



# STUDENT WORKBOOK

## Linear Inverted Pendulum Experiment for MATLAB®/Simulink® Users

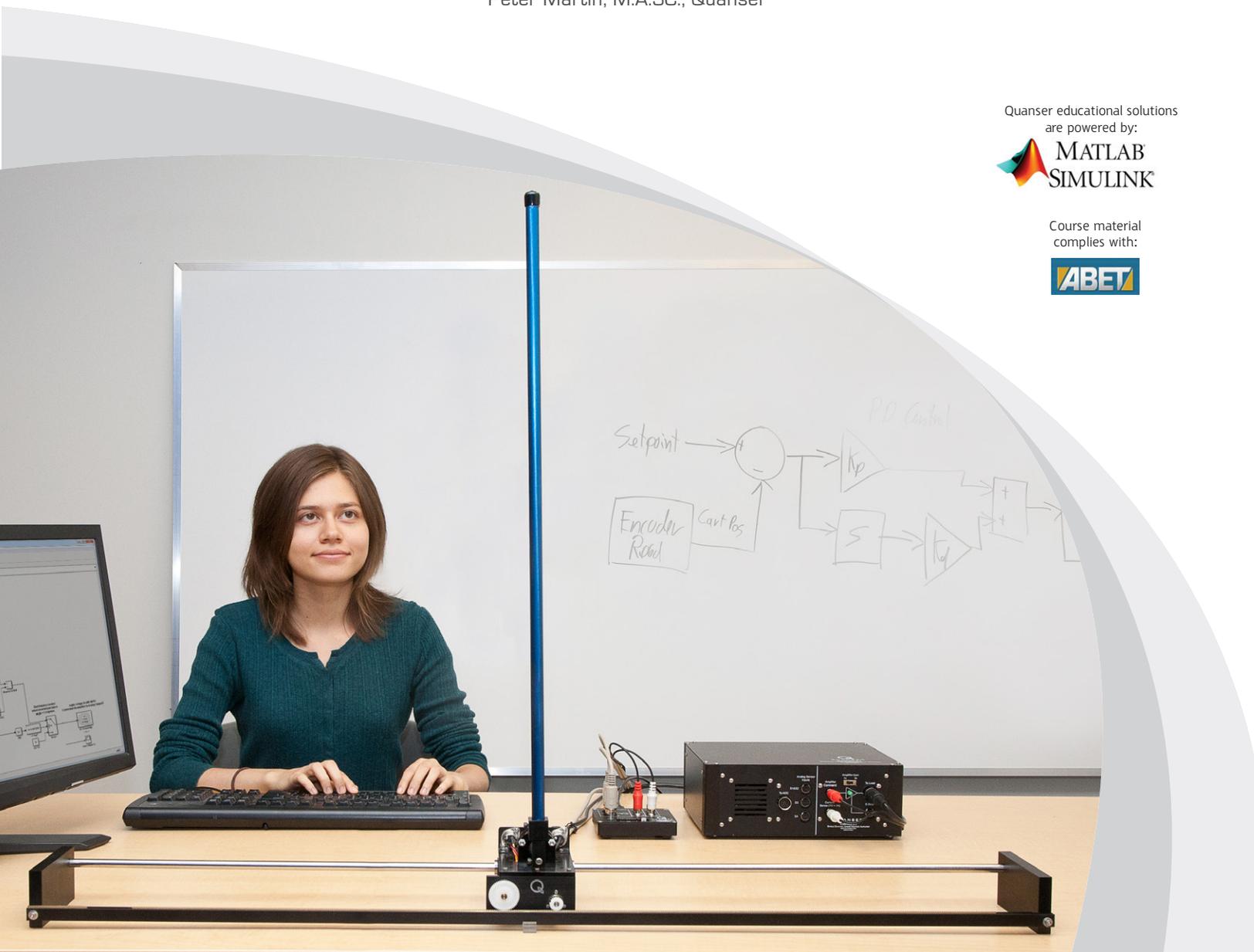
Standardized for ABET\* Evaluation Criteria

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# 1 INTRODUCTION

The objective of this laboratory is to investigate two inverted pendulum control problems using the IP02 linear servo plant. The first task is to balance the pendulum while tracking a commanded cart position. This task is accomplished using a state-feedback controller designed using the Linear Quadratic Regulator (LQR) methodology. The second task is to swing the pendulum into an upright position from its initial hanging, downward position. This task is accomplished using a non-linear energy-based control scheme.

## Topics Covered

- Obtaining the linear state-space representation of the open-loop system.
- Designing a state-feedback controller using the LQR algorithm.
- Developing a non-linear energy-based swing-up controller.
- Implementing the controllers on the Quanser linear pendulum plant and evaluating their performance.

## Prerequisites

In order to successfully carry out this laboratory, the user should be familiar with the following:

- The required software and hardware outlined in Section 4.
- State-space modeling fundamentals.
- Some knowledge of state-feedback.
- Basics of **QUARC®**.
- Laboratory described in the QUARC Integration [2] in order to be familiar using **QUARC®** with the IP02.

# 2 BALANCE CONTROL

## 2.1 Specifications

The response of the inverted pendulum should satisfy the following requirements:

- **Cart Position Rise Time:**  $t_r \leq 1.5$  s
- **Maximum pendulum angle deflection:**  $|\alpha| \leq 1.0$  deg
- **Maximum Control Effort (V):**  $|V_m| < 10$  V. The control effort should be minimized as much as possible within the specified hard limit.

**Note:** The previous specifications are given in response to a  $\pm 30$  mm square wave cart position setpoint.

## 2.2 Background

### 2.2.1 Linear State-Space Model

The linear Single Inverted Pendulum (SIP) model is shown in Figure 2.1. The pendulum pivot is on the IP02 cart, and is measured using the *Pendulum* encoder. The centre of mass of the pendulum is at length,  $l_p$ , and the moment of inertia about the centre of mass is  $J_p$ . The pendulum angle,  $\alpha$ , is zero when it is perfectly balanced in an inverted position and increases positively when rotated counter-clockwise (CCW). The positive direction of linear displacement of the cart,  $x_c$ , is to the right when facing the cart. The position of the pendulum centre of gravity is denoted as the  $(x_p, y_p)$  coordinate.

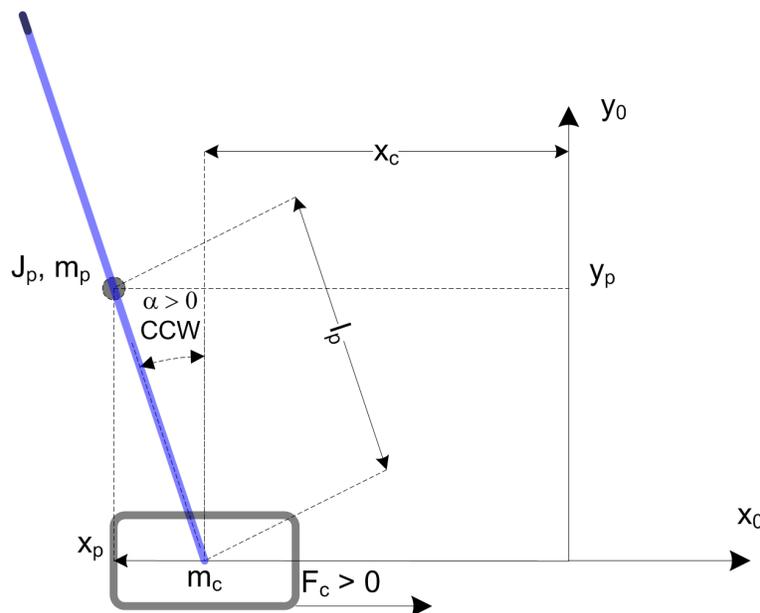


Figure 2.1: Linear Inverted Pendulum schematic

The Linear Pendulum Gantry Workbook [6] presents a modeling experiment where the linear state-space model that represents the pendulum gantry system is developed. The general relationship between the linear force applied to the cart and the dynamics of the pendulum is consistent for the inverted pendulum system. Thus, we can modify the

Equations of Motion (EOM) of the pendulum gantry to account for the orientation of the inverted pendulum shown in Figure 2.1 to yield the following equations for the acceleration of the cart and pendulum:

$$\ddot{x}_c = \frac{1}{J_T} \left( -(J_p + M_p l_p^2) B_{eq} \dot{x}_c - M_p l_p B_p \dot{\alpha} + M_p^2 l_p^2 g \alpha + (J_p + M_p l_p^2) F_c \right). \quad (2.1)$$

and

$$\ddot{\alpha} = \frac{1}{J_T} \left( -(M_p l_p B_{eq}) \dot{x}_c - (J_{eq} + M_p) B_p \dot{\alpha} + (J_{eq} + M_p) M_p l_p g \alpha + M_p l_p F_c \right). \quad (2.2)$$

where

$$J_T = J_{eq} J_p + M_p J_p + J_{eq} M_p l_p^2$$

and

$$J_{eq} = M_c + \frac{\eta_g K_g^2 J_m}{r_{mp}^2}. \quad (2.3)$$

For the linear inverted pendulum system, the state is defined

$$x^\top = [x_c \quad \alpha \quad \dot{x}_c \quad \dot{\alpha}]$$

We can thus define  $\dot{x}_1 = x_3$  and  $\dot{x}_2 = x_4$ . Substituting state  $x$  into the equations of motion found where  $x_c = x_1$ ,  $\alpha = x_2$ ,  $\dot{x}_c = x_3$ ,  $\dot{\alpha} = x_4$  gives

$$\dot{x}_3 = \frac{1}{J_T} \left( -(J_p + M_p l_p^2) B_{eq} x_3 - M_p l_p B_p x_4 + M_p^2 l_p^2 g x_2 + (J_p + M_p l_p^2) u \right)$$

and

$$\dot{x}_4 = \frac{1}{J_T} \left( -(M_p l_p B_{eq}) x_3 - (J_{eq} + M_p) B_p x_4 + (J_{eq} + M_p) M_p l_p g x_2 + M_p l_p u \right).$$

The  $A$  and  $B$  matrices in the  $\dot{x} = Ax + Bu$  equation are therefore:

$$A = \frac{1}{J_T} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & M_p^2 l_p^2 g & -(J_p + M_p l_p^2) B_{eq} & -M_p l_p B_p \\ 0 & (J_{eq} + M_p) M_p l_p g & -M_p l_p B_{eq} & -(J_{eq} + M_p) B_p \end{bmatrix} \quad (2.4)$$

and

$$B = \frac{1}{J_T} \begin{bmatrix} 0 \\ 0 \\ J_p + M_p l_p^2 \\ M_p l_p \end{bmatrix}. \quad (2.5)$$

In the output equation, only the position of the cart and pendulum angle is being measured. Based on this, the  $C$  and  $D$  matrices in the output equation are

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

and

$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

By substituting in the system parameters listed in the *IP02 User Manual* [3] and the *SIP and SPG User Manual* [5], we can calculate the eigenvalues of matrix  $A$ , which yields open-loop poles located at

$$OL = -16.26, -4.56, 4.84, \text{ and } 0.$$

## 2.2.2 Linear Quadratic Regular (LQR)

If (A,B) are controllable, then the Linear Quadratic Regular optimization method can be used to find a feedback control gain. Given the plant model in Equation 2.4 and Equation 2.5, find a control input  $u$  that minimizes the cost function

$$J = \int_0^{\infty} x(t)'Qx(t) + u(t)'Ru(t) dt, \quad (2.6)$$

where  $Q$  and  $R$  are the weighting matrices. The weighting matrices affect how LQR minimizes the function and are, essentially, tuning variables.

Given the control law  $u = -Kx$ , the state-space becomes

$$\begin{aligned} \dot{x} &= Ax + B(-Kx) \\ &= (A - BK)x \end{aligned}$$

## 2.2.3 Feedback Control

The feedback control loop that balances the linear pendulum is illustrated in Figure 2.2. The reference state is defined

$$x_d = [x_{cd} \ 0 \ 0 \ 0]$$

where  $x_{cd}$  is the desired cart position. The controller is

$$u = K(x_d - x). \quad (2.7)$$

Note that when  $x_d = 0$  then  $u = -Kx$ , which is the control used in the LQR algorithm.

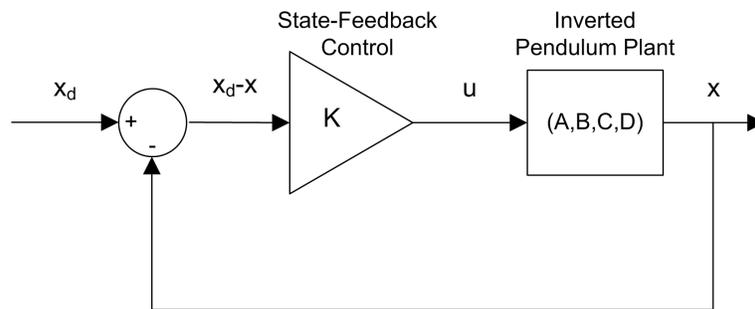


Figure 2.2: State-feedback control loop

When running this on the actual system, the pendulum begins in the hanging, downward position. We only want the balance control to be enabled when the pendulum is brought up around its upright vertical position. The controller is therefore

$$u = \begin{cases} K(x_d - x) & |x_2| < \epsilon \\ 0 & \text{otherwise} \end{cases}$$

where  $\epsilon$  is the angle about which the controller should engage. For example if  $\epsilon = 10$  degrees, then the control will begin when the pendulum is within  $\pm 10$  degrees of its upright position, i.e., when  $|x_2| < 10$  degrees.

## 2.3 Pre-Lab Questions

1. Based on the analysis of the system in Section 2.2.1, is the system stable, marginally stable, or unstable?
2. Designing a controller with the Linear Quadratic Regular (LQR) technique is an iterative process. In software, you have to select the  $Q$  and  $R$  matrices, generate the gain  $K$  using the LQR algorithm, and then simulate the system or implement the control to access the control performance. The relationship between changing  $Q$  and  $R$  and the closed-loop response is not evident. However, we can gain insight into how changing the different elements in  $Q$  and  $R$  will effect the response. We will only be changing the diagonal elements in  $Q$ , thus let

$$Q = \begin{bmatrix} q_1 & 0 & 0 & 0 \\ 0 & q_2 & 0 & 0 \\ 0 & 0 & q_3 & 0 \\ 0 & 0 & 0 & q_4 \end{bmatrix}. \quad (2.8)$$

Since we are dealing with a single-input system,  $R$  is a scalar value. Using the  $Q$  and  $R$  defined, expand the cost function given in Equation 2.6.

3. For the feedback control  $u = -Kx$ , the Linear-Quadratic Regular algorithm finds a gain  $K$  that minimizes the cost function  $J$ . Matrix  $Q$  sets the weight on the states and determines how  $u$  will minimize  $J$  (and hence how it generates gain  $K$ ). From your solution in Question 2, explain how increasing the diagonal elements,  $q_i$ , effects the generated gain  $K = [k_1 \ k_2 \ k_3 \ k_4]$ .
4. Explain the effect of increasing  $R$  has on the generated gain,  $K$ .

## 2.4 In-Lab Exercises

### 2.4.1 Control Simulation

#### Experiment Setup

The *s\_sip\_lqr* Simulink diagram shown in Figure 2.3 is used to simulate the closed-loop response of the linear inverted pendulum using the state-feedback control described in Section 2.2.3 with the control gain  $K$ .

The *Amplitude (m)* gain block is used to change the desired cart position. The state-feedback gain  $K$  is read from the Matlab workspace. The Simulink *State-Space* block reads the  $A$ ,  $B$ ,  $C$ , and  $D$  state-space matrices that are loaded in the Matlab workspace. The *Find State X* block contains high-pass filters to find the velocity of the rotary arm and pendulum.

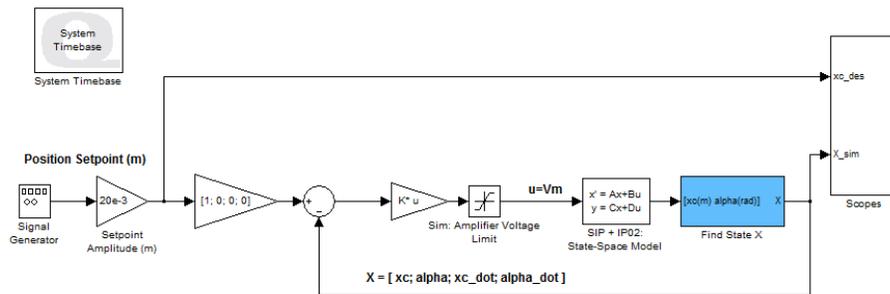


Figure 2.3: *s\_sip\_lqr* Simulink diagram used to simulate the state-feedback control

**Note:** Before you can conduct this experiment, you need to make sure that the lab files are configured according to your system setup. If they have not been configured already, go to Section 4.2 to configure the lab files first.

1. Open *setup\_ip02\_sip.m* and go down to the *LQR Control* section shown here:

```
% ##### LQR CONTROL #####  
if strcmp ( CONTROLLER_TYPE, 'MANUAL' )  
    Q = diag([1 1 1 1]);  
    R = 0.1;  
  
    [ K, S, EIG_CL ] = lqr( A, B, Q, R );
```

The  $Q$  and  $R$  are initially set to the default values of:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and}$$
$$R = 0.1.$$

2. These will not give you the desired response, but run the script to generate the default gain  $K$ .
3. Run *s\_ip02\_sip* to simulate the closed-loop response with this gain. See figures 2.4a and 2.4b for the typical response.

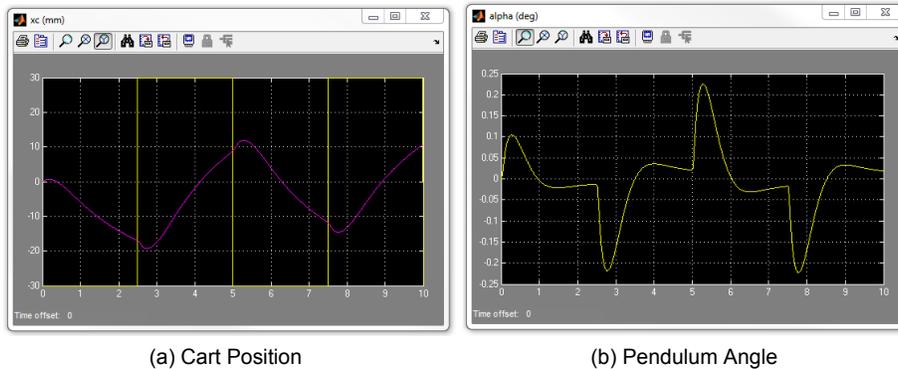


Figure 2.4: Default Simulated Closed-Loop Response

4. Vary the value of the  $R$  parameter and observe its effect on the response of the system.
5. If  $Q = \text{diag}[q_1, q_2, q_3, q_4]$ , vary each  $q_i$  independently and examine its effect on the gain and the closed-loop response. For example, when increasing  $q_3$ , what happens to  $x_c$  and  $\alpha$ ? Vary each  $q_i$  by the same order of magnitude and compare how the new gain  $K$  changes compared to the original gain. Keep  $R = 0.1$  throughout your testing. Summarize your results.
 

**Note:** Recall your analysis in pre-lab Question 3 where the effect of adjusting  $Q$  on the generated  $K$  was assessed generally by inspecting the cost function equation. You may find some discrepancies in this exercise and the pre-lab questions.
6. Find a  $Q$  and  $R$  that will satisfy the specifications given in Section 2.1. When doing this, don't forget to keep the DC motor voltage as small as possible within  $\pm 10$  V. This control will later be implemented on actual hardware. Enter the weighting matrices,  $Q$  and  $R$ , used and the resulting gain,  $K$ .
 

**Note:** Recall that the most important design objective for the controller is to maintain the pendulum in the balanced inverted position.
7. Plot the responses from the  $x_c$  (mm),  $\alpha$  (deg), and  $V_m$  (V) scopes in a Matlab figure. When the QUARC controller is stopped, these scopes automatically save the last 5 seconds of their response data to the variables  $\text{data\_xc}$ ,  $\text{data\_alpha}$ , and  $\text{data\_vm}$ . For  $\text{data\_xc}$ , the time is in  $\text{data\_xc}(:,1)$ , the setpoint (desired cart position) is in  $\text{data\_xc}(:,2)$ , and the simulated cart position is in  $\text{data\_xc}(:,3)$ . In the  $\text{data\_alpha}$  and  $\text{data\_vm}$  variables, the  $(:,1)$  holds the time vector and the  $(:,2)$  holds the actual measured data.
8. Measure the rise time of the simulated cart position response and the maximum pendulum angle. Does the response satisfy the specifications given in Section 2.1?

## 2.4.2 Control Implementation

In this section, the LQR based state-feedback controller that was designed and simulated in the previous sections is run on the actual IP02 Inverted Pendulum device.

### Experiment Setup

The  $q\_sip\_lqr$  Simulink diagram shown in Figure 2.5 is used to run the state-feedback control on the linear inverted pendulum.

The *Amplitude (m)* gain block is used to change the desired cart position. The state-feedback gain  $K$  is read from the Matlab workspace. The *SIP + IP02:Actual Plant* block interfaces with the IP02 cart motor and sensors of the system. The *SIP + IP02:State-Space Model* and *Find State X* blocks are used to simulate the response of the inverted pendulum using the linear state-space model presented in Section 2.2.1.

**Note:** Before you can conduct this experiment, you need to make sure that the lab files are configured according to your system setup. If they have not been configured already, go to Section 4.2 to configure the lab files first.

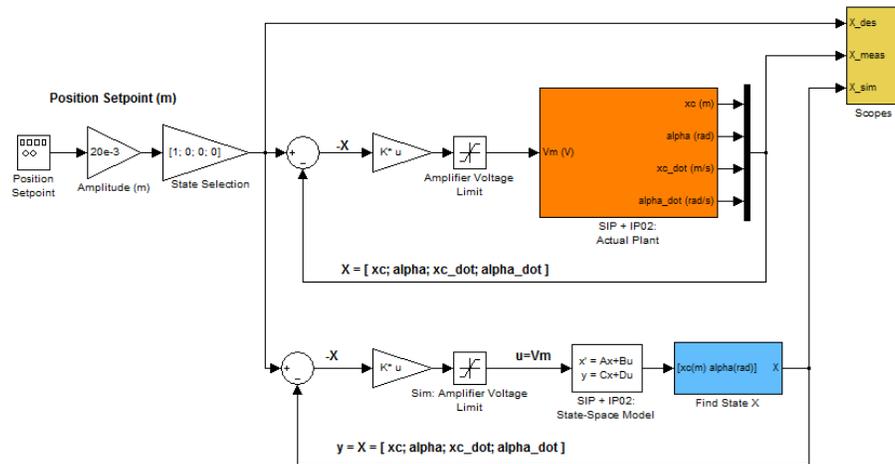


Figure 2.5: `q_sip_lqr` Simulink diagram used to implement the state-feedback control

1. Run the `setup_ip02_sip` script.
2. Make sure the gain  $K$  you found in Section 2.4.1 is loaded.
3. Open the `s_sip_lqr` Simulink diagram.
4. Turn ON the power amplifier.
5. Go to QUARC | Build to build the controller.
6. Ensure the pendulum is in the hanging down position and is motionless. Go to QUARC | Run to start the controller. Once it is running, manually bring up the pendulum to its upright vertical position. You should feel the voltage kick-in when it is within the range where the balance control engages. Once it is balanced, introduce the  $\pm 30$  mm cart position command by setting the *Amplitude (mm)* gain in the Simulink diagram to 30.
 

**Note:** Once the controller has engaged, do not attempt to manually lower the pendulum. If the pendulum or cart move outside of a safe workspace, the system watchdog should halt the controller automatically.
7. The response should look similar to your simulation. Once you have obtained a suitable response, go to QUARC | Stop to stop the controller. Be careful, as the pendulum will fall down when the controller is stopped. Similar to the simulation, the response data will be saved to the workspace. Use this to plot the cart, pendulum, and control input responses in a Matlab figure.
8. Measure the rise time of the cart position, pendulum deflection and voltage used. Are the specifications given in Section 2.1 satisfied for the implementation?
9. Shut off the power amplifier.

# 3 SWING-UP CONTROL

## 3.1 Background

In this section a nonlinear, energy-based control scheme is developed to swing the pendulum up from its hanging, downward position. The swing-up control described herein is based on the strategy outlined in [7]. Once upright, the control developed in Section 2 can be used to balance the pendulum in the upright vertical position.

### 3.1.1 Pendulum Dynamics

The dynamics of the pendulum can be redefined in terms of pivot acceleration as

$$J_p \ddot{\alpha} + M_p g l_p \sin(\alpha) = M_p l_p u \cos(\alpha). \quad (3.1)$$

The pivot acceleration,  $u$ , is the linear acceleration of the pendulum link base. The acceleration is proportional to the force applied to the IP02 cart and is expressed as

$$F_c = M_c u$$

where  $M_c$  is the mass of the cart (with the additional weight). If we take the equivalent mass of the cart to include the inertial force of the rotating motor armature,  $J_{eq}$ , defined in Equation 2.3 then

$$F_c = J_{eq} u. \quad (3.2)$$

The voltage-force relationship, which was found in the *Linear Pendulum Gantry Workbook* [6], is

$$F_c = \left( \frac{\eta_g K_g K_t}{R_m r_{mp}} \right) \left( -\frac{K_g K_m \dot{x}_c}{r_{mp}} + \eta_m V_m \right). \quad (3.3)$$

### 3.1.2 Energy Control

Ideally if the cart position is kept constant and the pendulum is given an initial position, it swings with a constant amplitude. However, because of friction there will be damping in the oscillation. The purpose of energy control is to control the pendulum in such a way that the friction is constant.

The potential and kinetic energy of the pendulum is

$$E_p = M_p g l_p (1 - \cos(\alpha)) \quad (3.4)$$

and

$$E_k = \frac{1}{2} J_p \dot{\alpha}^2.$$

The pendulum parameters are described in the Modeling section of the *Linear Pendulum Gantry Workbook* [6], and their values are given in the *SIP and SPG User Manual* [5]. Adding the kinetic and potential energy together give us the total pendulum energy

$$E = \frac{1}{2} J_p \dot{\alpha}^2 + M_p g l_p (1 - \cos \alpha). \quad (3.5)$$

Taking its time derivative we get

$$\dot{E} = \dot{\alpha} (J_p \ddot{\alpha} + M_p g l_p \sin \alpha). \quad (3.6)$$

To introduce the pivot acceleration  $u$  and eventually, our control variable, solve for  $\sin \alpha$  in Equation 3.1 to obtain

$$\sin(\alpha) = \frac{1}{M_p g l_p} (-J_p \ddot{\alpha} + M_p l_p u \cos(\alpha)).$$

Substitute this into  $\dot{E}$ , found in Equation 3.6, to get

$$\dot{E} = M_p l_p u \dot{\alpha} \cos \alpha$$

One strategy that will swing the pendulum to a desired reference energy  $E_r$  is the proportional control

$$u = (E - E_r) \dot{\alpha} \cos \alpha.$$

By setting the reference energy to the pendulum potential energy, i.e.,  $E_r = E_p$ , the control will swing the link to its upright position. Notice that the control law is nonlinear because the proportional gain depends on the pendulum angle,  $\alpha$ , and also notice that the control changes sign when  $\dot{\alpha}$  changes sign and when the angle is  $\pm 90$  degrees.

For energy to change quickly the magnitude of the control signal must be large. As a result, the following swing-up controller is implemented

$$u = \text{sat}_{u_{max}}(\mu(E - E_r)\text{sign}(\dot{\alpha} \cos \alpha)) \quad (3.7)$$

where  $\mu$  is a tunable control gain and  $\text{sat}_{u_{max}}$  function saturates the control signal at the maximum acceleration of the pendulum pivot,  $u_{max}$ . Taking the sign of  $\dot{\alpha} \cos \alpha$  allows for faster switching.

In order to translate the pivot acceleration into servo voltage, first solve for the voltage in Equation 3.3 to get

$$V_m = \frac{F_c R_m r_{mp}}{\eta_g K_g \eta_m K_t} + \frac{K_g K_m \dot{x}_c}{r_{mp}}$$

Then substitute the force-acceleration relationship given in Equation 3.2 to obtain the following

$$V_m = \frac{J_{eq} R_m r_{mp} u}{\eta_g K_g \eta_m K_t} + \frac{K_g K_m \dot{x}_c}{r_{mp}} \quad (3.8)$$

### 3.1.3 Self-Erecting Control

The energy swing-up control can be combined with the balancing control in Equation 2.7 to obtain a control law which performs the dual tasks of swinging up the pendulum and balancing it. This can be accomplished by switching between the two control systems.

Basically the same switching used in Section 2.2.3 to engage the balance controller when the pendulum is manually raised is used. Only instead of feeding 0 V when the balance control is not enabled, the swing-up control is engaged. The controller therefore becomes

$$u = \begin{cases} K(x_d - x) & |x_2| < \epsilon \\ \text{sat}_{u_{max}}(\mu(E - E_r)\text{sign}(\dot{\alpha} \cos \alpha)) & \text{otherwise} \end{cases} \quad (3.9)$$

## 3.2 Pre-Lab Questions

1. Evaluate the potential energy of the pendulum when it is in the downward and upright positions.
2. Compute the maximum acceleration deliverable by the IP02 cart. Assume the maximum equivalent voltage applied to the DC motor is 5 V such that

$$V_m - \frac{K_g K_m \dot{x}_c}{r_{mp}} = 5. \quad (3.10)$$

The IP02 motor parameters are given in the *IP02 User Manual* [3].

3. Find the controller acceleration when the pendulum is initially hanging down and motionless. From a practical viewpoint, what does this imply when the swing-up control is activated?
4. Assume the pendulum is starting to swing from the downward position in the positive direction. Calculate the acceleration the swing-up controller will generate when  $\mu = 5$ . Does this saturate the controller?

## 3.3 In-Lab Exercises

### Experiment Setup

The  $q\_sip\_swingup$  Simulink diagram shown in Figure 3.1 is used to run the swing-up control on the Quanser Linear Inverted Pendulum system. The *SIP + IP02: Actual Plant* subsystem contains QUARC blocks that interface with the system hardware and the feedback is implemented using a Simulink *Gain* block.

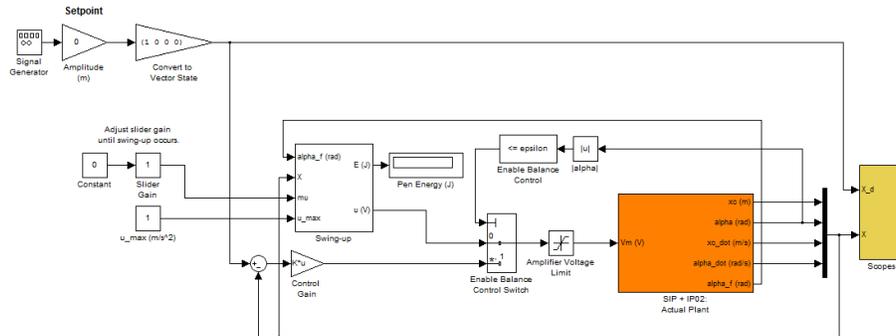


Figure 3.1:  $q\_sip\_swingup$  Simulink diagram used to run the swing-up control

**Note:** Before you can conduct this experiment, you need to make sure that the lab files are configured according to your system setup. If they have not been configured already, go to Section 4.2 to configure the lab files first.

**Note:** Before performing the inverted pendulum **Swing-Up** lab, please add the additional weight to the IP02 base unit.

1. Run the *setup\_ip02\_sip* script.
2. Make sure the gain  $K$  you found in Section 2.4.1 is loaded.
3. Open the  $q\_sip\_swingup$  Simulink diagram.
4. Turn ON the power amplifier.
5. Go to QUARC | Build to build the controller.
6. Ensure that the *Constant* block that feeds into the control gain  $\mu$  is set to **0** to disable the swing-up controller.
7. Run the controller and rotate the pendulum up to the upright position. While the inverted pendulum is balancing, record the total energy reading displayed in *Pen Energy (J)* numeric indicator. Is the value as expected?
 

**Note:** Once the controller has engaged, do not attempt to manually lower the pendulum. If the pendulum or cart move outside of a safe workspace, the system watchdog should halt the controller automatically.
8. Go to QUARC | Stop to stop the controller.
9. Set the *Constant* block that feeds into the control gain  $\mu$  to **1** to implement the self-erecting control in Equation 3.9, which includes both the swing-up and balance control.
10. Set the maximum acceleration, and proportional gain parameters to:

$$\begin{aligned} u_{max} &= 1 \text{ m/s}^2 \\ \mu &= 1 \end{aligned}$$

11. Ensure the pendulum is motionless in the gantry position, and the IP02 cart is in the centre of the track. Then go to QUARC | Run to start the controller.

The cart and pendulum should be moving back and forth slowly. Set  $u_{max}$  to the value you calculated in Section 3.2, and gradually increase  $\mu$  until the pendulum swings upright. Do not set value of  $u_{max}$  above the maximum

acceleration you found for the IP02. When the pendulum swings up to the vertical upright position, the balance controller should engage and balance the link. If the cart moves too far to one side of the track and the model stops, reset the cart in the centre of the track with the pendulum stationary and restart the model.

12. Plot the response of the arm and pendulum angles as well as the control voltage and record the swing-up parameters. Did the swing-up behave with the parameters you expected?

# 4 SYSTEM REQUIREMENTS

Before you begin this laboratory make sure:

- QUARC<sup>®</sup> is installed on your PC, as described in Reference [4].
- You have a QUARC compatible data-aquisition (DAQ) card installed in your PC. For a listing of compliant DAQ cards, see Reference [1].
- IP02 and amplifier are connected to your DAQ board as described Reference [5].

## 4.1 Overview of Files

File Name	Description
Linear Inverted Pendulum Workbook (Student).pdf	This laboratory guide contains an inverted pendulum and swing-up control experiment demonstrating LQR and energy-based feedback control of the Quanser linear inverted pendulum. The in-lab exercises are explained using the QUARC software.
setup_ip02_sip.m	The main <b>Matlab<sup>®</sup></b> script that sets the IP02 inverted pendulum control parameters. <b>Run this file only to setup the laboratory.</b>
config_ip02.m	Returns the configuration-based IP02 model specifications $R_m$ , $J_m$ , $K_t$ , $Eff_m$ , $K_m$ , $K_g$ , $Eff_g$ , $M$ , $r_{mp}$ , and $Beq$ .
config_sp.m	Returns the configuration-based pendulum specifications.
d_ip02_sip_lqr.m	Determines the control gain $K$ using LQR.
SPG_ABCD_eqns_student.m	Creates the student state space model of the linear inverted pendulum system.
s_sip_lqr.mdl	Simulink file that simulates the closed-loop pendulum gantry state-feedback control step response.
q_sip_lqr.mdl	Simulink file that implements the closed-loop IP02 lead speed controller using QUARC.
q_sip_swingup.mdl	Simulink file that implements the energy-based swing-up control

Table 4.1: Files supplied with the Linear Inverted Pendulum Laboratory.

## 4.2 Configuring the IP02 and the Lab Files

Before beginning the lab exercises the IP02 device, the  $q\_sip\_lqr$  Simulink diagram and the  $setup\_ip02\_sip.m$  script must be configured.

Follow these steps to get the system ready for this lab:

1. If you are performing the inverted pendulum **Balance Control** lab, setup the IP02 with the long pendulum and without the additional weight as described in [5]. If you are performing the **Swing-Up** lab, please add the additional weight to the IP02 base unit.
2. Load the **Matlab<sup>®</sup>** software.

- Browse through the Current Directory window in **Matlab®** and find the folder that contains the linear inverted pendulum files, e.g. `q_sip_lqr`.
- Double-click on the `q_sip_lqr.mdl` file to open the Simulink diagram shown in Figure Figure 2.5.
- Configure DAQ:** Double-click on the HIL Initialize block in the Simulink diagram and ensure it is configured for the DAQ device that is installed in your system. For instance, the block shown in Figure 2.5 is setup for the Quanser Q2-USB hardware-in-the-loop board. See the QUARC Installation Guide [4] for more information on configuring the HIL Initialize block.
- Go to the *Current Directory* window and double-click on the `setup_ip02_sip.m` file to open the setup script.
- Configure setup script:** The beginning of the setup script is shown below. Ensure the script is setup to match the configuration of your actual IP02 device. For example, the script given below is setup for an IP02 plant without the additional weight and long pendulum, and it is actuated using the Quanser VoltPAQ device with a gain of 1. See the IP02 User Manual [3] for more information on IP02 plant options and corresponding accessories.

Finally, make sure `MODELING_TYPE` is set to 'MANUAL'.

If you are performing the inverted pendulum **Balance Control** lab, configure the cart load settings as follows:

```
% if IP02: Type of Cart Load: set to 'NO_LOAD', 'WEIGHT'
  IPO2_LOAD_TYPE = 'NO_LOAD';
%IPO2_LOAD_TYPE = 'WEIGHT';
```

If you are performing the inverted pendulum **Swing-Up Control** lab, configure the cart load settings as follows:

```
% if IP02: Type of Cart Load: set to 'NO_LOAD', 'WEIGHT'
%IPO2_LOAD_TYPE = 'NO_LOAD';
IPO2_LOAD_TYPE = 'WEIGHT';
```

For both labs, make sure the script is configured as follows:

```
% Type of single pendulum: set to 'LONG_24IN', 'MEDIUM_12IN'
  PEND_TYPE = 'LONG_24IN';
% PEND_TYPE = 'MEDIUM_12IN';
% Turn on or off the safety watchdog on the cart position: set it to 1 , or 0
X_LIM_ENABLE = 1;          % safety watchdog turned ON
%X_LIM_ENABLE = 0;        % safety watchdog turned OFF
% Safety Limits on the cart displacement (m)
X_MAX = 0.3;              % cart displacement maximum safety position (m)
X_MIN = - X_MAX;         % cart displacement minimum safety position (m)
% Turn on or off the safety watchdog on the pendulum angle: set it to 1 , or 0
ALPHA_LIM_ENABLE = 1;    % safety watchdog turned ON
%ALPHA_LIM_ENABLE = 0;   % safety watchdog turned OFF
% Safety Limits on the pendulum angle (deg)
global ALPHA_MAX ALPHA_MIN
ALPHA_MAX = 20;          % pendulum angle maximum safety position (deg)
ALPHA_MIN = - ALPHA_MAX; % pendulum angle minimum safety position (deg)
% Amplifier Gain: set VoltPAQ amplifier gain to 1
K_AMP = 1;
% Amplifier Type: set to 'VoltPAQ' or 'Q3'
AMP_TYPE = 'VoltPAQ';
% AMP_TYPE = 'Q3';
% Digital-to-Analog Maximum Voltage (V); for MultiQ cards set to 10
VMAX_DAC = 10;
```

```
##### USER-DEFINED CONTROLLER DESIGN #####
% Type of Controller: set it to 'LQR_AUTO', 'MANUAL'
%CONTROLLER_TYPE = 'LQR_AUTO'; % LQR controller design: automatic mode
CONTROLLER_TYPE = 'MANUAL'; % controller design: manual mode
```

8. Run the script by selecting the Debug | Run item from the menu bar or clicking on the *Run* button in the tool bar.

# 5 LAB REPORT

When you prepare your lab report, you can follow the outline given in Section 5.1 to build the *content* of your report. Also, in Section 5.2 you can find some basic tips for the *format* of your report.

## 5.1 Template for Content

### I. PROCEDURE

#### I.1. Balance Control Experiment

1. Briefly describe the main goal of this experiment and the procedure.
  - Briefly describe the experimental procedure (Section 2.4.1), *Control Simulation*
  - Briefly describe the experimental procedure in Step 4 in Section 2.4.1.
  - Briefly describe the experimental procedure in Step 5 in Section 2.4.1.
  - Briefly describe the experimental procedure in Step 7 in Section 2.4.1.
  - Briefly describe the experimental procedure (Section 2.4.2), *Control Implementation*

#### I.2. Swing-Up Control Experiment

1. Briefly describe the main goal of this experiment and the procedure.

### II. RESULTS

Do not interpret or analyze the data in this section. Just provide the results.

#### II.1. Balance Control Experiment

1. Cart position, pendulum angle, and voltage responses from Step 7 in Section 2.4.1, *Control Simulation*
2. Cart position, pendulum angle, and voltage responses from Step 7 in Section 2.4.2, *Control Implementation*

#### II.2. Swing-Up Control Experiment

1. Cart position, pendulum angle, and voltage responses from Step 12 in Section 3.3

### III. ANALYSIS

Provide details of your calculations (methods used) for analysis for each of the following:

#### III.1. Balance Control Experiment

1. Step 6 in Section 2.4.1, *Control Simulation*.
2. Step 8 in Section 2.4.1, *Control Simulation*.
3. Step 8 in Section 2.4.2, *Control Implementation*.

### III.2. Swing-Up Control Experiment

1. Step 7 in Section 3.3.

### IV. CONCLUSIONS

Interpret your results to arrive at logical conclusions.

1. Steps 8 and 8 in Section 2.4.
2. Steps 12 in Section 3.3.

## 5.2 Tips for Report Format

### PROFESSIONAL APPEARANCE

- Has cover page with all necessary details (title, course, student name(s), etc.)
- Each of the required sections is completed (Procedure, Results, Analysis and Conclusions).
- Typed.
- All grammar/spelling correct.
- Report layout is neat.
- Does not exceed specified maximum page limit, if any.
- Pages are numbered.
- Equations are consecutively numbered.
- Figures are numbered, axes have labels, each figure has a descriptive caption.
- Tables are numbered, they include labels, each table has a descriptive caption.
- Data are presented in a useful format (graphs, numerical, table, charts, diagrams).
- No hand drawn sketches/diagrams.
- References are cited using correct format.

# REFERENCES

- [1] Quanser Inc. *QUARC User Manual*.
- [2] Quanser Inc. *IP02 QUARC Integration*, 2008.
- [3] Quanser Inc. *IP02 User Manual*, 2009.
- [4] Quanser Inc. *QUARC Installation Guide*, 2009.
- [5] Quanser Inc. *SIP and SPG User Manual*, 2009.
- [6] Quanser Inc. *Linear Pendulum Gantry Workbook*, 2012.
- [7] K. J. Åström and K. Furuta. Swinging up a pendulum by energy control. *13th IFAC World Congress*, 1996.

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