simulink controller diagram. Start by running the Matlab script called *setup_lab_amd1.m*. However, ensure beforehand that the *CONTROLLER_TYPE* flag is set to 'MANUAL'. This file initializes all the AMD-1 model parameters, user-defined parameters, and AMD-1's state-space matrices, as defined in pre-lab Assignment #3. First, the two PV controller gains, K_p and K_v , calculated in pre-lab Assignment #1 and satisfying the desired time requirements must be entered in the Matlab workspace. To assign K_p and K_v , type their values in the Matlab command window by following the Matlab notations used for the controller gains as presented in Table A.2 of Appendix A. Second, the state-feedback gain vector, K , must be calculated and entered in the Matlab workspace. Use Matlab to carry out the pole-placement calculations satisfying the design requirement expressed by Equation [3]. Third and last, the observer gain matrix, G , must also be calculated and entered in the Matlab workspace. Use Matlab to carry out the pole-placement calculations satisfying the design requirement expressed by Equation [4]. Refer to your in-class notes regarding the full-order observer design theory as needed.

Hint:

Pole-placement calculations can be achieved in Matlab by using the function '*place()*' or '*acker()*'.

CAUTION:



Have your lab assistant check your controller gain values. Do not proceed to the next step without his or her approval.

Step5. You are now ready to go ahead with compiling and running your actual switching-mode controller for the AMD-1 system. First, compile the real-time code corresponding to your diagram, by using the *QUARC* | *Build* option from the Simulink menu bar.

CAUTION:



Before starting your actual controller, manually move the linear cart (located on the top floor) to the middle of the track (i.e. mid-stroke position). Also make sure that the AMD-1 "ground" floor is properly clamped/mounted to a table.

Step6. Click on *QUARC* | *Start* from the Simulink diagram menu bar to start running the controller. This should start the AMD-1 control sequence and the floor deflection scope should read something as shown .

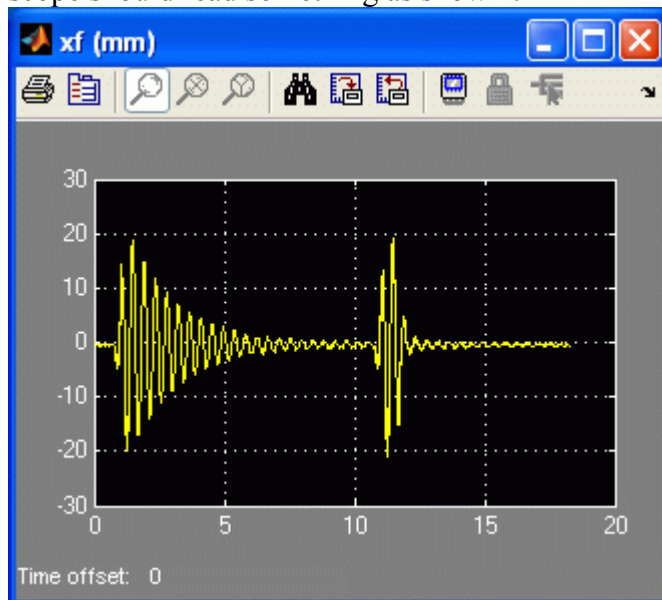


Figure 9: AMD-1 floor deflection estimate

As previously mentioned, the AMD-1's linear cart is commanded by a switching-mode controller. In excitation mode (i.e. Mode Switching Sequence equals to "1", or AMD OFF), the cart is subject to the PV position controller. In Active Mass Damping (AMD) mode (i.e. Mode Switching Sequence equals to "2", or AMD ON), the cart is subject to an observer-based state-feedback controller. The AMD-1 experiment is setup such that the cart excites the structural dynamics once every 10 seconds by following a full-period sinusoidal excitation at the structure's natural frequency. This is illustrated in Figure 9, above, by *Excitation #1* and *Excitation #2*. This initial and re-

peatable mass movement causes a vibration in the AMD-1 flexible structure which persists at its natural rate if the AMD mode/controller is not switched on, as depicted by the first 10 seconds in Figure 9. The AMD mode is only turned on after the second structural excitation. This causes the top floor vibrations to dampen out quickly, as shown by the last 10 seconds in Figure 9. In order to observe the structure natural behaviour versus its actively damped (or controlled) behaviour, the AMD mode is only switched on once every over 10-second period, while the excitation disturbance repeats itself once every 10-second period.

- Step7. In order to observe the system's real-time responses from the actual system, open the following scopes: xf_ddot (m/s^2), xf (mm), and *PV Position Controller/PV Control: xc* (mm). You should now be able to monitor on-the-fly, as the system goes through the synchronized excitation/AMD-control sequences, the measured top floor acceleration, the controller mode, the estimated floor deflection, and the linear cart setpoint and actual position, respectively. Furthermore, you can also open the sink Vm (V). This allows you to monitor on-line the actual commanded motor voltage, which is proportional to the control effort spent, sent to the power amplifier.
- Step8. Over an example run of 20 seconds, your actual AMD-1's floor acceleration and control mode sequences should look similar to the ones displayed in Figure 10, below. Corresponding to the same experimental run, the time records of the AMD-1's cart measured position and excitation setpoint are depicted in Figure 11, below.

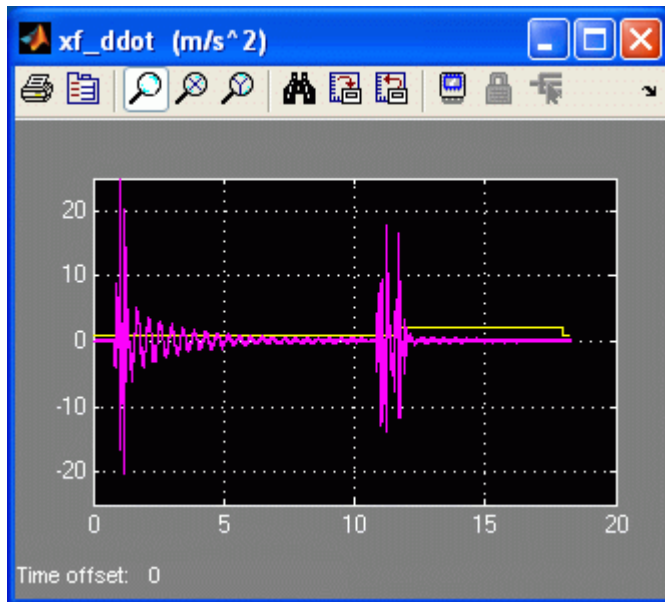


Figure 10: AMD-1 floor measured acceleration and controller mode

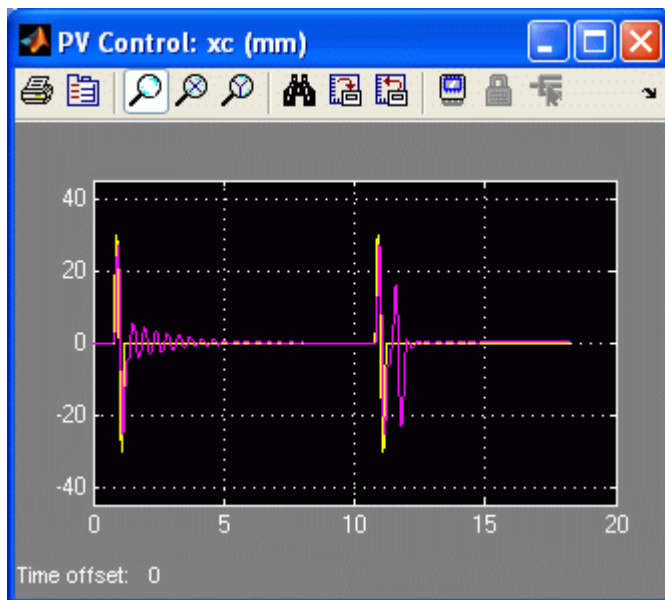


Figure 11: AMD-1 measured cart position response and excitation setpoint

- Step9. Compare the structure's actively damped and natural behaviours in terms of settling time and amplitude of the floor deflection (xf) and/or acceleration (xf_ddot). Assess the actual performance of your Active Mass Damping controller. Measure your floor estimated deflection response settling time. Are the design specifications satisfied? Explain.
- Step10. If your AMD-1 responses do not meet the desired design specifications, review your PV and/or observer and/or state-feedback gain calculations and/or alter the closed-loop pole locations until they do. If you are still unable to achieve the required performance level, ask your T.A. for advice.
- Step11. Include in your lab report your final values for K_p , K_v , K , and G as well as the resulting plots of the actual system responses (e.g. floor acceleration, estimated deflection, controller mode, cart position, command voltage). Each scope is configured to save its data to a Matlab variable, e.g. scope xf (mm) saves it to variable $data_xf$. This can be used to generate Matlab figures to display the measured response more clearly. Ensure to properly document all your results and observations.
- Step12. You can move on and begin your report for this lab.

Appendix A. Nomenclature

Table A.1, below, provides a complete listing of the symbols and notations used in the Active Mass Damper – One Floor (AMD-1) mathematical modelling, as presented in this laboratory. The numerical values of the system parameters can be found in Reference [1].

<i>Symbol</i>	<i>Description</i>	<i>Matlab / Simulink Notation</i>
M_{tf}	Structure Top Floor Mass	Mtf
M_r	Rack Mass	Mr
f_n	Floor (a.k.a. Roof) Natural Frequency	fn
K_f	Floor (a.k.a. Roof) Linear Stiffness Constant	Kf
M_f	Floor/Roof (i.e. floor 1 + rack) Total Mass	Mf
x_f	Floor Horizontal Deflection (Relative To The Ground)	xf
$\frac{\partial}{\partial t} x_f$	Floor Horizontal Velocity (Relative To The Ground)	xf_dot
$\frac{\partial^2}{\partial t^2} x_f$	Floor Horizontal Acceleration (Relative To The Ground)	xf_ddot
M_c	Total Mass of the Cart System	Mc
V_m	Cart Motor Armature Voltage	Vm
I_m	Cart Motor Armature Current	Im
R_m	Cart Motor Armature Resistance	Rm
L_m	Cart Motor Armature Inductance	Lm
K_t	Cart Motor Torque Constant	Kt
ζ_m	Cart Motor Efficiency	Eff_m
K_m	Cart Back-ElectroMotive-Force (EMF) Constant	Km
J_m	Cart Rotor Moment of Inertia	Jm
B_{eq}	Equivalent Viscous Damping Coefficient, as seen at the Motor Pinion	Beq
K_g	Cart Planetary Gearbox Gear Ratio	Kg
ζ_g	Cart Planetary Gearbox Efficiency	Eff_g

<i>Symbol</i>	<i>Description</i>	<i>Matlab / Simulink Notation</i>
r_{mp}	Cart Motor Pinion Radius	r_mp
F_c	Cart Driving Force Produced by the Motor (i.e. Control Force)	Fc
x_c	Cart Linear Position (Relative To The Floor)	xc
$\frac{\partial}{\partial t} x_c$	Cart Linear Velocity (Relative To The Floor)	xc_dot
V_T	Total Potential Energy of the AMD-1 System	
Tt_c	Cart's Translational Kinetic Energy	
Tr_c	Cart Rotor's Rotational Kinetic Energy	
Tt_f	Structure Top Floor's Translational Kinetic Energy	
T_T	Total Kinetic Energy of the AMD-1 System	
Q_{x_c}	Generalized Force Applied on the Generalized Coordinate x_c	
Q_{x_f}	Generalized Force Applied on the Generalized Coordinate x_f	

Table A.1 AMD-1 Model Nomenclature

Table A.2, below, provides a complete listing of the symbols and notations used in the PV position controller design, as used in this laboratory.

<i>Symbol</i>	<i>Description</i>	<i>Matlab / Simulink Notation</i>
PO	Percent Overshoot	PO
t_p	Peak Time	tp
ω_{nc}	Cart Undamped Natural Frequency	wn_c
ζ_c	Cart Damping Ratio	zeta_c
K_p	Cart Proportional Gain	Kp
K_v	Cart Velocity Gain	Kv
x_{c_d}	Cart Desired Position (i.e. Reference Signal)	xc_d
t	Continuous Time	
s	Laplace Operator	

Table A.2 PV Cart Controller Nomenclature

Table A.3, below, provides a complete listing of the symbols and notations used in the full-order observer and state-feedback controller design, as presented in this laboratory.

<i>Symbol</i>	<i>Description</i>	<i>Matlab / Simulink Notation</i>
A, B, C, D	State-Space Matrices of the AMD-1 System	A, B, C, D
X	Actual State Vector	X
Y	Actual Output Vector	Y
U	System Input	U
K	State-Feedback Gain Vector	K
X_o	Estimated State Vector	Xo
Y_o	Estimated Output Vector	Yo
G	Full-Order State Observer Gain Matrix	G
W_o	Observer's Observability Matrix	Wo
X_e	State Vector Estimation Error	Xe

Table A.3 Control Loop Nomenclature

Appendix B. AMD-1 Equations Of Motion (EOM)

This Appendix derives the general dynamic equations of the Active Mass Damper – One Floor (AMD-1) system. The Lagrange's method is used to obtain the dynamic model of the system. In this approach, the single input to the system is considered to be F_c . Furthermore, it is reminded that the reference frame used is defined in Figure 3, on page 8.

To carry out the Lagrange's approach, the Lagrangian of the system needs to be determined. This is done through the calculation of the system's total potential and kinetic energies.

Let us first calculate the system's total potential energy V_T . The potential energy in a system is the amount of energy that that system, or system element, has due to some kind of work being, or having been, done to it. It is usually caused by its vertical displacement from normality (gravitational potential energy) or by a spring-related sort of displacement (elastic potential energy). Here, there is no gravitational potential energy since both AMD-1 cart and structure are assumed to stay at a constant elevation (i.e. no vertical displacement from normality), for small angular structure oscillations. However as represented in Figure 3, the AMD-1 top floor is modelled as a linear spring-mass system. Therefore, the AMD-1's total potential energy is only due to its elastic potential energy. It results that the total potential energy of the AMD-1 plant can be fully expressed as:

$$V_T = \frac{1}{2} K_f x_f(t)^2 \quad [\text{B.1}]$$

It can be seen from Equation [B.1] that the total potential energy can be expressed in terms of the system's generalized coordinates alone.

Let us now determine the system's total kinetic energy T_T . The kinetic energy measures the amount of energy in a system due to its motion. Here, the total kinetic energy is the sum of the translational and rotational kinetic energies arising from the motorized linear cart (since the cart's direction of translation is orthogonal to that of the rotor's rotation) and the translational kinetic energy of the flexible structure's floor. In other words, the total kinetic energy of the AMD-1 system can be formulated as below:

$$T_T = Tt_c + Tr_c + Tt_f \quad [\text{B.2}]$$

First, the translational kinetic energy of the motorized cart can be expressed as a function of its centre of gravity's linear velocity, as shown by the following equation:

$$Tt_c = \frac{1}{2} M_c \left(\left(\frac{d}{dt} x_c(t) \right) + \left(\frac{d}{dt} x_f(t) \right) \right)^2 \quad [\text{B.3}]$$

Second, the rotational kinetic energy due to the cart's DC motor can be characterized by:

$$Tr_c = \frac{1}{2} \frac{J_m K_g^2 \left(\frac{d}{dt} x_c(t) \right)^2}{r_{mp}^2} \quad [\text{B.4}]$$

Third and last, the structure floor's translational kinetic energy can be characterized as follows:

$$Tt_f = \frac{1}{2} M_f \left(\frac{d}{dt} x_f(t) \right)^2 \quad [\text{B.5}]$$

Thus by replacing Equations [B.3], [B.4], and [B.5] into Equation [B.2], the system's total kinetic energy results to be such as:

$$T_T = \left(\frac{1}{2} M_c + \frac{1}{2} \frac{J_m K_g^2}{r_{mp}^2} \right) \left(\frac{d}{dt} x_c(t) \right)^2 + M_c \left(\frac{d}{dt} x_f(t) \right) \left(\frac{d}{dt} x_c(t) \right) + \left(\frac{1}{2} M_c + \frac{1}{2} M_f \right) \left(\frac{d}{dt} x_f(t) \right)^2 \quad [\text{B.6}]$$

It can be seen from Equation [B.6] that the total kinetic energy can be expressed in terms of the generalized coordinates' first-time derivatives.

Let us now consider the Lagrange's equations for our system. By definition, the two Lagrange's equations, resulting from the previously-defined two generalized coordinates, x_c and x_f , have the following formal formulations:

$$\left(\frac{\partial}{\partial t} \frac{d}{dt} L \right) - \left(\frac{\partial}{\partial x_c} L \right) = Q_{x_c} \quad [\text{B.7}]$$

and:

$$\left(\frac{\partial}{\partial t} \frac{d}{dt} L \right) - \left(\frac{\partial}{\partial x_f} L \right) = Q_{x_f} \quad [\text{B.8}]$$

In Equations [B.7] and [B.8], above, L is called the Lagrangian and is defined to be equal to:

$$L = T_T - V_T \quad [\text{B.9}]$$

For our system, the generalized forces can be defined as follows:

$$Q_{x_c}(t) = F_c(t) - B_{eq} \left(\frac{d}{dt} x_c(t) \right) \quad \text{and} \quad Q_{x_f}(t) = 0 \quad [\text{B.10}]$$

It should be noted that the (nonlinear) Coulomb friction applied to the linear cart has been neglected. Furthermore, the viscous damping force applied to the structure floor has also been neglected.

Calculating Equation [B.7] results in a more explicit expression for the first Lagrange's equation, such that:

$$\frac{(M_c r_{mp}^2 + J_m K_g^2) \left(\frac{d^2}{dt^2} x_c(t) \right)}{r_{mp}^2} + M_c \left(\frac{d^2}{dt^2} x_f(t) \right) = F_c - B_{eq} \left(\frac{d}{dt} x_c(t) \right) \quad [\text{B.11}]$$

Likewise, calculating Equation [B.8] also results in a more explicit form for the second Lagrange's equation, as shown below:

$$M_c \left(\frac{d^2}{dt^2} x_c(t) \right) + (M_c + M_f) \left(\frac{d^2}{dt^2} x_f(t) \right) + K_f x_f(t) = 0 \quad [\text{B.12}]$$

Finally, solving the set of the two Lagrange's equations, as previously expressed in Equations [B.11] and [B.12], for the second-order time derivative of the two Lagrangian coordinates results in the following two equations:

$$\frac{d^2}{dt^2} x_c(t) = \frac{r_{mp}^2 M_c x_f(t) K_f}{M_c r_{mp}^2 M_f + J_m K_g^2 M_c + J_m K_g^2 M_f} + \frac{r_{mp}^2 (-M_c B_{eq} - M_f B_{eq}) \left(\frac{d}{dt} x_c(t) \right)}{M_c r_{mp}^2 M_f + J_m K_g^2 M_c + J_m K_g^2 M_f} \quad [\text{B.13}]$$

$$+ \frac{r_{mp}^2 (M_c F_c + M_f F_c)}{M_c r_{mp}^2 M_f + J_m K_g^2 M_c + J_m K_g^2 M_f}$$

and:

$$\frac{d^2}{dt^2} x_f(t) = - \frac{(M_c r_{mp}^2 + J_m K_g^2) x_f(t) K_f}{M_c r_{mp}^2 M_f + J_m K_g^2 M_c + J_m K_g^2 M_f} + \frac{M_c B_{eq} r_{mp}^2 \left(\frac{d}{dt} x_c(t) \right)}{M_c r_{mp}^2 M_f + J_m K_g^2 M_c + J_m K_g^2 M_f} \quad [\text{B.14}]$$

$$- \frac{M_c F_c r_{mp}^2}{M_c r_{mp}^2 M_f + J_m K_g^2 M_c + J_m K_g^2 M_f}$$

Equations [B.13] and [B.14] represent the Equations Of Motion (EOM) of the system. It can be noticed, in the case of the AMD-1 system, that the EOM are linear.

As a remark, if both B_{eq} and J_m are neglected, Equations [B.13] and [B.14] become:

$$\frac{d^2}{dt^2} x_c(t) = \frac{K_f x_f(t)}{M_f} + \frac{(M_c + M_f) F_c}{M_c M_f} \quad [\text{B.15}]$$

and:

$$\frac{d^2}{dt^2} x_f(t) = - \frac{K_f x_f(t)}{M_f} - \frac{F_c}{M_f} \quad [\text{B.16}]$$