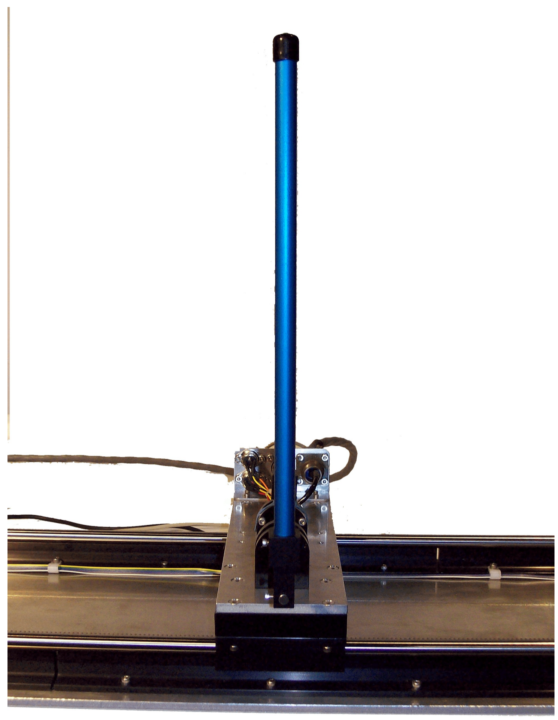
*Linear Motion Servo Plant: HFLC*

# Linear Experiment #3: LQR Control

## *Single Inverted Pendulum (SIP)*



**All of Quanser's systems have an inherent open architecture design. It should be noted that the following experimental setup, accompanying files, and configuration are merely one of the many possible uses of this product.**

# Laboratory Manual

# Table of Contents

# 1. Laboratory Objectives

The challenge in this laboratory is to design a state-feedback control system that balances a Single Inverted Pendulum (SIP) and regulates the position of the HFL cart to a desired setpoint.

# 2. Prerequisites

To successfully carry out this laboratory, the prerequisites are:

i) To be familiar with your HFLC main components (e.g. actuator, sensors), your data acquisition card (e.g. Q8), and your power amplifier (e.g. UPM), as described in References [1], [2], and [3], respectively.

ii) To be familiar in using QuaRC to control and monitor the plant in real-time and in designing their controller through Simulink. Otherwise, review Reference [4] as needed.

iii) To be familiar with the complete wiring and operating procedure of your HFLC servo plant and dedicated UPM, as discussed in Reference [1].

# 3. References

[1] High Fidelity Linear Cart (HFLC) User Manual.
[2] Q4/Q8 User Manual.
[3] Universal Power Module User Manual
[4] QuaRC Help files (type `doc quarc` in Matlab).

# 4. Experiment Design Files

Table 1, below, lists and describes the various computer files coming with the experiment.

| *File Name* | *Description* |
| --- | --- |
| HFLC SIP Control.pdf | The laboratory manual for the *HFLC - Linear Experiment #3: Single Inverted Pendulum (SIP) Control* laboratory. It contains information to setup and configure the laboratory, gives a summary of the system model, outlines the control design and its implementation, as well as the experimental procedure. |

| File Name | Description |
|---|---|
| HFLC SIP Equations.mws | Maple worksheet used to analytically derive the system's nonlinear equations of motion as well as its state-space model. Waterloo Maple 9, or a later release, is required to open, modify, and execute this file. |
| HFLC SIP Equations.html | HTML presentation of the *HFLC SIP Equations.mws* file. It allows to view the content of the Maple file without having Maple 9 installed. No modifications to the equations can be performed when in this format. |
| quanser_tools.mws | Executing this worksheet generates the *quanser* repository containing the *Quanser_Tools* package. The two package files are named: *quanser.ind* and *quanser.lib*. The *Quanser_Tools* module defines the generic procedures used in Lagrangian mechanics and resulting in the determination of a given system's equations of motion and state-space representation. It also contains data processing routines to save the obtained state-space matrices into a Matlab-readable file. |
| quanser_tools.rtf | Rich Text Format presentation of the quanser_tools.mws file. It allows to view the content of the Maple worksheet without having Maple 9 installed. No modifications to the Maple procedures can be performed when in this format. |
| setup_lab_hflc_sip.m | The main Matlab script that calls *setup_hflc_configuration.m*, *setup_sp_configuration.m*, and *HFLC_SIP_ABCD_eqns.m* to set the model, control, and configuration parameters. **Run this file only to setup the laboratory.** |
| setup_hflc_configuration.m | Returns the HFLC model parameters *Jm*, *Kt*, *Eff_m*, *M*, *r_mp*, and *Beq*, the encoder resolution constant *K_EC* and *K_EP*, the UPM amplifier current gain *Ka*, and the voltage and current saturation of the UPM. |
| setup_sp_configuration.m | Returns the Single Pendulum (SP) model parameters *Mp*, *Lp*, *lp*, *Jp*, and *Bp* according to the pendulum specified in *setup_lab_hflc_sip.m*. |

| ***File Name*** | ***Description*** |
|---|---|
| d_hflc_limits.m | Computes the maximum velocity, *v_max*, and the maximum acceleration, *a_max*, of the high-fidelity linear cart when using the UPM-180-25B. These limits are imposed on the desired cart position. |
| HFLC_SIP_ABCD_eqns.m | Matlab script file generated using the Maple worksheet *HFLC SIP Equations.mws*. It sets the *A*, *B*, *C*, and *D* matrices for the state-space representation of the HFLC-plus-SIP open-loop system used in Simulink models *s_hflc_sip.mdl* and *q_hflc_sip.mdl*. |
| d_hflc_sip_lqr.m | Matlab function that designs the controller gain *K* using LQR. It is used in the Simulink models *s_sip_lqr_hflc.mdl* and *q_sip_lqr_hflc.mdl*. |
| s_hflc_sip.mdl | Simulink file that simulates the closed-loop HFLC-plus-SIP system using its nonlinear equation of motion model and an LQR controller. |
| q_hflc_sip.mdl | Simulink file that implements the real-time state-feedback SIP controller for the HFLC system. |

Table 1 Laboratory Design Files

# 5. HFLC+SIP Controller Design

## 5.1. HFLC+SIP Modeling

As already mentioned in Table 1, above, the Maple worksheet named *HFLC SIP Equations.mws* (or its HTML equivalent *HFLC SIP Equations.html*) derives both nonlinear and linear models of the HFLC+SIP system. The free body diagram that accompanies the dynamic model derivation of the system is given in Figure 1.

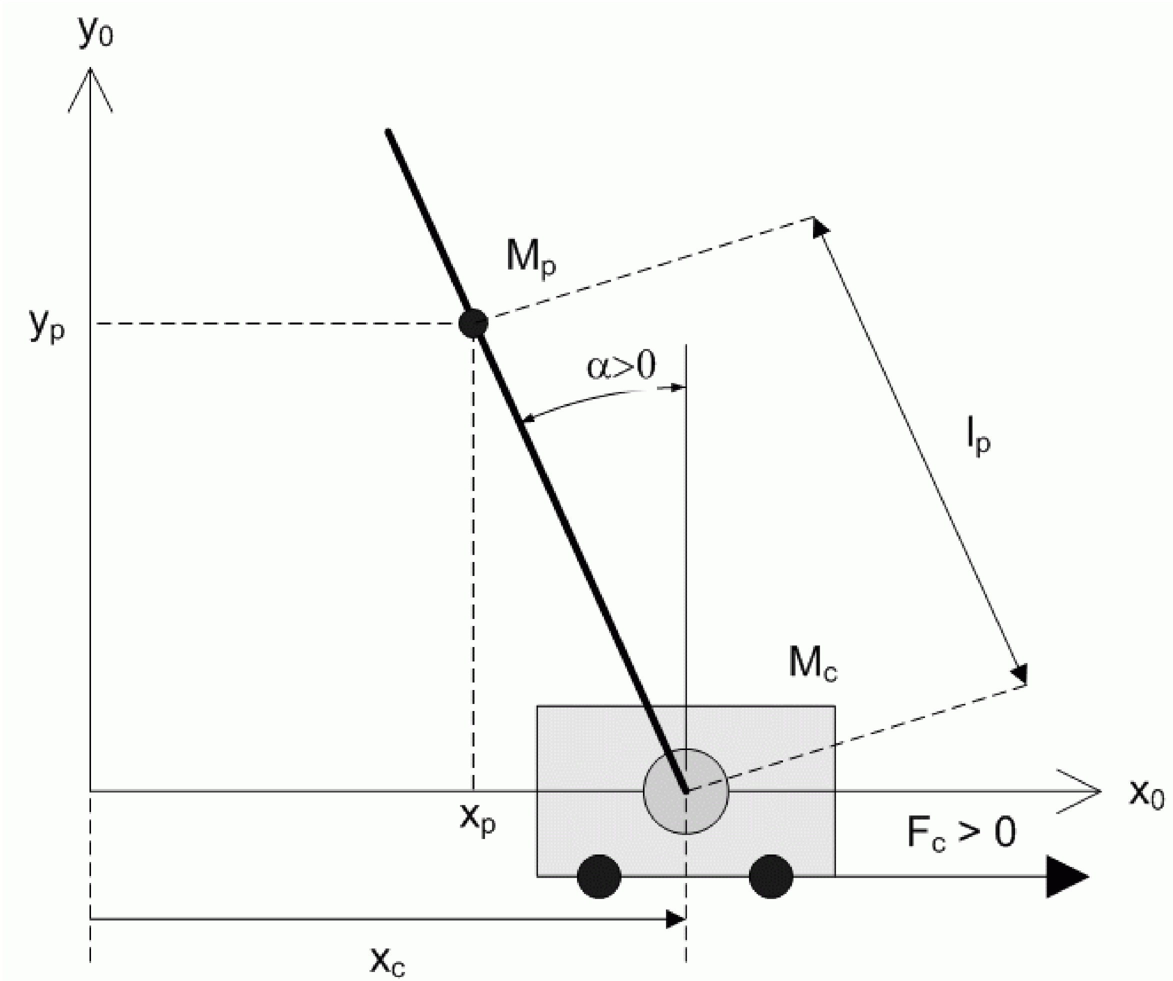



Figure 1 Free body diagram of HFLC+SIP system.

In summary, the linear state-space representation of the open-loop HLFC+SPG system is

$$\frac{\partial}{\partial t} x = A\,x + B\,F_c \qquad [1]$$

with the state

$$x^T = \left[ x_c, \alpha, \frac{\partial}{\partial t} x, \frac{\partial}{\partial t} \alpha \right], \qquad [2]$$

the cart force

$$F_c = \frac{\eta_m K_t I_m}{r_{mp}}, \qquad [3]$$

and the state-space matrices

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{g\,M_p}{M_c} & 0 & 0 \\ 0 & \frac{g\,(M_c + M_p)}{l_p\,M_c} & 0 & 0 \end{bmatrix}, \qquad [4]$$

and

$$B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M_c} \\ \frac{1}{l_p\,M_c} \end{bmatrix}. \qquad [5]$$

Only the cart position and pendulum angle are measured. Thus the output state-space equation is

$$\frac{\partial}{\partial t} y = C\,x + \mathrm{D}\,F_c \qquad [6]$$

where

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \qquad [7]$$

and

$$\mathrm{D} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \qquad [8]$$

The model parameters $r_{mp}$, $M_c$, $M_p$, $K_t$, $l_p$, and $\eta_m$ are defined in Reference [1]. This model does not take into account Coulomb friction (i.e. stiction), pendulum viscous damping $B_p$, cart visous damping $B_{eq}$, rotor moment of inertia $J_m$, and pendulum inertia $J_p$. The system input is the force of the cart and the control input is the motor armature current $I_m$, i.e. the controller enters $I_m$ in [3].

The laboratory can also be performed using pendula of different sizes. Further, additional parameters such as inertial and viscous damping may be included in the modeling.

Reference [1] gives the modeling parameters for a small 7-inch pendulum, the 12-inch medium pendulum above, and a 24-inch long pendulum. It also includes the inertia and viscous damping attributes of the pendula.

## 5.2. State-Feedback Controller Design

The inverted pendulum is an unstable, non-minimum phase system. The cart is to follow a reference position while balancing the inverted pendulum. That is, the cart position, $x_c$, tracks a desired setpoint position, $x_{c,d}$, and the upright pendulum angle, $\alpha$, is stabillized about 0. Assuming no actuator saturation, i.e. $I_m = u$, consider the state-feedback law

$$u = -K\zeta \qquad [9]$$

where

$$\zeta^T = \left[ x_c - x_{c,d}, \alpha, v_x, v_\alpha, \int x_c - x_{c,d}\, dt \right] \qquad [10]$$

is the augmented error state. The cart velocity, $v_x$, is found using the high-pass filter

$$V_x(s) = \mathrm{D}_1(s)\, X_c(s) \qquad [11]$$

and the angular rate of the pendulum, $v_\alpha$, is calculated using

$$V_\alpha(s) = \mathrm{D}_2(s)\, \alpha(s) \qquad [12]$$

where the high-gain observers $D_1(s)$ and $D_2(s)$ are defined in Reference [1]. The closed-loop system implemented is depicted in Figure 2.

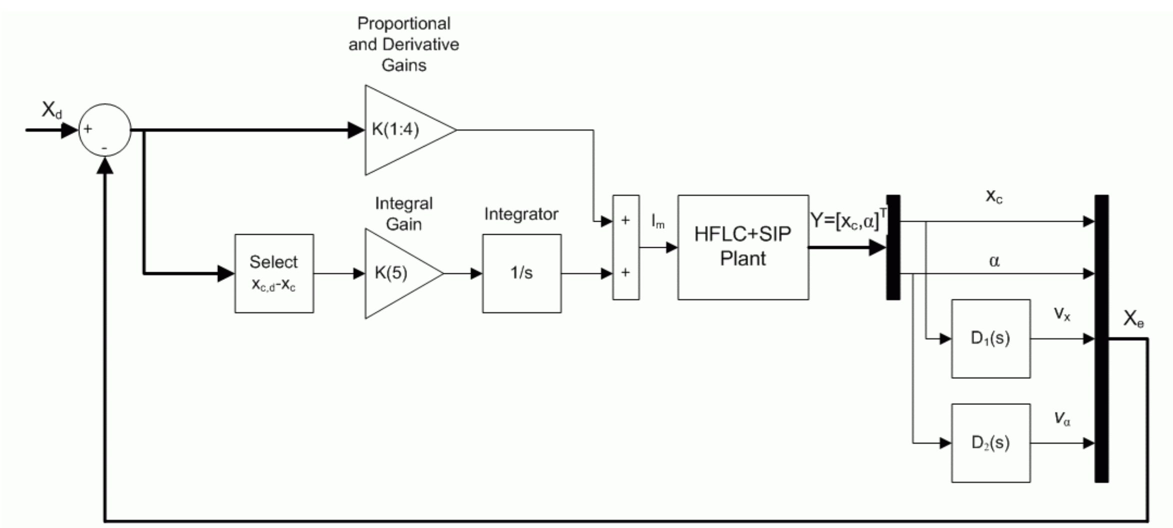



Figure 2 Closed-loop HFLC+SIP system.

The vector gain used in state-feedback [9] is designed using the Linear-Quadratic Regular (LQR) technique. This is an optimization method that minimizes the cost function

$$J = \int_0^\infty \zeta^T Q \zeta + u^T R u \, d\zeta \tag{13}$$

Given the system state-space matrices and using the Matlab *lqr* command with the weighting matrices

$$Q = \begin{bmatrix} 100 & 0 & 0 & 0 & 0 \\ 0 & 20 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.001 \end{bmatrix} \tag{14}$$

and

$$R = 1 \tag{15}$$

generates the control gain

$$K = \left[ -4.54 \left[ \frac{A}{m} \right], 10.63 \left[ \frac{A}{rad} \right], -2.97 \left[ \frac{A\,.\,s}{rad} \right], 1.76 \left[ \frac{A\,.\,s}{m} \right], -0.1 \left[ \frac{A}{m\,s} \right] \right] \tag{16}$$

With the controller defined in [9], the ζ state converges towards zero. Thus the cart position approaches its setpoint, $x_c(t) \rightarrow x_{c,d}(t)$, and the upright pendulum angle is regulated about zero, $\alpha(t) \rightarrow 0°$. The integration term improves the steady-state error of the cart position. Note that control gain *K* has opposing signs for the cart and the pendulum because they have opposite direction conventions (i.e. cart follows the right-hand convention and pendulum is positive when CCW).

# 6. In-Lab Procedure

## 6.1. Experimental Setup Components

To setup this experiment, the following hardware and software are required:

- **Power Module:** Quanser UPM 180-25B, or equivalent.
- **Data Acquisition Board:** Quanser Q4/Q8, or equivalent.
- **Linear Motion Servo Plant:** Quanser High Fidelity Linear Cart (HFLC), as shown in Figure 3 below.

- **Single Pendulum:** Quanser Medium 12-inch, shown in Figure 3 below, or Quanser Long 24-inch pendulum.
- **Real-Time Control Software:** The QuaRC-Simulink configuration, as detailed in Reference [4], or equivalent.

For a complete and detailed description of the main components comprising this setup, please refer to the corresponding manuals.

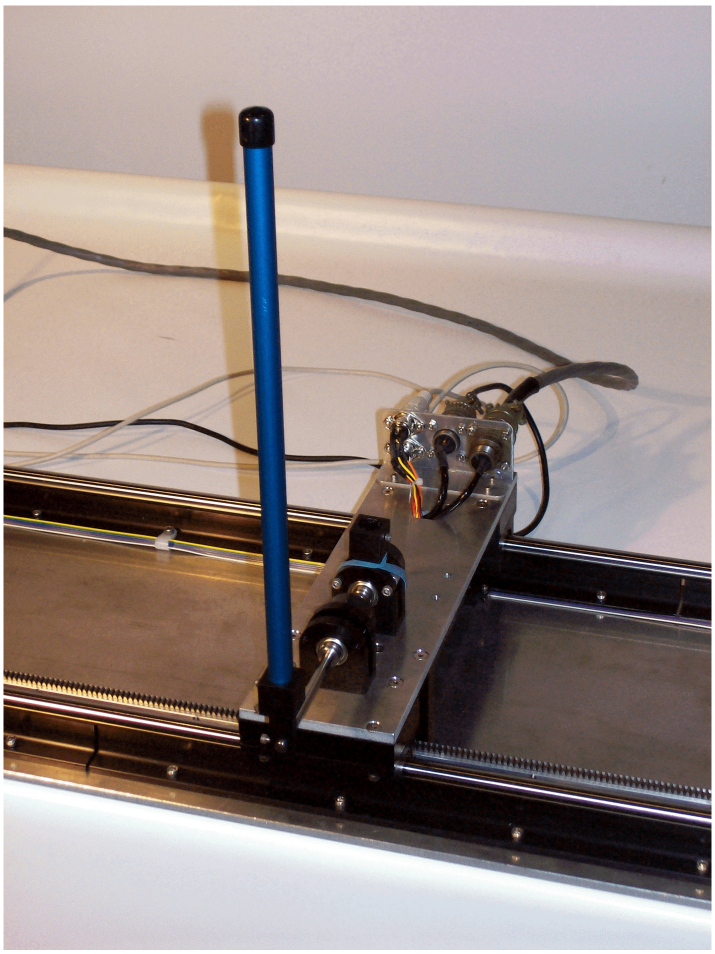


Figure 3 HFLC+SIP system when balance controller is running.

## 6.2. System Wiring and Hardware Setup

### 6.2.1. HFLC System Wiring And Setup

Refer to Reference [1] for the setup information required to carry out this control laboratory. Reference [1] provides the specifications and a description of the main components composing the HFLC system. It also fully describes the wiring conventions and the default wiring procedure for the HFLC servo plant. More importantly, Reference [1] gives some safety operating guidelines and the start-up procedure to properly initialize the UPM-180-25B after power up.

## 6.3. Controller Simulation

The Simulink model entitled *s_hflc_sip.mdl* is shown in Figure 4 and is used to simulate the HFLC+SIP system. The *HFLC+SIP Non-linear EOM* subsystem block includes the nonlinear equations of motion of the system developed in the Maple worksheet *HFLC SIP Equations.mws*. The cart setpoint position can be changed in the *SIP Setpoint* block and designed is implemented in the *HFLC+SIP Control* block.

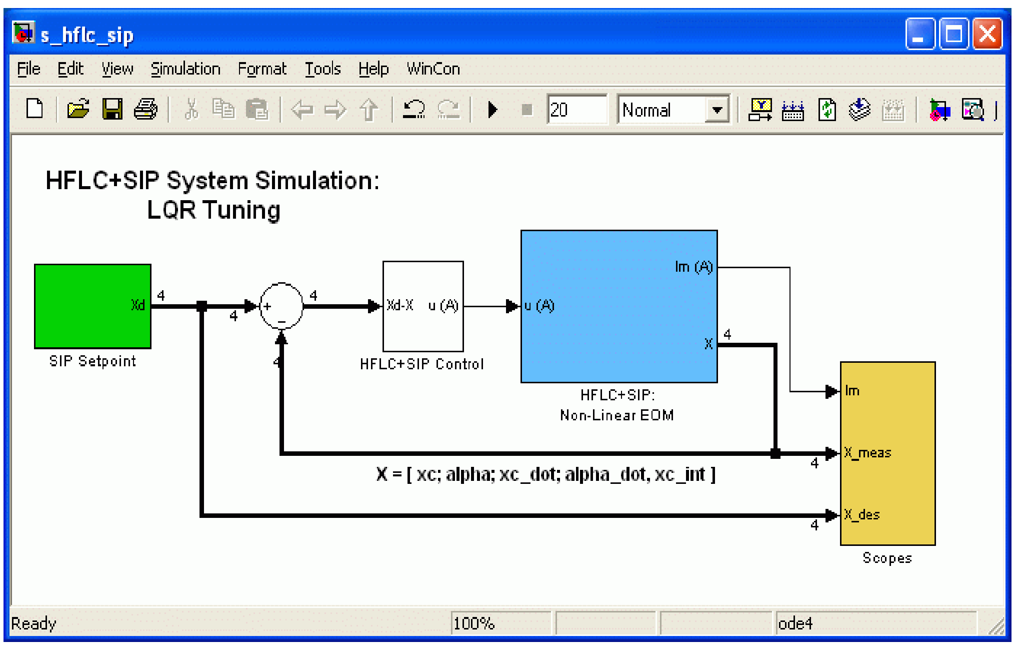


Figure 4 Simulink model used to simulate the HFLC+SIP system.

### 6.3.1. Objectives

- Investigate the closed-loop performance of the controller using the nonlinear model of the HFLC+SIP system.
- Ensure the controller does not saturate the actuator before implementing it on the actual system.

### 6.3.2. Procedure

Follow these steps to simulate the controller:

Step 1. Open Simulink model *s_hflc_sip.mdl* shown in Figure 4 above.
Step 2. Run the Matlab script *setup_lab_hflc_sip.m* to set the model parameters and the control gain *K*.
Step 3. Click on the black arrow located in the tool bar (or click on *Simulation-Start* in the menu) to simulate the closed-loop system. As shown in Figure 5, the purple trace in the *xc (mm)* scope is the simulated cart position and the yellow trace is the desired position. The position of the cart should track the commanded square position signal, $x_c(t) \rightarrow x_{c,d}(t)$, while balancing the pendulum. Therefore the upright pendulum angle, shown in Figure 6, should be stabilized about zero, $\alpha(t) \rightarrow 0$.

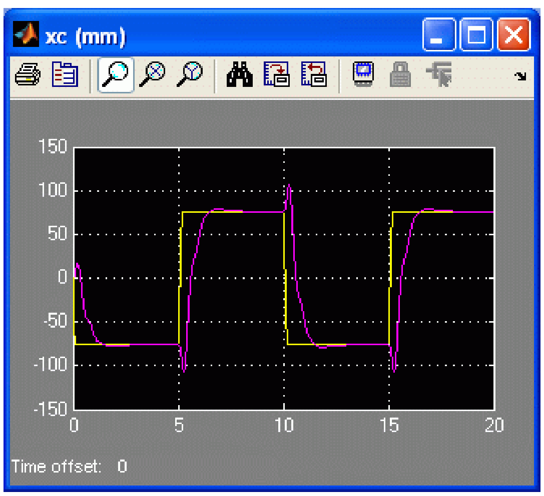



Figure 5 Simulated cart response.

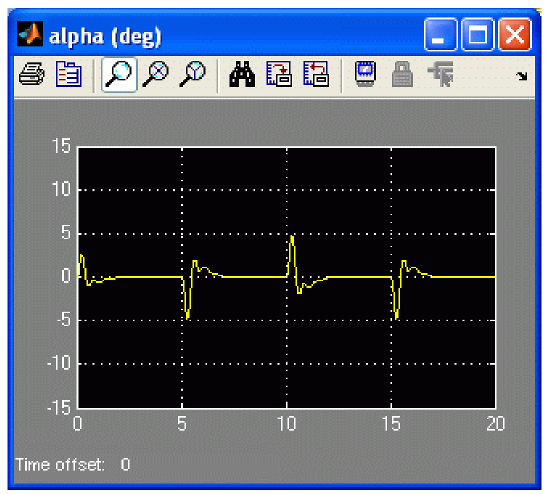



Figure 6 Simulated pendulum response.

Step 4. Verify that the motor input current shown in the *Im (A)* scope is relatively smooth and not saturated, as depicted in Figure 7.

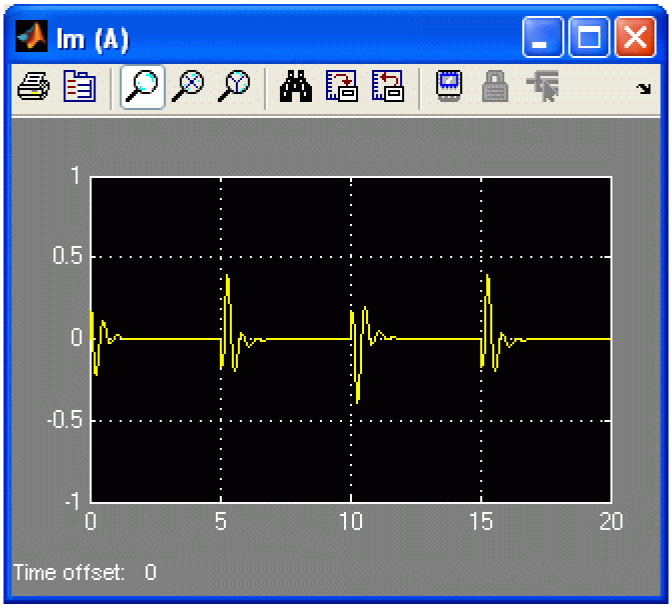


Figure 7 Simulated control input.

## 6.4. Controller Real-Time Implementation

The *q_hflc_sip.mdl* Simulink model shown in Figure 8 implements the previously designed balance and includes the HFLC subsystem discussed in Reference [1] that is used to interface with the actual HFLC+SIP plant.

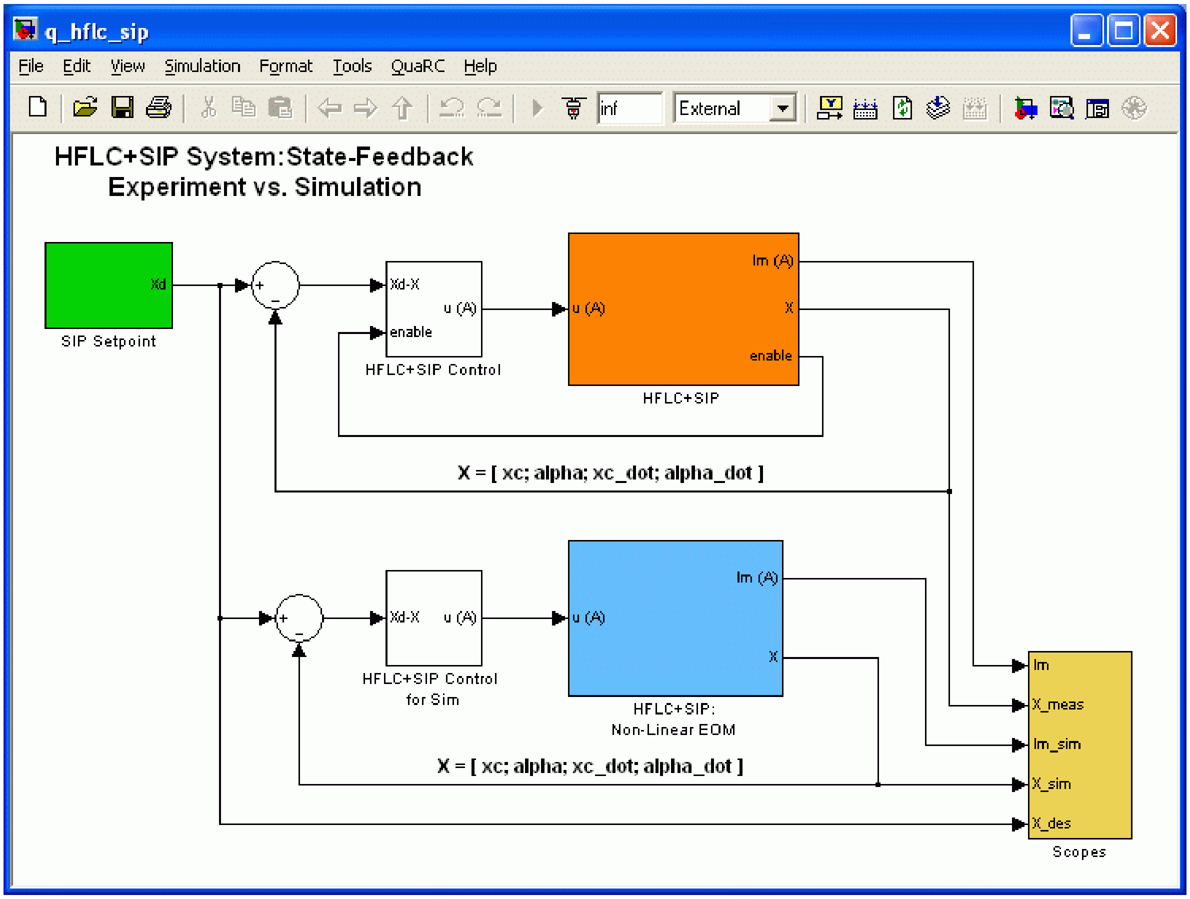



Figure 8 Simulink model that controls the HFLC+SIP system.

Similarly to the cart safety limits found in the *HFLC* subsystem (as described in Reference [1]), a watchdog is placed in the *HFLC+SIP* subystem shown in Figure 9. It stops the real-time controller if the pendulum angle goes beyond the ALPHA_MIN and ALPHA_MAX limits, which are specified in the *setup_lab_hflc_spg.m* file (given that ALPHA_LIM_ENABLE = 1).

As shown below in Figure 10, the pendulum begins in the gantry position. The control design is based on a model that defines $\alpha = 0$ when the pendulum is in the upright position. Since $\alpha = 0$ is set when WinCon is started an offset of $-\pi$ is applied to angle $\alpha$. The controller is activated when the pendulum is rotated in the upright position until it is within the tolerance specified in the box labeled "desired initial alpha (rad): upright position". By default, this tolerance is set to 0 radians.

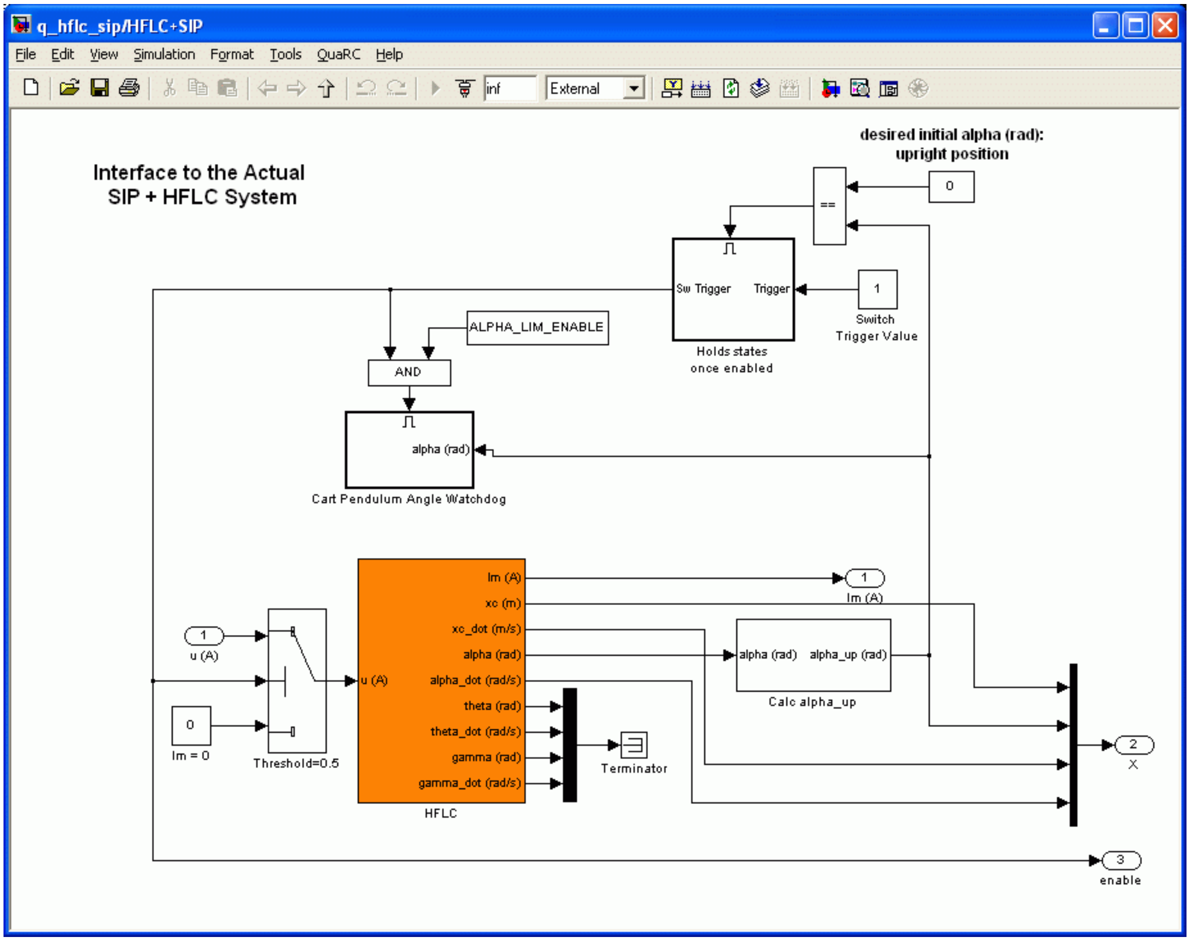


Figure 9 HFLC+SIP subsystem.

## 6.4.1. Objectives

- To implement with WinCon the previously designed controller in order to balance the Single Inverted Pendulum (SIP) on the linear track.
- Compare the actual and simulated responses by running the simulation in parallel.

## 6.4.2. Experimental Procedure

Follow the steps described below to implement the designed LQR controller in real-time and observe its effect on the actual HFLC plant:

Step 1. Open Simulink model *q_hflc_sip.mdl* shown in Figure 8 above.
Step 2. Execute the design file *setup_lab_hflc_sip.m* to setup the workspace before

compiling the diagram and running it in real-time with WinCon. This file sets the state-space model of the HFLC+SIP system. It also calculates the feedback gain vector *K* and enters it in the Matlab workspace. Further, various parameters used in the *HFLC* subsystem such as the filter cutoff frequencies, cart position safety limits and PWM amplifier gain must also be loaded prior to compilation.

Step 3. The real-time code corresponding to the diagram can now be built by selecting the *QuaRC | Build* item from the Simulink menu bar. After successful compilation and download you should be able to run your controller on the actual system in real-time.

Step 4. If not already opened, double-click on the sinks *xc (mm)* and *alpha (deg)* found under the *Scopes* subsystem. The *xc (mm)* scope displays the cart setpoint, the measured cart position, and the simulated cart position generated by the nonlinear EOMs. The *alpha (deg)* scope displays the measured and simulated inverted pendulum angle positions.

Step 5. **Ensure the real-time code is ran safely by manually moving the HFL cart to the middle of the track (i.e. home position) so it is free to move on both sides.** The UPM *Home* green light should go on.

Step 6. Verify that the suspended pendulum is not moving before starting the controller, as shown in Figure 10.

Step 7. Start your real-time controller by clicking on QuaRC | Start from the Simulink model tool bar. When you start the controller, the *OK* and *Enable* LED's on the UPM will illuminate.

Step 8. Manually rotate the pendulum **counter-clockwise** in the upright position until the controller is activated (i.e. when the cart begins moving), as illustrated in Figure 10.

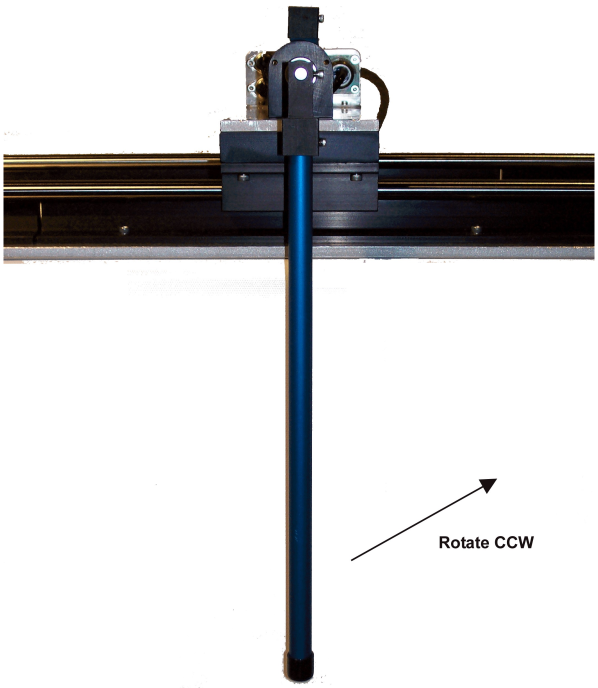



Figure 10 Rotate pendulum counter-clockwise in the upright position to enable controller.

Step 9. Release the pendulum as soon as the controller is activated. Any obstructions are not compensated for and may lead to instability.

Step 10. **Press the E-Stop button if the HFLC system does not behave as expected to cut power to the HFLC DC motor**. The *OK* LED on the UPM will turn off.

Step 11. Figure 11 depicts the measured and simulated responses of the cart position and pendulum angle and Figure 12 is the corresponding control input.

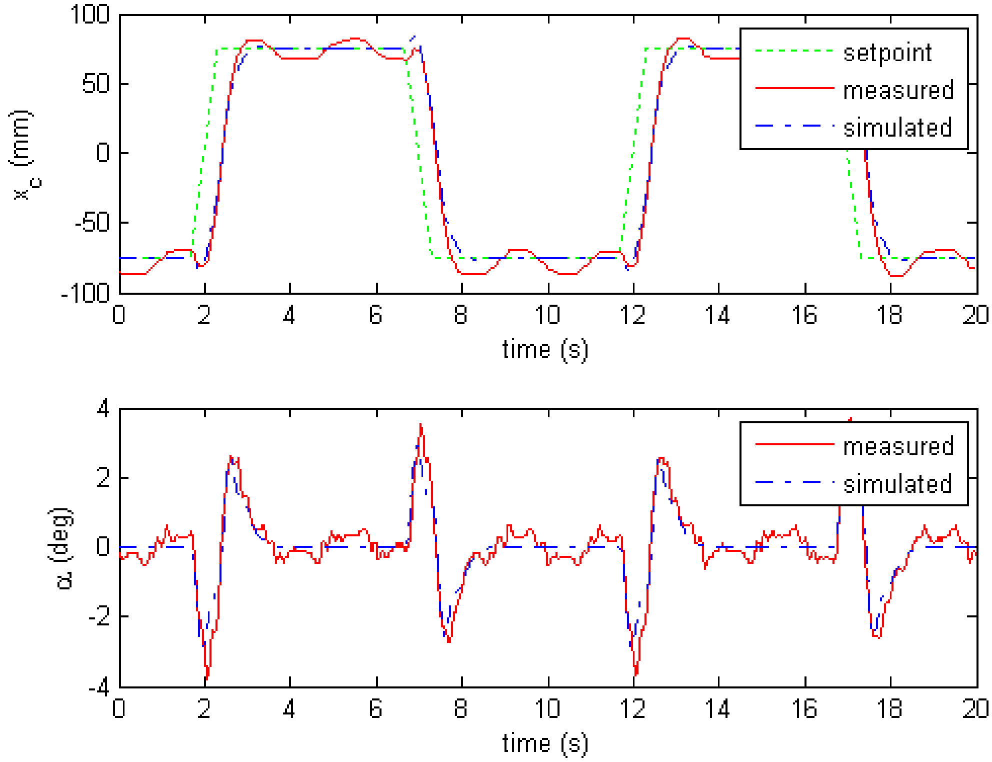



Figure 11 Measured and simulated HFLC+SIP closed-loop response.

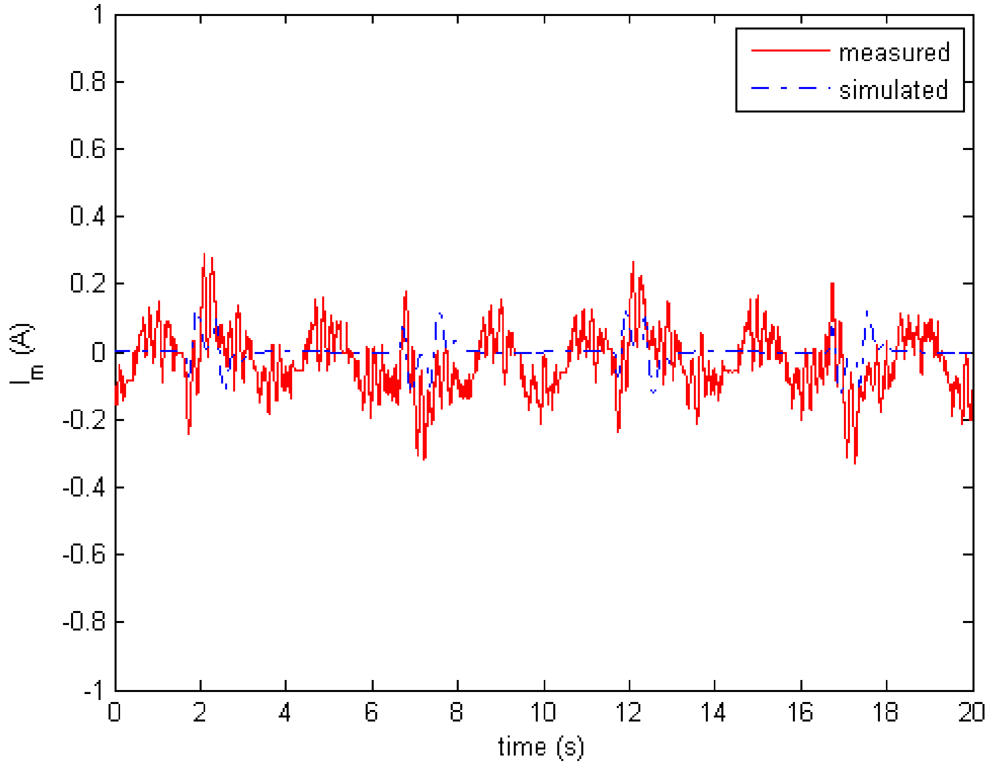



Figure 12 Measured and simulated control input.

Step 12. Click on the Stop button in the Simulink model tool bar to stop running the real-time controller. The *OK* and *Enable* LED's on the UPM will turn off.