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STUDENT WORKBOOK

Gyro/Stable Platform Experiment for MATLAB®/Simulink® Users

Standardized for ABET* Evaluation Criteria

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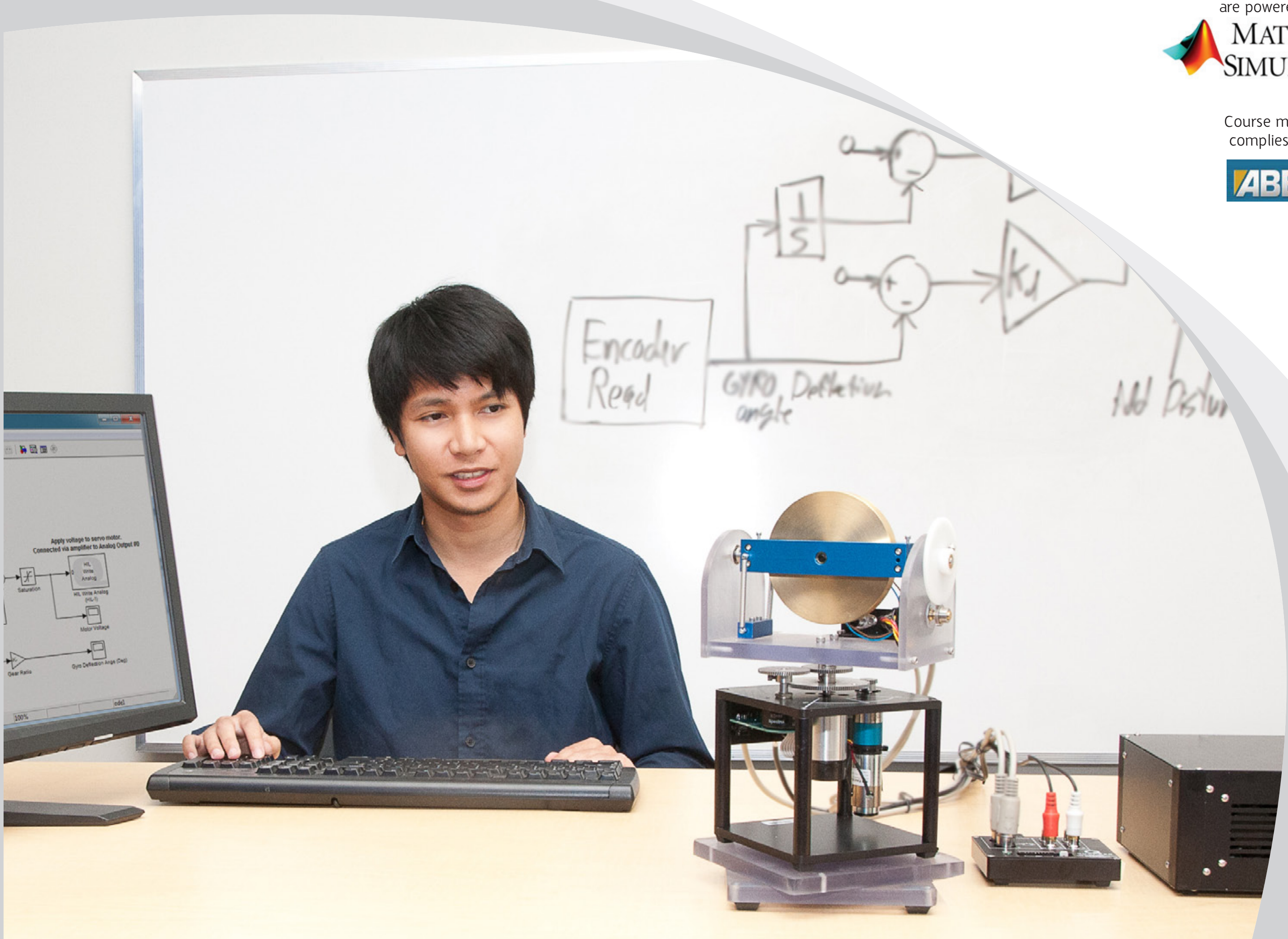

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CONTENTS

1 INTRODUCTION

The objective of this experiment is to design a controller that maintains the direction of the gyroscope module while the top base plate is rotated relative to the bottom base plate. While the disk spins, the SRV02 is used to apply the correct amount of counter torque and maintain the gyroscope heading in the event of disturbances (i.e., rotation of the bottom support plate).

Gyroscopes are used in many different devices, e.g., airplanes, large marine ships, submarines, and satellites.

Topics Covered

- Modeling the system from first principles.
- Design a PID-based controller.
- Implement the designed controller on the device. Test if the gyroscope module maintains its headings when a disturbance is added.

Prerequisites

In order to successfully carry out this laboratory, the user should be familiar with the following:

- Transfer function fundamentals.
- Basics of Simulink®.
- QUARC Integration lab detailed in Appendix A in the SRV02 Workbook [5].

2 BACKGROUND

2.1 Modeling

2.1.1 Servo Model

The Servo Base Unit (SRV02) open-loop transfer function is given by

$$P(s) = \frac{\Theta_l(s)}{V_m(s)} = \frac{K}{s(\tau s + 1)} \tag{2.1}$$

where $\Theta_l(s) = \mathcal{L}[\theta_l(t)]$ is the load gear position and $V_m(s) = \mathcal{L}[v_m(t)]$ is the applied motor voltage. The system steady-state gain and time constant are given by:

$$K = 1.53 \text{ rad/s/V},$$

and

$$\tau = 0.0486 \text{ s}.$$

Note: The model parameters, K and τ, were computed for the SRV02 with the GYRO-E module mounted. If desired, you can conduct an experiment to find more precise values of K and τ for your particular servo. See *SRV02 Modeling* laboratory in [5] for more information.

2.1.2 Gyroscope Gain

In order to derive a model of the system, an understanding of gyroscopic principles is required. For a detailed derivation of the dynamic equations, see the textbook references [1], [8], [9] given in the *References* section.

Consider the simplified model shown in Figure 2.1. The inertial disc, or flywheel, spins at a relatively constant velocity, ω_f. When the base rotates at a speed of ω_b, the resulting gyroscopic torque about the sensitive axis is

$$\tau_g = \omega_b L_f \tag{2.2}$$

where

$$L_f = J_f \omega_f$$

is is the angular momentum of the flywheel and J_f is its moment of inertia. The springs mounted on the gyroscope counteract the gyroscopic torque, τ_g, by the following amount

$$\tau_s = K_r \alpha \tag{2.3}$$

where K_r is the *rotational* stiffness of the springs.

Given that the spring torque equals the gyroscopic torque, $\tau_s = \tau_g$, we can equate equations 2.2 and 2.3 to obtain the expression

$$K_r \alpha = \omega_b J_f \omega_f. \tag{2.4}$$

The base speed is proportional to the deflection angle through the gain G_g,

$$\omega_b = G_g \alpha. \tag{2.5}$$

By examining 2.4 and 2.5, we find that the *gyroscopic sensitivity gain* is given by

$$G_g = \frac{\omega_b}{\alpha} = \frac{K_r}{J_f \omega_f}. \tag{2.6}$$

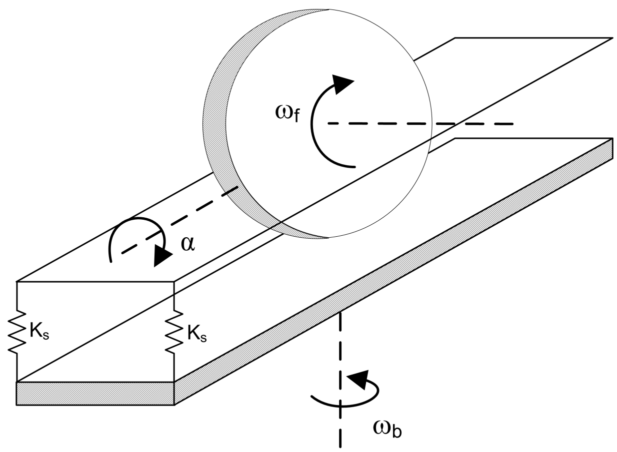


Figure 2.1: Simplified rotary gyroscope model.

Thus the deflection at the gyroscope sensitive axis is directly proportional to the speed of rotational speed of the base (in the steady state). This means that the deflection angle, α, can be used to measure the rotation of the platform relative to the base *without* a direct measurement. **Note:** the dynamics in the sensitive axis are ignored and a more complete model would include these dynamics as $\alpha(s)/\omega_b(s)$.

2.1.3 Joint Stiffness

The two springs are attached as shown in Figure 2.2. The stiffness at the axis of rotation is derived in the following fashion. Assume the springs have a spring constant K_s and an un-stretched length L_u. The length of the springs at the normal position, i.e., $\alpha = 0$, is given by L. If the axis is rotated by an angle α, then the two forces about the sensitive axis are given by (for small α)

$$F_1 = K_s \Delta L_1 = K_s(L - L_u - \alpha R)$$

and

$$F_2 = K_s \Delta L_2 = (L - L_u + \alpha R).$$

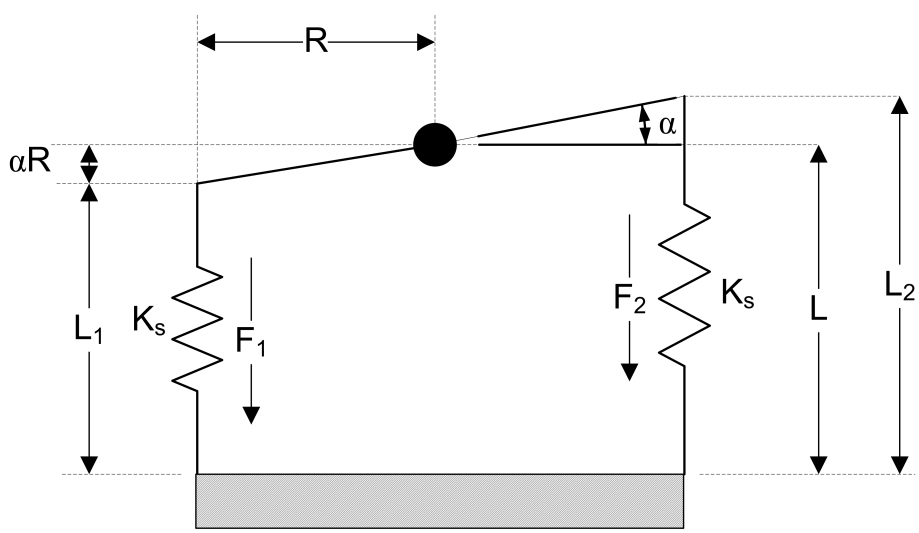


Figure 2.2: Forces acting on springs.

The spring torque about the pivot due to the two forces is

$$\tau_s = R(F_2 - F_1) = 2R^2 K_s \alpha.$$

The rotational stiffness is given by

$$K_r = \frac{\tau_s}{\alpha} = 2R^2 K_s. \tag{2.7}$$

2.2 Control Design

2.2.1 Desired Position Control Response

The block diagram shown in Figure 2.3 is a general unity feedback system with compensator (controller) $C(s)$ and a transfer function representing the plant, $P(s)$. The measured output, $Y(s)$, is supposed to track the reference signal $R(s)$ and the tracking has to match to certain desired specifications.

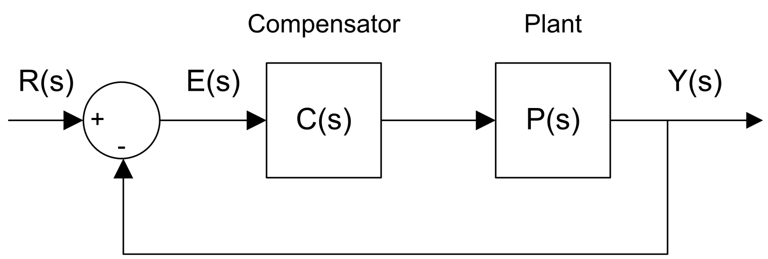


Figure 2.3: Unity feedback system.

The output of this system can be written as:

$$Y(s) = C(s)\,P(s)\,\left(R(s) - Y(s)\right)$$

By solving for $Y(s)$, we can find the closed-loop transfer function:

$$\frac{Y(s)}{R(s)} = \frac{C(s)\,P(s)}{1 + C(s)\,P(s)}$$

When a second order system is placed in series with a proportional compensator in the feedback loop as in Figure 2.3, the resulting closed-loop transfer function can be expressed as:

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\,\omega_n\,s + \omega_n^2} \tag{2.8}$$

where ω_n is the natural frequency and ζ is the damping ratio. This is called the *standard second-order* transfer function. Its response properties depends on the values of ω_n and ζ.

2.2.2 Control Specifications

The desired time-domain specifications for stabilizing the gyroscope are:

$$\omega_n = 6\pi \text{ rad/s} \tag{2.9}$$

or 3 Hz, and

$$\zeta = 0.7. \tag{2.10}$$

2.2.3 GYRO PD Controller

To stabilize the heading of the gyroscope, we will develop a Proportional-Derivative (PD) controller depicted in Figure 2.4.

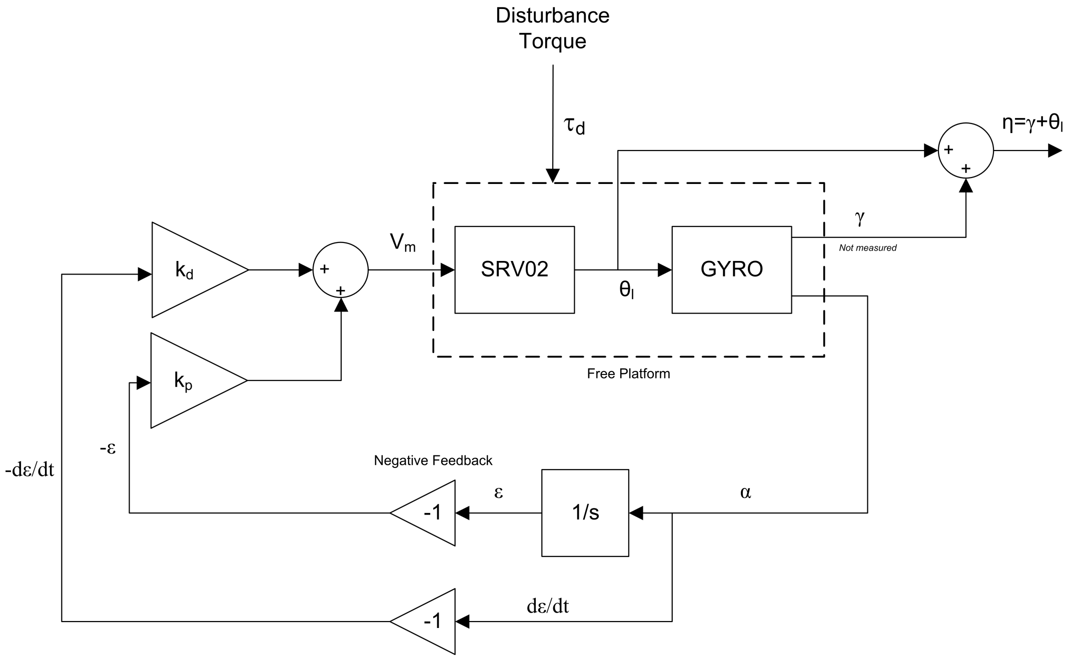


Figure 2.4: Gyroscope PD control block diagram

Assume that the support plate (and servo) rotate relative to the base plate by the angle γ (not measured) and that the gyro module rotates relative to the servo module by the angle θ_l (measured), the total rotation of the gyro module relative to the base plate can be expressed by

$$\eta = \gamma + \theta_l. \tag{2.11}$$

We want to design a controller that maintains the gyro heading, i.e., keeps $\eta = 0$, independent of γ and we can only use the measurement from the gyro sensor, α. In other terms, we want to stabilize the system such that $\dot{\eta} \to 0$. Differentiating Equation 2.11 gives

$$\dot{\eta} = \dot{\gamma} + \dot{\theta}_l.$$

Given that $\dot{\eta} = \omega_b$ and the gyro gain definition in Equation 2.5, this becomes

$$G_g \alpha = \dot{\gamma} + \dot{\theta}_l.$$

Taking the Laplace and solving for $\alpha(s)/s$ we have

$$\frac{\alpha(s)}{s} = \frac{1}{G_g}(\gamma(s) + \Theta_l(s)).$$

Introducing the new variable

$$\epsilon(s) = \frac{\alpha(s)}{s},$$

which is the integral of the deflection angle, the gyro transfer function can be changed to the following

$$\epsilon(s) = \frac{1}{G_g}(\gamma(s) + \Theta_l(s)).$$

Add the SRV02 dynamics given in Section 2.1.1 into $\Theta_l(s)$ to introduce our control variable $V_m(s)$

$$\epsilon(s) = \frac{1}{G_g}\left(\gamma(s) + \frac{K}{s(\tau s + 1)} V_m(s)\right). \tag{2.12}$$

Adding the PD control

$$V_m(s) = -(k_p + k_d\, s)\epsilon(s)$$

to 2.12 and solving for $\epsilon(s)/\gamma(s)$ we obtain the closed-loop transfer function

$$\frac{\epsilon(s)}{\gamma(s)} = \frac{s(\tau s + 1)}{G_g \tau s^2 + (K k_d + G_g)s + K k_p}. \quad (2.13)$$

3 PRE-LAB QUESTIONS

1. Find the steady-state speed of the flywheel, ω_f, given the motor equation

$$v_{g,m} = i_{g,m} R_{g,m} + k_{g,m} \omega_f \tag{3.1}$$

   where $i_{g,m}$ is the nominal current, $v_{g,m}$ is the nominal voltage, $R_{g,m}$ is the motor resistance, and $k_{g,m}$ is the back-emf constant. The motor parameter values are given in the Gyroscope User Manual [7].

2. Find the value of the gyroscope sensitivy gain, G_g. The flywheel moment of inertia is

$$J_f = \frac{1}{2} m_f r_f^2 = 0.00103 \text{ N-m-s}^2\text{/rad}.$$

   Note that the inertia unit N-m-s^2/rad is equivalent to kg-m^2. Refer to the Gyroscope User Manual for parameter values.

3. The closed-loop transfer function was found in 2.13. Find the PD control gains, k_p and k_d, in terms of ω_n and ζ. **Hint:** Remember the standard second order system equation.

4. Based on the nominal SRV02 model parameters, K and τ given in Section 2.1.1, calculate the control gains needed to satisfy the time-domain response requirements given in Section 2.2.2.

4 LAB EXPERIMENTS

4.1 Control Implementation

In this secction, the gyroscopic control developed in Section 2.2 is implemented on the actual system. The goal is to see if the gyro module can maintain its heading when a disturbance is added by the user, i.e., the base plate is rotated.

The *q_gyro* Simulink diagram shown in Figure 4.1 is used to run the PD control on the Quanser Rotary Gyroscope system. The *SRV02 Gyroscope* subsystem contains QUARC blocks that interface with the DC motor and sensors of the system. The PI controller developed in Section 2.2 is implemented using a Simulink *Gain* and *Integrator* blocks.

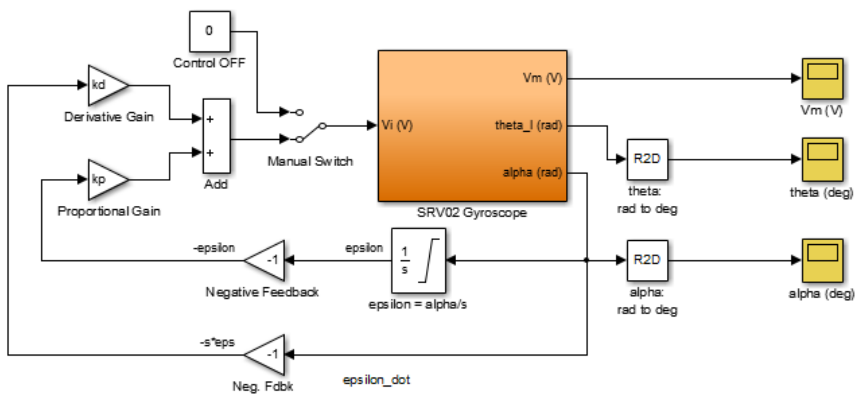


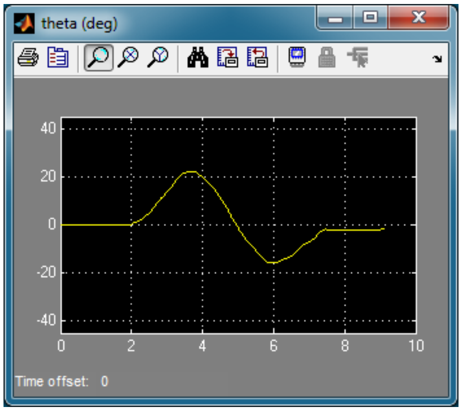
Figure 4.1: q_gyro Simulink diagram used the model

Experiment Setup

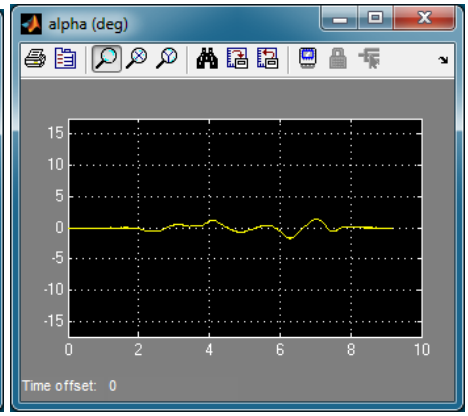
IMPORTANT: Before you can conduct this experiment, you need to make sure that the lab files are configured according to your system setup. If they have not been configured already, then go to Section 5 to configure the lab files first.

Follow these steps to run gyroscope control:

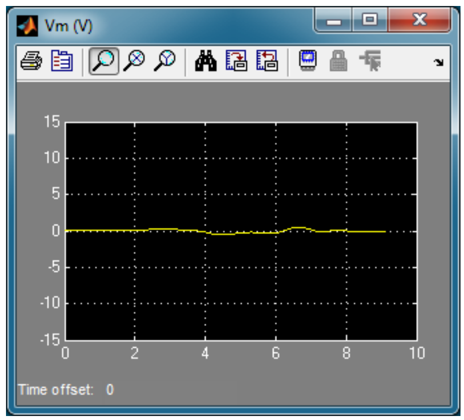
1. The amplifier should be turned ON and the disc should be rotating, as discussed in Section 5.
2. Run the *setup_gyro.m* script.
3. Open the *q_gyro* Simulink diagram.
4. Make sure the Manual Switch is in **downward** position to enable the PD control.
5. Go to QUARC | Build to build the controller.
6. Go to QUARC | Start to run the controller.
7. Manually rotate the bottom base plate about 45 degrees (or any other set angle). The GYRO module should be maintaining its heading. Example scope responses are given in Figure 4.2.
8. Stop the controller once you have obtained a representative response.
9. Plot the responses from the *theta (deg)*, *alpha (deg)*, and *Vm (V)* scopes in a Matlab figure. The response data is saved in variables *data_theta*, *data_alpha*, and *data_vm*.
10. Return the base plate to its original location (i.e., before you rotated it).



(a) SRV02 Angle



(b) GYRO Deflection Angle



(c) SRV02 Voltage

Figure 4.2: Typical Rotary Gyroscope response when PD control is ON

11. Start the QUARC controller again.
12. Turn OFF the PD control by setting the Manual Switch in the **upward** position, i.e., 0 V is applied to the motor.
13. Rotate the bottom base plate by the same amount as previously done, e.g., 45 degrees counter-clockwise. Plot the response.
14. Examine how the GYRO module responds when you rotate the base plate. Explain the resulting responses when the PD control is ON and OFF. Based on your observations, explain what the PD control is actually doing and how it relates to gyroscopes.

5 SYSTEM REQUIREMENTS

Required Software

- Microsoft Visual Studio (MS VS)
- Matlab® with Simulink®, Real-Time Workshop, and the Control System Toolbox
- QUARC®

See the QUARC®software compatibility chart in [4] to see what versions of MS VS and Matlab are compatible with your version of QUARC and for what OS.

Required Hardware

- Data acquisition (DAQ) device that is compatible with QUARC®. This includes Quanser DAQ boards such as Q2-USB, Q8-USB, QPID, and QPIDe and some National Instruments DAQ devices. For a full listing of compliant DAQ cards, see Reference [2].
- Quanser SRV02-ET rotary servo.
- Quanser Rotary Gyroscope (attached to SRV02).
- Quanser VoltPAQ-X1 power amplifier, or equivalent.

Before Starting Lab

Before you begin this laboratory make sure:

- QUARC® is installed on your PC, as described in [3].
- DAQ device has been successfully tested (e.g., using the test software in the Quick Start Guide or the *QUARC Analog Loopback Demo*).
- Rotary Gyroscope and amplifier are connected to your DAQ board as described Reference [6].

5.1 Overview of Files

File Name	Description
Gyroscope User Manual.pdf	This manual describes the hardware of the GYRO-E system and explains how to setup and wire the system for the experiments.
Gyroscope Workbook (Student).pdf	This laboratory guide contains pre-lab questions and lab experiments demonstrating how to design and implement controllers for both the joint space and work space on the GYRO-E plant using QUARC®.
setup_gyro.m	The main Matlab script that sets the SRV02 motor and sensor parameters, the SRV02 configuration-dependent model parameters, and the GYRO-E sensor parameters. **Run this file only to setup the laboratory.**
config_srv02.m	Returns the configuration-based SRV02 model specifications Rm, kt, km, Kg, eta_g, Beq, Jeq, and eta_m, the sensor calibration constants K_POT, K_ENC, and K_TACH, and the amplifier limits VMAX_AMP and IMAX_AMP.
calc_conversion_constants.m	Returns various conversions factors.
d_model_param.m	Calculates the SRV02 model parameters *K* and *tau* based on the device specifications *Rm, kt, km, Kg, eta_g, Beq, Jeq,* and *eta_m.*
q_gyro.mdl	Simulink file that implements the PD controller on the GYRO-E system using QUARC®.

Table 5.1: Files supplied with the Rotary Gyroscope

5.2 Experiment Setup

Before beginning the in-lab procedure outlined in Section 4, the q_gyro Simulink diagram and the setup_gyro.m script must be configured.

Follow these steps:

1. Setup the Rotary Servo Base Unit, i.e., SRV02, with the Gyroscope module as detailed in the Gyroscope User Manual ([7]).
2. Load the Matlab® software.
3. Browse through the *Current Directory* window in Matlab and find the folder that contains the file *setup_gyro.m*.
4. Open the *setup_gyro.m* script.
5. **Configure setup_gyro.m script**: When used with the GYRO-E, the SRV02 has the gyroscope module load and has to be in the high-gear configuration. Make sure the script is setup to match this setup:
   - EXT_GEAR_CONFIG to 'HIGH'
   - LOAD_TYPE to 'GYRO'
   - K_AMP to 1 (unless your amplifier gain is different)
   - AMP_TYPE to your amplifier type (e.g., VoltPAQ).
   - Ensure other parameters such as ENCODER_TYPE, TACH_OPTION, and VMAX_DAC match your system configuration.
   - CONTROL_TYPE to 'STUDENT'.

6. Run setup_gyro.m to setup the Matlab workspace.
7. Enter the PD controller gains, k_p and k_d, you found in Section 3 as *kp* and *kd* in Matlab.
8. Enter the gyro gain you calcultated in Section 3 as *Gg* in Matlab.
9. Open the *q_gyro.mdl* Simulink diagram, shown in Figure 4.1.
10. **Configure DAQ:** Ensure the HIL Initialize block in the *SRV02 Gyroscope* subsystem is configured for the DAQ device that is installed in your system. See Reference [2] for more information on configuring the HIL Initialize block.
11. Turn ON the amplifier (e.g., VoltPAQ-X1). The flywheel on the GYRO-E module should begin spinning. Wait till it reaches its steady-state speed.

6 LAB REPORT

For the gyroscope experiment, follow the outline corresponding to that experiment to build the *content* of your report. Also, in Section 6.2 you can find some basic tips for the *format* of your report.

6.1 Template for Content (Gyroscope)

I. PROCEDURE

1. Briefly describe the main goal of the experiment.
2. Briefly describe the experiment procedure in Step 9 in Section 4.1.
3. Briefly describe the experiment procedure in Step 13 in Section 4.1.

II. RESULTS

Do not interpret or analyze the data in this section. Just provide the results.

1. Gyroscope control ON response, Step 9 in Section 4.1.
2. Gyroscope control OFF response, Step 13 in Section 4.1.

III. ANALYSIS
Provide details of your calculations (methods used) for analysis for each of the following:

1. Effect of having the PD control on and off, Step 14 in Section 4.1.

IV. CONCLUSIONS
Interpret your results to arrive at logical conclusions for the following:

1. How does this relate to an actual gyroscope system, Step 14 in Section 4.1.

6.2 Tips for Report Format

PROFESSIONAL APPEARANCE

- Has cover page with all necessary details (title, course, student name(s), etc.)
- Each of the required sections is completed (Procedure, Results, Analysis and Conclusions).
- Typed.
- All grammar/spelling correct.
- Report layout is neat.
- Does not exceed specified maximum page limit, if any.
- Pages are numbered.
- Equations are consecutively numbered.
- Figures are numbered, axes have labels, each figure has a descriptive caption.
- Tables are numbered, they include labels, each table has a descriptive caption.
- Data are presented in a useful format (graphs, numerical, table, charts, diagrams).
- No hand drawn sketches/diagrams.
- References are cited using correct format.

REFERENCES

[1] Robert H. Cannon. *Dynamics of Physical Systems*. McGraw Hill Book Company, 1967.

[2] Quanser Inc. *QUARC User Manual*.

[3] Quanser Inc. *QUARC Installation Guide*, 2009.

[4] Quanser Inc. *QUARC Compatibility Table*, 2010.

[5] Quanser Inc. SRV02 lab manual. 2011.

[6] Quanser Inc. *SRV02 Rotary Flexible Link User Manual*, 2011.

[7] Quanser Inc. *SRV02 Gyroscope User Manual*, 2012.

[8] Carl Machover. *Basics of Gyroscopes*. John F. Rider, 1960.

[9] Paul H. Savet. *Gyroscopes: Theory and Design*. McGraw Hill Book Company, 1961.

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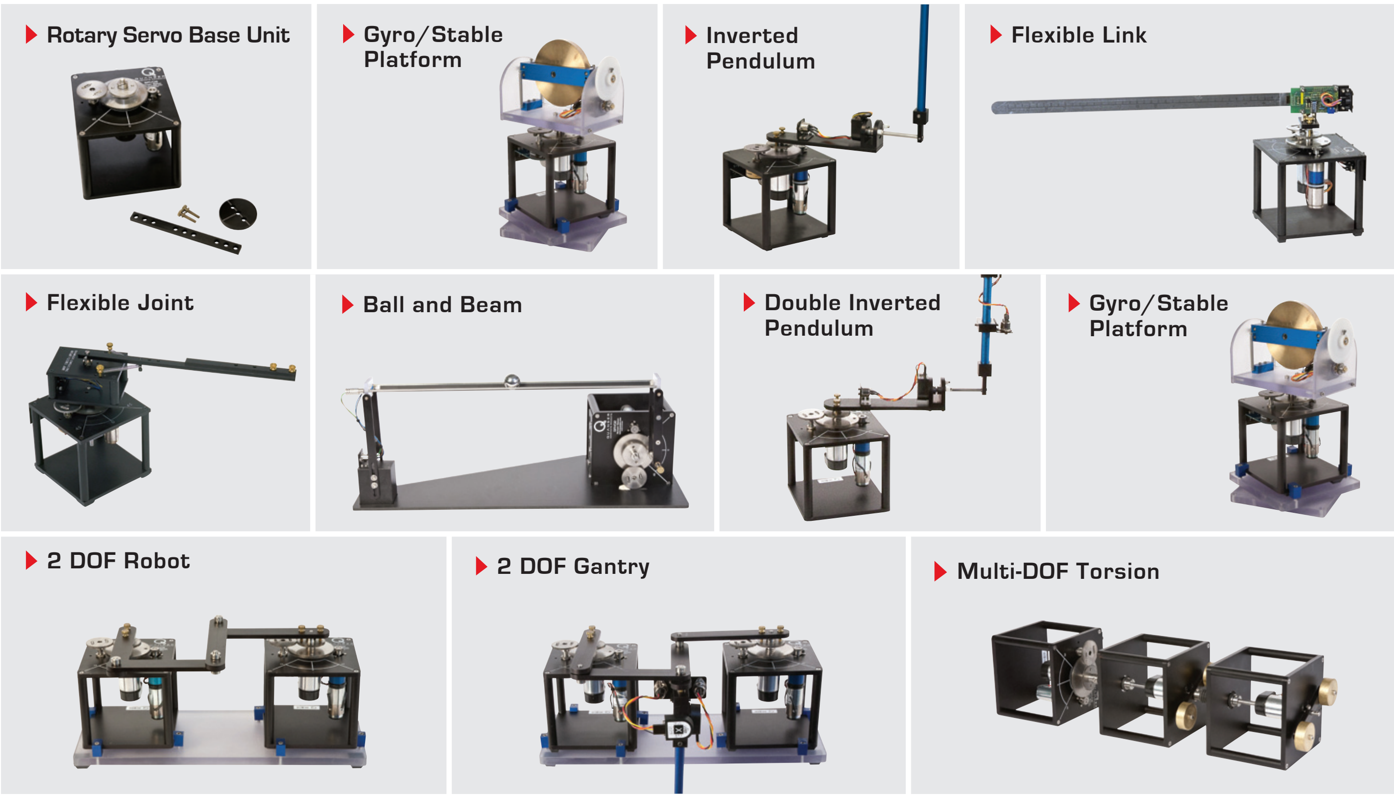

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